Private or Public Law Enforcement? The case of Digital Piracy Policies with illegal non-monitored behaviors*

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Abstract

Should rights be publicly or privately enforced in the case of digital piracy? The emergence of large-scale anti-piracy laws and the existence of illegal non-monitored channels raise important issues for the design of anti-piracy policies. In this paper, we study the impact of these policies in two enforcement settings (public vs. private) when an illegal non-monitored channel is provided as an outside adoption option for users. Considering market outcomes, we show that optimal price and monitoring strategies leads to deter the illegal non-monitored channel while accommodating to the presence of the illegal monitored one. Public enforcement is shown to generate higher monitoring levels and lower price levels, as well as higher legal welfare than private enforcement does. However, we identify potential conflicts of interest between the legal seller and the social planner when the efficiency of illegal non-monitored channel is low. Insights about the role of supply-side anti-piracy policies and their interaction with demand-side policies are also provided. Although demand-side and supply-side policies are found to be substitutes in the public enforcement setting, they turn to be complements in the private enforcement one when the efficiency of the illegal non-monitored channel is high.

JEL Classification: D23, D78, K42

Keywords: copyright infringement, law enforcement, digital piracy, monitoring, illegal behavior deterrence

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Highlights:

- We consider supply-side and demand-side anti-piracy policies in two different enforcement settings (public vs. private).
- Users adopt digital goods from either a legal channel, an illegal monitored channel or an illegal non-monitored channel.
- We show that supply-side policies may have counter-productive effects and damage legal welfare.
- Supply-side and demand-side policies are not always substitutes. Supply-side policies may require reinforcement of demand-side policies.
- Public enforcement is socially optimal but a coordination problem may arise between the legal seller and the social planner.

1 Introduction

In its last report on digital music, the IFPI (International Federation of the Phonographic Industry) estimates that *"[in 2013] as many as a third of all internet users (32%) still regularly access unlicensed sites* and underlines that *[their] markets remain rigged by illegal free music"* (2013 IFPI *Digital Music Report*). For concerned industries, such persisting trends about online piracy are found to damage the development of cultural industries and several legislation initiatives have been implemented to tackle this issue. At the international level, several countries agreed on common grounds to reinforce or better protect the rights of copyright holders (*e.g.*, *Anti-Counterfeiting Trade Agreement* in 2011). While the ratification and the implementation of the ACTA treaty is still an ongoing process, several countries have implemented anti-piracy measures on their own. These can be sorted in two categories, namely supply-side measures and demand-side measures. Supply-side measures target various players that - voluntarily or not - provide access to illegal supply. Examples of supply-side interventions are the ban of Megaupload by the US Government in January 2012 or voluntary agreements between search engines, streaming websites and copyright holders for not displaying nor ranking illegal content that exhibits copyright violations.

Demand-side measures focus on individuals who share and consume illegal content. The most famous implementation of these measures is the so-called "three-strike-law". It consists in first warning an individual who infringes copyright first and second in suing this individual when infringement is repeated. The practical enforcement of this type of measures greatly varies among countries. As well as New Zealand, France was among the first countries to implement such a measure and did it through a dedicated "independent administrative authority" (Hadopi as for Haute Autorité pour la Diffusion des Oeuvres et la Protection des Droits sur Internet) enacted by the French Government in 2008. In 2013, a similar measure has been implemented in the US (under the Copyright Alert System). Compared to the French implementation, the number of warnings in the US is larger (six instead of three warnings) and the penalty incurred is somewhat different (monetary fine in France, restrictions on the Internet connection in the US).[1] However, still comparing the two systems, the most notable distinguishing feature is that the US system only relies on private players since it is actually implemented by a consortium of Internet access providers and copyright holders[2]. Considering other initiatives, the same distinction between private based (*e.g.* Ireland) or public-based enforcement mechanisms (*e.g.* South Korea, United Kingdom) arises.

The academic debate about the effects of illegal downloading is very intense (see Wald-

[1]When enacted on 2008, the punishment under the Hadopi system was both a monetary fine and a shutdown of infringers' Internet connection. No individual has been condemned to a shutdown and this punishment has been given up in 2013. However, some infringers have been suited and condemned to a monetary fine. In the US, the Center for Copyright Information coordinates efforts of ISP's and artist associations (*e.g.* RIAA). Its last report on May 2014 highlights 1.3 million alerts sent after ten months and "filling of 265 challenges" only.

[2]Despite, note that looking inside the blackbox of the French system, copyright holders are not totally external to the policy enforcement mechanism since they are found to have a strong influence on the list (number and names) of the titles monitored under the supervision of the authority.

fogel (2011) for a presentation of key issues). This debate is both about the theoretical mechanisms surrounding piracy (Belleflamme and Peitz (2012)) and about how these effects precisely affect the industry from an empirical perspective (for instance, see Hammond (2014)). Focusing on the theoretical literature, the design of anti-piracy policies has already been well documented (see *e.g.* Banerjee et al. (2008), Choi et al. (2010) or Arai (2011)). However, less is said about the exact effects of various types of enforcement policies which are currently being implemented. Second, most approaches oppose legal to illegal practices, the latter always being potentially monitored, yet at some cost and at some degree. However, a classical result in the literature on the economics of 'crime' is that monitoring some conducts affects the efficiency of legal conducts (Becker, 1968) as well as provides incentives for generating new illegal conducts and bypassing existing regulations (Leung (1991)). The latter effect is also empirically documented in the case of digital piracy (Arnold et al. (2014)) but its impact on policy design and implementations has not been directly addressed. Our paper introduces both dimensions. First, we fully integrate the opportunity for digital pirates to use an illegal non-monitored channel as well as both legal and illegal monitored channels to access to digital goods. The illegal non-monitored channel cannot be monitored through demand-side interventions. However, some supply-side actions may target this channel. We analyze the impacts of such interventions and their interactions with the demand-side policies implemented on the monitored channel. Second, we contrast two implementation settings (namely, private vs. public enforcement settings) from which we derive the expected effect for both types of policy enforcement.

In this context, we show that the optimal strategy of the legal seller and monitoring authority always leads to the deterrence of the illegal non-monitored channel, whatever the enforcement setting considered is. However, this channel plays the role of a potential incumbent and impacts on the price charged by the legal seller. Since supply-side anti-piracy policies target illegal non-monitored channels by attempting to degrade their efficiency, we discuss the impact of the latter on price and monitoring strategies.

Our analysis reveals interesting insights about how supply-side and demand-side policies interact. In particular, we show that those supply-side policies targeted to non-monitored illegal channels sometimes generate the need to more control other channels monitored by demand-side initiatives. In those cases, there is no substitution between these two policies. Rather, supply-side policies need to be accompanied by demand-side ones. This depends on two factors, namely the type of enforcement setting and the *ex ante* efficiency of the alternative channel (that is, before supply-side intervention is undertaken). When enforcement is public, there is some substitution in the use of these two policies. In contrast, when enforcement is private, this substitution only holds when the efficiency of the alternative channel is rather low before the supply-side policy is implemented. Thus, the *ex ante* level of efficiency of the alternative channel (that is, before policy action is performed) also plays a role.

Comparing the two enforcement settings, we find that the legal seller is not necessarily better off in the private enforcement setting compared to the public enforcement setting because the monitoring authority provides a higher effort in the public enforcement setting. This conveys a positive externality on legal sellers when the illegal non-monitored channel is weakly efficient. We also consider the effect of the two regimes on welfare and we highlight potential coordination issues when an enforcement setting is designed.

Section 2 surveys the theoretical and empirical literature on digital piracy and anti-piracy enforcement. Section 3 introduces the model. Section 4 presents the optimal strategies of both the legal seller and monitoring authority, as well as their impacts on market outcomes and welfare. Section 5 discusses the results and concludes.

2 Related Literature

We borrow from different topics in the literature. A first strand deals with digital piracy, more specifically with theoretical models of digital piracy.[3] In short, this literature aims to identify the impacts of digital piracy on the users' behaviors and on the profit of copyright-holders, with a special emphasis on the conditions for piracy to be profitable for copyright holders. The standard framework is that of a monopolist selling a digital good that exhibit network externalities of various types (direct or indirect sampling effects, club and network effects, provision of online or offline complementary items (see *e.g.* Dewenter et al. (2012)). At the same time, this monopolist faces some users' communities that can to cheaply reproduce the original good. The general findings of this literature is that introducing piracy often increases market segmentation. Since piracy may positively impacts on the users' willingness-to-pay, it is found to be profitable when there are substantial spillovers between the legal and illegal goods, depending on the artists' intrinsic characteristics[4]. However, piracy may decrease incentives to innovate and be detrimental the supply variety in the long run (Piolatto and Schuett (2012)). This framework has been extended to account for competition (cf. Belleflamme and Picard (2007)) or for the intervention of commercial illegal players instead of users' communities (Banerjee (2006), Martinez-Sanchez (2010)). It has also been extended to the case of two-sided markets (see *e.g.* Rasch and Wenzel (2013) in the case of software piracy).

A second more general topic is related to the economic analysis of illegal activities and public law enforcement to prevent people from engaging in these activities. Since Becker's influential article (Becker (1968)), many studies have been interested in analyzing in the agents' incentives to engage into illegal activities, as well as in the type of sanctions which should be used to enforce public law efficiently (see Polinsky and Shavell (2000) for a survey on this issue). This general framework has further been applied to many settings. One of these is that of maritime piracy (see *e.g.* Leeson (2010)). One relevant framework for our purpose is Guha (2012) who studies the effect of the monitoring intensity exerted by public authorities and the amount of piracy behaviors. Interestingly, they conclude to the existence of multiple equilibria where less monitoring may lead to higher efficiency. Although this setting is specific to maritime piracy, it raises many insights that can be translated to digital piracy. In particular, it shows that increasing anti-piracy monitoring may also depress returns to the pirates' alternative occupation. Considering digital

[3] See Belleflamme and Peitz (2014) for an excellent survey on digital piracy. See also Peitz and Waelbroeck (2006) for a survey on both theoretical and empirical aspects about digital piracy.

[4] See Bacache et al. (orth) for an empirical investigation

piracy, there are two types of alternative occupations which correspond either to the legal purchase of a digital good or to the use of some non-monitored ways for acquiring it. Our paper echoes in some sense to this paper by introducing both possibilities.

Other papers combine the two previous topics. In the context of commercial digital piracy, Banerjee (2003) examines the government's role in restricting commercial piracy and shows that monitoring may not lead to the social optimal outcome. However, the model targets the software industry for which commercial piracy may be more widespread than that for cultural goods. In a more general setting, Banerjee (2011) shows that, in some cases, it may be better for a monopolist facing piracy to expand her output beyond the monopoly level rather than to invest in private detection mechanisms. Cremer and Pestieau (2009) also consider the effect of the degree of anti-piracy enforcement on welfare and distinguish three levels of enforcement. The highest level is that chosen by the private monopoly. The next level is the one chosen by a welfare-maximizing monopoly. The lowest level, which can be zero, is the level of monitoring chosen by the monitoring authority when the good is sold and priced by a profit-maximizing monopoly.

This framework has been extended in several dimensions. A first extension concerns the interplay between private and public anti-piracy policies. In the context of digital experience goods, Banerjee et al. (2008) shows how public and private policies may interact in the case of software commercial piracy. There may be both a public regulation (*e.g.*, fines imposed to illegal users when they are detected) and a private one (*e.g.*, by investments into anti-copying technologies). The interplay between these two regulations may exhibit some coordination failure. More precisely, it is shown that if monitoring is socially optimal, the anti-copying investment subgame perfect equilibrium does not guarantee the prevention of copying. Conversely, if not monitoring is socially optimal, the anti-copying investment subgame perfect equilibrium may guarantee the prevention of copying. Choi et al. (2010) considers a close issue in the context of competition and shows that the optimal private protection level depends on the degree of substitution between private protection devices.

A second extension is about the type of penalty incurred when being detected for copyright infringement. Investigating the software market, Arai (2011) asks which court (i.e, civil vs. penal) should be concerned with piracy and demonstrates that welfare is larger when fines are always enforced but rather small. A last extension introduces the possibility to target specific categories of users when enforcing anti-piracy policies. Whenever possible, Harbaugh and Khemka (2010) shows that a targeted strategy may be more relevant for both legal sellers and users than a non-targeted one which would consist in monitoring all the categories of users indistinctly.

Our paper adds to this theoretical literature on digital piracy regulation.[5] It differentiates from the current literature according to three reasons. First, although many papers address the issue of digital piracy regulation, we did not identify papers that consider the existence of an outside option to monitored piracy other than legal purchase or no

[5]Note that other papers try to assess the impacts of anti-piracy policy regulations using an experimental or empirical setting. See *e.g.* Adermon and Liang (2014), Arnold et al. (2014), Bhattacharjee et al. (2006), Danaher et al. (orth) or Maffioletti and Ramello (2004).

consumption. However, some anecdotal evidences and empirical studies point out the existence of such alternatives (Arnold et al. (2014), Suire et al. (2012)). Second, we develop a theoretical framework in which we catch the interdependency between some supply-side policies and demand-side policies. To our knowledge, these two types of policies have not been addressed in a single setting. Third, while many papers investigate the intensity and characteristics of digital piracy monitoring, few ask about the impact of alternative enforcement settings, that is, public vs. private implementations. Our model attempts to fill the gap on these three issues.

3 The Model

We introduce three types of agents in the model. Internet users consume digital cultural goods (*e.g.*, music or movies). The monitoring authority is in charge of monitoring some - but not all - illegal channels (*i.e.*, peer-to-peer protocol-based activities). The legal seller provides the official and unique legal version available of the good.

3.1 Users

There is a continuum of N Internet users (further users) labeled by i (with $i = 0, ...N$). Without loss of generality, let us suppose that $N = 1$ (hence, market shares are equal to the number of users). The users wish to consume digital content and derive from that a gross utility equal to v ($v > 0$). To do so, each user i selects one among three different distribution channels (further, channels).

The first potential strategy is to purchase the product legally (further labeled by B as for *buying*). This strategy leads to utility level $U_{B,i} = v - p$ with $i = 0, ...N$ where p ($p \geq 0$) denotes the price users pay when purchasing *via* the legal channel.

The second possibility for a user to adopt digital content is to use a channel which activity is monitored through a demand-side policy. We denote this strategy by *illegal monitored channel* (further labeled by M) adoption. Hence, the use of the illegal monitored channel is risky and leads to a fine f ($f > 0$) for user i when detected. We suppose that users are risk-neutral and face probability ϕ ($\phi \in [0, 1]$) of being detected using channel M. Further, we suppose that users have heterogeneous skills when downloading illegally. This is depicted by an individual expertise coefficient e_i that we assume to be uniformly distributed between 0 and $N = 1$. This translates into probability e_i for user i to download the searched good successfully when using the illegal non-monitored channel, thus getting v. With remaining probability $(1 - e_i)$, user i does not access to the digital good she expects to (*e.g.*, file is broken or damaged) and gets 0. Although a file that is downloaded from channel M may not meet the expectations of user i, the latter may be detected by the monitoring authority, then having to pay fine f ($f > 0$). Using channel M thus provides a utility level $U_{M,i} = ve_i - \phi f$.[6]

[6]This falls from $U_{M,i} = (((1 - \phi)e_i v + (1 - e_i)0) + \phi\,(e_i(v - f) + (1 - e_i)(-f))) = ve_i - \phi f$.

Finally, users may choose downloading or copying techniques to acquire digital files. These practices may be very diverse (*e.g.*, hand-to-hand copy from hardware drives, private virtual networks, newsgroups, illegal streaming websites, *etc.*). All these *alternative* practices (further labeled by A) share the common feature of not being currently monitored by the demand-side policy. As for the illegal monitored channel, the utility user i derives from adopting from channel A also depends on the users' individual expertise. However, as opposed to both channels B and M, we consider that the illegal non-monitored channel is less efficient than both channels B and M (*e.g.*, lower availability, lower download speed access, higher access restrictions). Introducing parameter $\alpha \in]0, 1[$ to capture such a lower relative efficiency for channel A, we express the utility function of the user i who uses channel A by $U_{A,i} = \alpha v e_i$.

Note that by definition this channel is not monitored through demand-side policies. However, supply-side policies may target this channel. The expected impact of these measures is to damage the efficiency of this channel so as to deteriorate the users' ability to find items that would match their preferences on these channels. Because the actual diversity of practices covered by the alternative channels, supply-side policies may only have an incremental impact. We thus use marginal variations of α to derive the impact of supply-side policies on non-monitored channels.[7].

To focus on relevant cases only, we also assume $f > v > p \geq 0$ for punishment to be credible to potential infringers when caught. This rules out the possibility for some users not to adopt from neither channel B, M and A, and the market is therefore always fully-served.

3.2 The legal seller and the monitoring authority

3.2.1 The legal seller

Several distribution strategies are likely to be used by content providers (*i.e.*, artists, resellers and platforms). Since we are not here interested in the internal organization of the music and/or movie industries, we only consider a single profit-motivated seller who distributes on the legal market at price p. The revenue she derives from legal sales writes as pm_B, where m_B is the market share of the legal seller. Because production costs are essentially fixed costs, we normalize these costs to zero.

3.2.2 The monitoring authority

The monitoring authority is in charge of monitoring some but not all illegal activities.[8] To do so, a monitoring strategy is implemented on the illegal monitored channel to track

[7] From now on, the term "supply-side policy" will only refer to the supply-side policy implemented on the alternative channel A while the term "demand-side policy" will only refer to the demand-side policy implemented on the monitored channel M

[8] Note that depending on the legal framework, this control authority may take diverse forms (authority, agency, *etc.*). By convenience, we further refer to the "monitoring authority" throughout the paper.

potential infringers. When caught, an infringer incurs fine f ($f > 0$) assessed in monetary terms. Note that this fine may incorporate several types of sanctions such as a Internet connection shutdown/restriction, legal penalties possibly including other administrative or moral costs.

In the case of demand-side policy, monitoring consists in sampling users on the illegal monitored channel with probability ϕ. This measures the monitoring intensity enforced by the monitoring authority. Monitoring is costly and this cost depends on the monitoring intensity. Let us note by $\varsigma(\phi) = (a/2)\phi^2$ this monitoring cost, where a ($a > 0$) is a measure for the relative efficiency of the screening technology.

In the private enforcement setting, the legal seller is in charge of monitoring channel M. The monitoring intensity is deduced from a profit-maximizing scheme where the benefits (that is, the revenues from the legal market) are balanced with respect to the monitoring costs. The objective of the legal seller is thus to maximize $\pi_{PR} = pm_B - \varsigma(\phi)$ with respect to ϕ, where π_{PR} is the profit of the legal seller in the private enforcement setting.

The public enforcement setting applies when the monitoring authority formulates its monitoring strategy according to its own objective. To do so, it considers the whole benefits from the legal market and maximizes the welfare generated by legal activities (*i.e.* the sum of the surplus of the legal users CS_B and legal revenues pm_B) while supporting the cost of this policy (*i.e.* $\varsigma(\phi)$). The objective of the monitoring authority is hence to maximize $H_{PU} = pm_B + CS_B - \varsigma(\phi)$ with respect to ϕ. As opposed to the case of private enforcement setting, the profit of the legal seller here reduces to her revenues $\pi_{PU} = pm_B$.

3.3 Timing of the game

The game consists in three successive stages. At stage 1, the monitoring level is defined by the legal seller in the private enforcement setting whereas it is defined by the monitoring authority in the public enforcement setting. At stage 2, the legal seller selects her pricing strategy p. At stage 3, users adopt from one of the three channels. This defines a sequential game with perfect information that we solve by backward induction. We first consider the last stage of the game so the users' strategies are conditional to p and ϕ. We further focus on the case in which the market is likely to be shared by channels B, M and A. This applies when $\phi < p/f$ and $(v - p)/(v - p + \phi f) < \alpha < (v - \phi f)/v$ (see proof in Appendix 7.1). These conditions generate lower and upper bounds for the strategic variables p and ϕ.

4 Results

Solving backwards, we here present the results of the public and private enforcement settings. When presenting the two equilibrium outcomes, we pay particular attention to the impact of the efficiency of the alternative channel. Our motivation for that is that the supply-side policies which target the alternative channel are designed to impact on its efficiency. Hence, looking at parameter α enables us to consider the interaction between some demand-side and supply-side anti-piracy policies. Because monitoring technologies are costly to deploy at a large scale, we next present the results for high values for cost parameter a (*i.e.*, $a > \frac{4f^2}{v}$) up to this point.

4.1 Public enforcement setting

In the public enforcement setting, the monitoring authority is in charge of the monitoring activity. Remind that the monitoring authority maximizes the whole surplus generated by the legal market minus monitoring costs. This includes the legal seller's profit but also the legal users' surplus. Hence, the interests of the monitoring authority and those of the legal sellers are not necessarily aligned.

4.1.1 Equilibrium in the Public enforcement setting

Proposition 1. *In the public enforcement setting, the monitoring authority enforces anti-piracy policy with optimal monitoring level* $\phi^*_{PU} = \left(\frac{f}{a(1-\alpha)}\right)$ *and the legal seller charges optimal price* $p^*_{PU} = \left(v - \left(\frac{\alpha}{a}\right)\left(\frac{f}{1-\alpha}\right)^2\right)$ *(proof in Appendix)* ■

Corollary. *In the public enforcement setting, illegal non-monitored channel* A *is deterred from the market at equilibrium. Market shares are* $m^*_{B,PU} = \frac{f^2}{av(1-\alpha)^2}$, $m^*_{A,PU} = 0$ *and* $m^*_{M,PU} = 1 - m^*_{B,PU}$ *. The optimal profit of the legal seller amounts to* $\pi^*_{PU} = \frac{f^2}{av}\left(\frac{1}{1-\alpha}\right)^2\left(v - \frac{\alpha f^2}{a}\left(\frac{1}{1-\alpha}\right)^2\right)$.

Proposition 1 and its corollary characterize the equilibrium in the public enforcement setting. First note that the illegal non-monitored channel ends up being deterred from the market in equilibrium. This is an interesting result since the monitoring authority does not drive any *direct* action against it. One striking point is that even if channel A cannot be directly monitored, the strategies of both legal seller and monitoring authority indirectly influence the use of channel A. At the same time, our results point out that when the monitoring authority enforces its optimal level ϕ^*_{PU} in the public enforcement setting, the illegal monitored channel keeps on operating. The mechanisms at stake can be illustrated as follows. Remind that increasing monitoring is more and more costly and that we focus on high values for a (*i.e.*, $a > \frac{4f^2}{v}$). The monitoring authority has therefore some incentives not to fully monitor the illegal monitored channel because there exists a threshold for ϕ_{PU} beyond which the marginal costs spent for extra monitoring are not overcome by the marginal gains (in terms of legal welfare) which result from higher monitoring. Setting such a moderate level for monitoring intensity ϕ_{PU} allows channel M to take some market share over channel A, all other things being equal. On the legal seller's side, it is profit-enhancing for her to decrease her price p_{PU} so as to attract some extra users from channel A whose level for expertise e_i is low, all other things being equal. Putting these two effects together, we eventually find that optimal levels for ϕ_{PU} and p_{PU} are so that the market share of channel A vanishes. It is too costly for channel M to take the whole market share of channel A on its own on the one side and, similarly, the decrease in price that is required for the legal seller to take the whole market share of channel A on her own does not lead to an optimal profit-maximizing outcome on the other side.

4.1.2 Impact of supply-side policies

For clarity purpose we first present the impact of the efficiency of the alternative channel on outcomes. Since supply-side policies are set to lower efficiency level, the impact of these policies are then deduced by symmetry.

Proposition 2. *In the public enforcement setting equilibrium, the efficiency of the illegal non-monitored channel A (α) has (i) a negative impact on price and (ii) a positive impact on monitoring level. (proof in Appendix)* ■

Corollary. *In the public enforcement setting equilibrium, the efficiency of the illegal non-monitored channel A (α) has (i) a negative impact on the market share of channel M, and (ii) a positive impact on the market share of channel B.*

Define legal welfare as the sum of the seller's profit and the surplus of legal sellers minus monitoring costs. We derive the following corollary from proposition 2.

Corollary. *Consider the public enforcement setting equilibrium. (i) The efficiency of channel A has a positive (resp. negative) impact on the surplus of the users of channel B (resp. M). (ii) The efficiency of channel A has a positive effect on profit if* $0 < \alpha_1 < \alpha$, *negative otherwise (with* $\alpha_1 \equiv \frac{1}{4av}\left(3f^2 + 4av - f\sqrt{9f^2 + 32av}\right)$, $0 < \alpha_1 < 1$*)). (iii) The efficiency of channel A has a positive effect on legal welfare.*

Panel (a) of Figure 1 summarizes these different effects. Comparative statics on α reveals to what extent the legal seller benefits from a higher level of efficiency of the illegal non-monitored channel in this setting. Despite she needs to lower price, the legal seller benefits from higher sales volumes. Such an increase in sales results is achieved through higher enforcement level that is not costly to her in the public enforcement setting. Taking the twin effect of $\alpha > \alpha_1$ on both p^*_{PU} and $m^*_{B,PU}$ into account, we find mixed results about the impact of α on the profit of the legal seller. Greater efficiency for channel A is detrimental to the legal seller's profit only when channel A is relatively efficient ($\alpha > \alpha_1$, *Ib* area), beneficial otherwise ($\alpha < \alpha_1$, *Ia* area). Indeed, when channel A is highly efficient, the legal seller needs to consent to relatively large price cuts to deter channel A. For these high values the negative price effect associated with an increase in α overcomes the positive volume effect. Considering instead lower values of α in the public enforcement setting, the negative price effect turns to be more than compensated by the positive volume effect. It eventually drives an increase in profit.

Considering the surplus of legal buyers, we can show that this surplus positively depends on α. A straightforward explanation is that an increase in α both lowers price and increases the market share of B. Conversely, the surplus of the users of the illegal monitored channel negatively depends on α because of the ensuing decrease of the market share of M on the one hand and of a higher enforcement level on the other hand. Interestingly, legal welfare always increases with α. This means that the positive effect on the legal users' surplus overcomes the negative effect on profit when the illegal non-monitored channel is shown to be highly efficient. For high values for α, an increase in the efficiency of channel A leads to larger size the legal channel and generates a net surplus transfer from the legal seller to her users.

The effects of supply-side anti-piracy policies. By symmetry, consider negative marginal variations of α. We can deduce the effects of a supply-side policy in the public enforcement setting. We should thus expect higher price and lower monitoring levels after this policy is implemented. This ends us with a reduction of the size of the legal channel together with lower consumer surplus for the users who adopt the legal supply. Besides, legal welfare is damaged after policy implementation. This statement applies whatever the initial efficiency of the alternative channel is. All in all, one can see that the effect of these policies may turn out to be counterproductive. In this setting, the unique rationale for supply-side policies would be an increase in the profit of the legal seller. This however only holds in the case in which the alternative channel was initially highly efficient ($\alpha > \alpha_1$). Such effects were previously hard to predict in a framework that did not explicitly consider the non-monitored illegal channel as an outside option for users and that did not endogenize the interplay with demand-side policies.

4.2 Private enforcement setting

In the private enforcement setting, the monitoring activity is delegated to the the legal seller. Recall that the objective of the legal seller is now defined by the profit she can derive from her sales minus the monitoring costs she has to support. Note that, as opposed to the public enforcement setting, the interests of the legal users are not anymore taken into account in the objective-maximization scheme of the legal seller, as the latter here strictly aligns to her own interests.

4.2.1 Equilibrium in the Private enforcement setting

Proposition 3. *In the private enforcement setting, the legal seller enforces anti-piracy policy with optimal monitoring level* $\phi^*_{PR} = \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}$ *and charges optimal price* $p^*_{PR} = v\left(\frac{\alpha f^2 + av(1-\alpha)^2}{2\alpha f^2 + av(1-\alpha)^2}\right)$. *(proof in Appendix)* ■

Corollary. *In the private enforcement setting, the illegal non-monitored channel is deterred the market in equilibrium. Market shares are* $m^*_{B,PR} = \frac{f^2}{av(1-\alpha)^2 + 2f^2\alpha}$, $m^*_{A,PR} = 0$ *and* $m^*_{M,PR} = 1 - \frac{f^2}{av(1-\alpha)^2 + 2f^2\alpha}$. *The optimal profit of the legal seller amounts to* $\pi^*_{PR} = \frac{vf^2}{2\left(2f^2\alpha + av(1-\alpha)^2\right)}$.

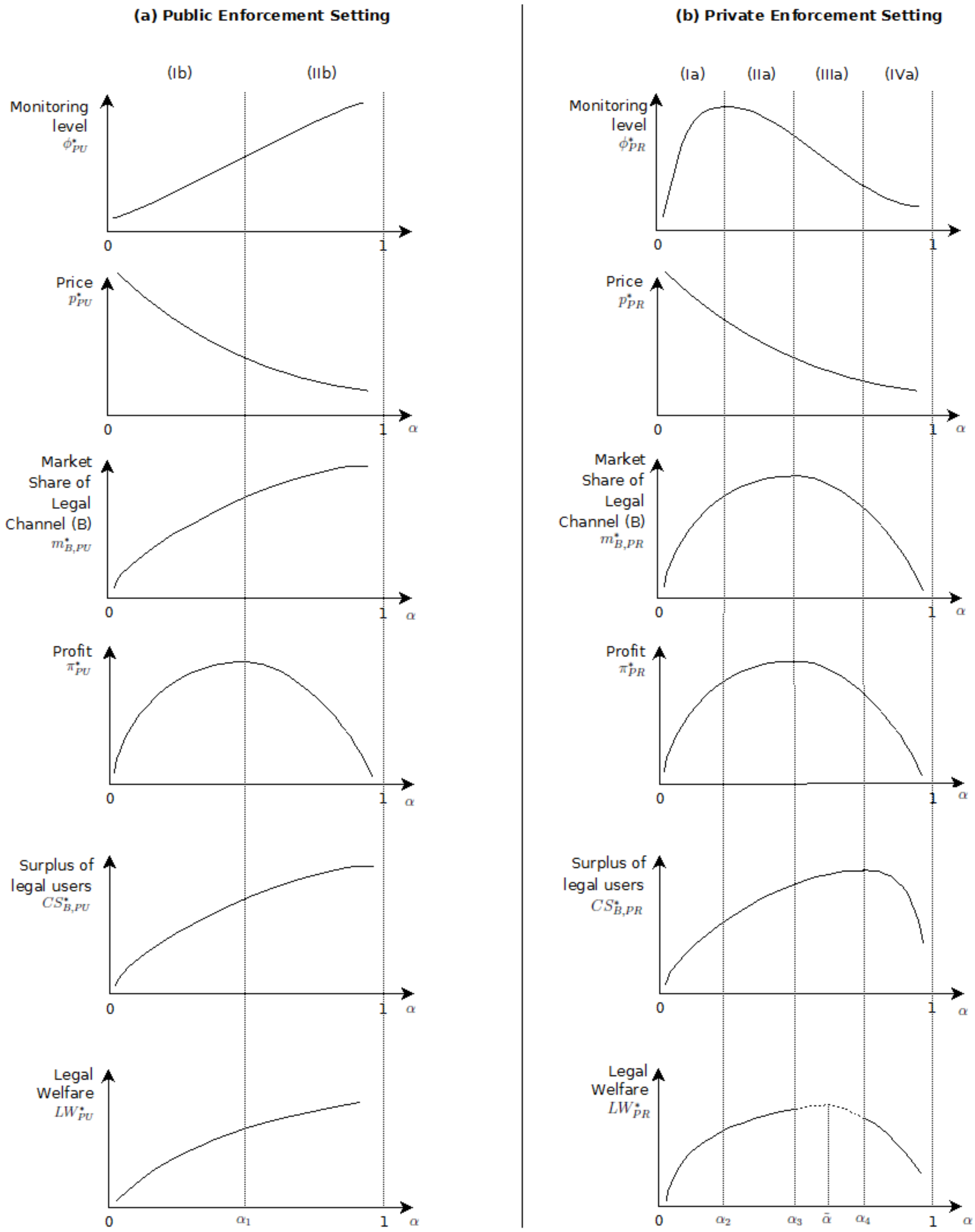

Figure 1: Sketch of Comparative statics on α in the public (panel a) vs. private (panel b) enforcement setting

Similarly to the public enforcement setting, both price and monitoring strategies are defined so that channel A is deterred from the market, whatever channel A's efficiency α is. The underlying intuitions of this result remain the same than those in the public enforcement setting. Remind however that the legal seller now selects both strategic variables p_{PR} and ϕ_{PR} to maximize her profit function $\pi_{PR} = pm_B - \varsigma(\phi)$. The specification of $\varsigma(\phi)$ implies that she has no interest in providing full-monitoring effort. The legal seller sets optimal monitoring level ϕ^*_{PR} so that channel M gains market share over channel A. Monitoring - yet not entirely - channel M allows at the same time the legal seller to gain some extra users from channel A whose expertise e_i is low. The ability of the legal seller to manipulate ϕ_{PR} allows her to soften the decrease in price p_{PR} she needs to gain any extra user from channel A. In equilibrium the combination of the two strategies of the legal seller leads to the deterrence of the illegal non-monitored channel.

4.2.2 Impact of supply-side policies

As for Section 4.1.2, we first analyze the impact of the efficiency of the alternative channel on outcomes and we then deduce the impact of supply-side policies. Expected effects are summarized and illustrated in Panel (b) of Figure 1.

Proposition 4. *In the private enforcement setting equilibrium, the efficiency of the illegal non-monitored channel has (i) a negative effect on price for any value of* $\alpha \in [0, 1]$ *and (ii) a positive effect on monitoring level if* $0 < \alpha < \alpha_2$*, negative otherwise (with* $\alpha_2 \equiv 1 - \sqrt{\frac{2f^2}{av}}$, $0 < \alpha_2 < 1$*). (proof in Appendix)* ■

Corollary. *Consider the private enforcement setting equilibrium. The efficiency of the illegal non-monitored channel positively impacts on the market share of channel B and negatively impacts on the market share of channel M if* $0 < \alpha < \alpha_3$ *(with* $\alpha_3 \equiv \frac{av - f^2}{av}$, $0 < \alpha_3 < 1$*). Otherwise, it negatively impacts on the market share of channel B and positively impacts on the market share of M.*

Corollary. *Consider the private enforcement setting equilibrium. (i) The efficiency of channel A has a positive (resp. negative) impact on profit if* $0 < \alpha < \alpha_3$ *(resp.* $\alpha_3 < \alpha < 1$*). (ii) The efficiency of channel A has a positive (resp. negative) impact on the surplus of the users of channel B if* $0 < \alpha < \alpha_4$ *(resp.* $\alpha_4 < \alpha < 1$*), with* $\alpha_4 \equiv \frac{1}{3av}\left(av - f^2 + \sqrt{f^4 - 2avf^2 + 4a^2v^2}\right)$ $(0 < \alpha_4 < 1)$*. (iii) The efficiency of channel A has a positive impact on legal welfare if* $0 < \alpha < \widetilde{\alpha}$*, negative otherwise, with* $0 < \widetilde{\alpha} < 1$.

Similarly to the public enforcement setting, the optimal price p^*_{PR} charged by the legal seller negatively depends on α in the private enforcement setting. However, the impact of α on the optimal monitoring level ϕ^*_{PR} is here no longer monotonic. Improving the efficiency of the alternative channel (*i.e.*, increasing values for α) has a positive impact on monitoring level ϕ^*_{PR} only when channel A is weakly efficient ($\alpha < \alpha_2$, *Ia* area). In the opposite situation (that is, when channel A is highly efficient, *i.e.* $\alpha > \alpha_2$), a legal seller facing more efficient channel A chooses to *soften* her monitoring effort while keeping on decreasing her price. Since more enforcement is costly to the legal seller, the latter has to substitute monitoring effort to price at this point.

However, when α is higher (*i.e.* $\alpha > \alpha_3$, see *IIIa* and *IVa* areas), the legal seller has to consent quite high price cuts that are, at some point, detrimental to her. Remind that, in the public enforcement setting, these price cuts had a positive effect on legal users and provided some incentives for the monitoring authority to increase its monitoring effort. This is no longer the case here since the surplus of legal users is excluded from the objective function of the legal seller. Thus, when α is relatively high ($\alpha > \alpha_4$), the surplus of the legal consumers declines. Legal welfare increases if $\alpha < \alpha_3$ and decreases if ($\alpha > \alpha_4$). [9]

The effects of supply-side anti-piracy policies. Supply-side anti-piracy policies towards the alternative channel translate into negative variations of α, as they consist in harming the efficiency the alternative channels. As suggested by the previous analysis on the impact of α, one can see that the effect of these policies is far more ambiguous compared to the public enforcement setting. For highly efficient alternative channel (see *IVa* area), there is a rationale for such supply-side anti-piracy policies. However, there is less support for such policies in the *IIIa* area in which the surplus of the legal users and legal welfare may move to opposite direction. In the *Ia* and *IIa* areas, we once again find the results of the private enforcement setting according to which supply-side anti-piracy policies may harm both legal users and the legal seller.

To sum up, the effects of supply-side anti-piracy policies on the alternative illegal channel are far more ambiguous compared to those in the public enforcement setting. Whereas their impact on price is the same (*i.e.* increase in price) in both public and private enforcement settings, their impact on monitoring level is positive if the initial value for channel A's efficiency is high (i.e. $\alpha < \alpha_2$), negative otherwise. For highly efficient alternative channel (*IVa* area), we find support for such supply-side anti-piracy policies. There is however less support for them in the *IIIa* area where legal users' surplus and the legal welfare may move into opposite directions. In the *Ia* and *IIa* areas, we once again obtain the same results than in the private enforcement setting, according to which supply-side anti-piracy policies may harm both the legal users and the legal seller.

4.3 Equilibrium comparison of the public vs. private enforcement settings

We here compare the equilibrium outcomes of public and private enforcement settings. Doing so, we are fully able to assess the impact of the type of enforcement setting on price, monitoring level, profit and legal welfare.

Proposition 5. *The legal seller reaches out a higher profit level in the public enforcement setting compared to that achieved in the private enforcement setting when the efficiency of channels A is relatively low (i.e.,* $0 < \alpha < \widehat{\alpha}$, *with* $0 < \widehat{\alpha} < 1$*). (proof in Appendix)* ■

Proposition 5 raises an apparent paradox. It may sound paradoxical that the legal

[9] Since the legal welfare function is continuous, we can state that legal welfare reaches a maximum on the range $[\alpha_3, \alpha_4]$. However, this maximum may not be unique. This is illustrated by a dotted line on Figure 1.

seller may get higher profit in the public enforcement setting than in the private one. To explain this paradox, first recall the position of the legal seller. In the private enforcement setting, she is free to set the monitoring level so as to maximize profit. However, she has to pay for this monitoring effort. Conversely, in the public enforcement setting, she cannot manipulate the monitoring level but when some monitoring effort is made, she does not have to pay for it. monitoring authority. Second, consider Proposition 6.

Proposition 6. *Whatever v, a, f and α, in the private enforcement setting, as compared to the public enforcement setting, (i) the legal seller charges higher price, (ii) lower monitoring level is implemented and (iii) the market share of the legal channel is lower. (proof in Appendix)* ■

In the public enforcement setting, the monitoring authority provides higher monitoring effort towards illegal channel M compared to the level that is provided by the legal seller in the private enforcement setting. This is because the surplus of the legal users is included into her objective function and this surplus increases with higher monitoring levels. Put differently, the monitoring policy here generates a positive externality. The legal seller benefits from a positive externality that pushes the monitoring authority to provide a higher monitoring level while incurring no direct cost doing so. Since the monitoring authority also cares about legal users' surplus, monitoring intensity is higher than that implemented by the legal seller. In some sense, the legal seller internalizes monitoring costs in the private enforcement setting. The monitoring strategy is here only defined as a balance only between her revenue and the monitoring costs that are needed to secure this revenue. This provides lesser incentives to supervise channel M in the private enforcement setting. In turn, higher monitoring effort in the public enforcement setting allows the legal seller to relax her price while increasing her market share at the same time. Switching from the private enforcement setting to the public enforcement one is profit-enhancing only if the alternative channel is weakly efficient (*i.e.*, $0 < \alpha < \widehat{\alpha}$). As the efficiency of the illegal alternative channel is found to be high, this is no longer true because the monitoring authority tends to monitor more than the legal seller would do. Remember that the rationale for this higher level is that the latter is costly but generates a welfare-improving transfer to legal users. This transfer is yet detrimental to the legal seller. The rationale for such high levels hence vanishes in the case of private enforcement since the legal seller would not choose a monitoring level that would be detrimental to herself.

Proposition 7. *Whatever values for v, a, f and α are, level for the surplus of legal users is higher in the public enforcement setting than that in the private enforcement one. The same applies for legal welfare. (proof in Appendix)* ■

Considering legal welfare, proposition 7 clearly states the superiority of the public enforcement setting over the private one. Indeed, in the public enforcement setting, the legal seller charges a lower price whereas the monitoring authority chooses a higher monitoring level compared to that chosen by the legal seller in the private enforcement setting. The market share of the legal seller is higher in the public enforcement setting and lowering price thus leads to higher levels for the surplus of legal users. Although the legal seller's profit may be in some cases lower in the public enforcement setting than that in the private enforcement one, such a loss is lower compared to the benefit the legal users generate from switching from the private enforcement framework to the public enforcement setting.

Put together, propositions 5 and 7 together underline potential coordination issues between the legal seller and the legal welfare-driven social planner when the efficiency of channel α is high. Indeed, when $\widehat{\alpha} < \alpha < 1$, the legal seller is better off operating in the private enforcement setting whereas such a social planner prefers the public enforcement setting to the private enforcement one. However, when the efficiency of channel α is relatively low (*i.e.*, $0 < \alpha < \widehat{\alpha}$), both prefer the public enforcement setting compared to the private enforcement one. An explanation is that the legal seller wants to supervise the illegal monitored channel by herself when the alternative channel exhibits a high level of efficiency. It is so because price sensitivity of legal buyers is higher when their outside option (A) is highly efficient. Hence, there is less of a need to cut price in the private enforcement setting compared to the public one. Although the market share of the legal seller turns to be lower than in the public enforcement setting, the legal seller here better benefits from a slight decrease in price when channel A is shown to be somehow efficient. This is nevertheless detrimental to legal welfare since monitoring channel M the private enforcement way leads to lower level for legal consumers' surplus which are not overcome by lower monitoring costs. When the efficiency of channel α is relatively low (*i.e.*, $0 < \alpha < \widehat{\alpha}$), the legal seller is better off letting the monitoring authority supervising channel M because the monitoring effort she would have to provide is too high compared to the gain she could get from better softening the decrease in price of her products. In this case, both social - legal welfare - and private - profit - objective coincide.

5 Discussion and Conclusion

Should some rights be publicly or privately enforced? What if some illegal behaviors could *never* be monitored through direct - demand-side - interventions? This article proposes a theoretical framework to understand these two issues in the case of digital piracy. Whereas previous research considered that illegal channels can always be directly monitored - yet imperfectly and at some costs -, we consider here demand-side anti-piracy policies cannot fully monitor all illegal channels (*e.g.* illegal streaming, offline hand-to-hand file copies, *etc.*). The design of public policies about copyrights infringements thus has to take the presence of such outside option into account. Such illegal alternative channels can nevertheless be targeted through supply-side policies, yet imperfectly as well. We build a theoretical model able to analyze the interplay between anti-piracy policies, sellers' pricing and user legal and illegal adoption behaviors. The first original feature or our approach lies in the contrast between a private and a public enforcement setting. The second original feature lies in in the existence of an outside illegal option for file adoption. The third one is to study the interplay between some supply-side and demand-side policies.

Focusing on a framework in which monitoring costs are relatively high and users are able to adopt from either a legal channel, an illegal monitored channel and an illegal non-monitored channel, we show that the optimal strategies of the legal seller and of the monitoring authority always lead to deter the illegal not-monitored channel out. Although this channel is deterred in equilibrium, it nevertheless plays the role of a potential incumbent and it impacts on the price charged by the legal seller. We emphasize on three resulting issues.

The role of supply-side anti-piracy policies. Legal authorities are aware of the potential use of alternative channels. Some supply-side anti-piracy policies targeted to these channels have been implemented to degrade their efficiency and discourage users from their use. We show that these policies may lead to unexpected - sometimes counter-productive - effects. When copyrights are enforced a public way, we show that these policies always harm legal welfare (where legal welfare is defined by the sum of the legal seller's revenues, the legal users' surplus minus monitoring costs). An explanation of this result is that the legal seller has some incentive to increase price. These measures are therefore not neutral *vis-à-vis* the distribution of welfare. Although legal users are always harmed by these measures, the legal seller may benefit from them, especially when the illegal alternative channel was initially highly efficient. When copyrights are enforced the private way, the impact of supply-side policies depends on the level of efficiency of the illegal non-monitored channel before these policies are undertaken. As this level of efficiency is here rather low, we retrieve the results of the private enforcement setting. Yet, for higher level of efficiency for the illegal non-monitored channel, this conclusion may not hold. Our findings then cast some doubts on the legitimacy of the supply-side anti-piracy measures which target channels that are not monitored by demand-side anti-piracy policies. It allows to endogenize pricing and monitoring strategies, whose joint effect may have opposite effects than that initially expected (*i.e.* discouraging piracy, increasing the size of the legal market).

The interplay between supply-side and demand-side policies. In our setting, supply-side policies target channels that are not monitored through demand-side interventions. Intuition may predict that the effects of these demand-side and supply-side policies on the legal supply would go in the same direction. Hence, these policies may be considered as rough substitutes. However, this reasoning does not account here for all the strategic interdependencies between supply-side policies, demand-side policies and the strategy of the legal seller. Our model captures these interdependencies. It reveals that in the public enforcement setting, when some supply-side policies are implemented, applying demand-side policies leads to decrease monitoring intensity. This suggests the two policies to be substitutes in the public enforcement setting. This result does not entirely hold in the private enforcement case, as the two policies are found to be substitutes when the efficiency of the illegal non-monitored channel is low. Otherwise, policies are complements, the legal seller increasing her monitoring level to keep her consumers.

The impact of the enforcement regime (public vs. private) As far as legal welfare is used as a relevant criterion to address this issue, our results clearly point out to the superiority of the public enforcement setting. However, public enforcement generates higher monitoring levels compared to private enforcement. The legitimacy for higher control levels is that they enable the legal seller to decrease price, which benefits to legal users also. Is the legal seller better off in this situation? On the one hand, the legal seller cuts monitoring costs in the public enforcement setting. On the other hand, she benefits from a larger legal market share. If the seller had to choose between public enforcement and private one, she should then plea for public enforcement. However, since her ability to raise price is lower in the case of public enforcement, public enforcement is found to benefit the legal seller only if the alternative illegal channel is weakly efficient. If not, there is a conflict of interests between a social planner - who would always prefer public enforcement - and the legal seller - who would here prefer private to public enforcement. Since any issue related to digital piracy is frequently surrounded by a high lobbying intensity from both sides, our paper delineates the situations in which some consensus may be reached out to other rather conflictual situations. Note however that the choice between private and public enforcement is not neutral towards public finance and that public enforcement generates higher legal welfare compared to private one, yet also generating non-financed public deficit. While such a consideration is beyond the scope of the present study, it may however interfere in the choice between public and private enforcement in a context of scarcity for public resources.

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7 Appendix

7.1 Adoption outcomes

We here focus on the last stage of the game, thus considering given levels for ϕ and p, as well as a given set of parameters (α, f, v). $\widetilde{e}$ represents the location of the indifferent user between B and M, whereas $\widehat{e}$ (resp. $\overline{e}$) represents that of the indifferent user between B and A (resp. A and M). From the specifications of the utility functions, we find that $\widetilde{e} = (v - p + \phi f)/v$, $\widehat{e} = \frac{v-p}{\alpha v}$ and $\overline{e} = \frac{\phi f}{v(1-\alpha)}$. To identify the adoption outcomes, we have to define the relative rankings of $\widetilde{e}$, $\widehat{e}$ and $\overline{e}$ on the unit line. Doing so, we can then deduce the optimal user's choice as a function of her location e_i for each area on this line. Under our assumptions, we show that only two rankings between $\widetilde{e}$, $\widehat{e}$ and $\overline{e}$ are possible, namely $\widehat{e} \leq \widetilde{e} \leq \overline{e}$ and $\overline{e} \leq \widetilde{e} \leq \widehat{e}$. Moreover, for the market to be both fully-served and shared by B, A and M, we show that $\widetilde{e}$, $\widehat{e}$, $\overline{e}$, 0 and 1 have to be ranked so that $0 < \widehat{e} \leq \widetilde{e} < \overline{e} < 1$ or $0 < \widehat{e} < \widetilde{e} \leq \overline{e} < 1$. Both rankings apply when ϕ, p, α, f and v are defined as follows: $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha < \frac{v-\phi f}{v}$. This expresses the two conditions which have to be simultaneously fulfilled for the three channels to cover and share the market.

Otherwise, when ϕ, p, α, f and v are defined so that $\phi < \frac{p}{f}$ and $\alpha < \frac{v-p}{v-p+\phi f}$ (resp. $\frac{p}{f} < \phi < 1$ and $0 < \alpha < \frac{v-p}{v}$), the market is shared by channels B and M (resp. only served by channel B). For all other range of values for ϕ, p, α, f and v, the market is fully-served and shared by channels B and A.

7.2 Public enforcement setting

Proof of proposition 1. Consider the market to be fully-covered and shared by the three channels, that is, $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha < \frac{v-\phi f}{v}$.

At the second stage of the game, the legal seller maximizes her profit w.r.t p. The profit function writes as $\pi(p,\phi) = p\widehat{e} = \left(\frac{1}{\alpha v}\right)\left(-p^2 + pv\right)$. It is straightforward to see that $\pi(p,\phi)$ is an inverted U-shaped function for $p \in \mathbb{R}^+$ which reaches out its optimal state at $p = \frac{v}{2}$, with $\pi(0,\phi) = \pi(v,\phi) = 0$. Let us remember that conditions $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha < \frac{v-\phi f}{v}$ have to be taken into account when identifying the optimal pricing strategy of the legal seller. These two conditions can be expressed as $\phi f < p$ and $v - \left(\frac{\alpha}{1-\alpha}\right)\phi f < p$. From here, we have to consider two possible cases: $\phi f < v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$ and $\phi f > v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$.

When $\phi f < p$ and $\phi f < v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$, the two conditions can be rewritten and combined so that $v - \left(\frac{1}{1-\alpha}\right)\phi f < p < v$. We once again have to take two subcases into account. In subcase 1a, we have $v - \left(\frac{\alpha}{1-\alpha}\right)\phi f < \frac{v}{2}$, which can be rewritten as $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi$. From the shape of function $\pi(p,\phi)$ for $p \in \mathbb{R}$, it is straightforward to see that the optimal pricing strategy of the legal seller is $p^* = \frac{v}{2}$. In subcase 1b, we have $v - \left(\frac{\alpha}{1-\alpha}\right)\phi f > \frac{v}{2}$, which can be rewritten as $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) > \phi$. In this subcase, the optimal pricing strategy of the legal seller can expressed by $p^*(\phi) = v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$. We proceed in a similar way when $\phi f < p$ and $\phi f > v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$. The combination of these two conditions leads to $v - \left(\frac{\alpha}{1-\alpha}\right)\phi f < \phi f < p < v$. Two subcases have to be considered. In subcase 1c, we have $\phi f < \frac{v}{2}$, which can be reexpressed as $\phi < \frac{1}{2}\frac{v}{f}$. Again, from the shape of function $\pi(p,\phi)$ for $p \in \mathbb{R}$, the optimal pricing strategy of the legal seller is given by $p^* = \frac{v}{2}$. In subcase 1d, ϕ is defined so that $\phi f > \frac{v}{2}$, which can be rewritten as $\phi > \frac{v}{2f}$. This latter subcase leads to $p^*(\phi) = \phi f$.

We now move to the first stage of the game. Here, the monitoring authority maximizes its objective function w.r.t. monitoring intensity ϕ. This function is defined as the sum of legal consumers' and seller's surpluses minus the cost the monitoring authority invests to monitor channel M. Formally, the objective function of the monitoring authority is given by $H(p,\phi) = \pi(p,\phi) + CS_B(p,\phi) - \varsigma(\phi)$, with $\varsigma(\phi) = \frac{a}{2}\phi^2$. Here, as we come from the second stage of the game, p is given as an expression of ϕ. To identify the optimal strategy (p^*, ϕ^*) of the legal seller and the monitoring authority, we successively deal with subcases 1a, 1b, 1c and 1d.

Subcase 1a refers to the subcase in which $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha)$. It results from the latter condition that α is here defined so that $\frac{1}{2} < \alpha < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{v}{2\alpha} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1a, we show that ϕ has to be defined so that $0 < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha) < \frac{1}{2}\frac{v}{f}$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{v}{2\alpha} - \frac{a}{2}\phi^2$ on $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha)$. This leads it to define $\phi^* = \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$. The optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)$ defines the solution of the public enforcement setting in subcase 1a. Here, we obtain $\pi^* = \frac{v}{4\alpha}$ and $H^* = \frac{v}{2\alpha} - \frac{a}{2}\left(\frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)^2$.

Subcase 1b refers to the subcase in which $\phi < \min\left\{\frac{v}{f}(1-\alpha), \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)\right\}$. By substituting $p^*(\phi) = v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{f}{1-\alpha}\phi - \frac{a}{2}\phi^2$. One can show that $H(\phi) = \frac{f}{1-\alpha}\phi - \frac{a}{2}\phi^2$ is an inverted U-shaped function for $\phi \geq 0$ which reaches out its optimal state at $\phi = \frac{f}{a(1-\alpha)}$, with $H(0) = H\left(\frac{2f}{a(1-\alpha)}\right) = 0$. Let us first consider subcase 1b1 in which $0 < \phi < \frac{v}{f}(1-\alpha) < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$. From the latter expression of ϕ, α is here defined so that $0 < \alpha < \frac{1}{2}$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{f}{1-\alpha}\phi - \frac{a}{2}\phi^2$ on $0 < \phi < \frac{v}{c}(1-\alpha) < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$. The monitoring authority sets $\phi^* = \frac{v}{f}(1-\alpha)$ when $\frac{v}{f}(1-\alpha) < \frac{f}{a}\left(\frac{1}{1-\alpha}\right)$ and $\phi^* = \frac{f}{a}\left(\frac{1}{1-\alpha}\right)$ when $\frac{v}{f}(1-\alpha) > \frac{f}{a}\left(\frac{1}{1-\alpha}\right)$. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ (resp.$(p^*, \phi^*) = \left(v - \alpha\left(\frac{1}{1-\alpha}\right)^2\left(\frac{f^2}{a}\right), \frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)$) defines the solution of the public enforcement setting in subcase 1b1 when $0 < a < \frac{f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$ (resp. $a > \frac{f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$). From here, we obtain $\pi^* = v(1-\alpha)$ and $H^* = v - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$ when $0 < a < \frac{f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$, whereas we have $\pi^* = \frac{f^2}{av}\left(\frac{1}{1-\alpha}\right)^2\left(v - \frac{\alpha f^2}{a}\left(\frac{1}{1-\alpha}\right)^2\right)$ and $H^* = \frac{f^2}{a}\left(\frac{1}{1-\alpha}\right)^2 - \frac{a}{2}\left(\frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)^2$ when $a > \frac{f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$. Let us secondly consider subcase 1b2 in which $0 < \phi < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \frac{v}{c}(1-\alpha)$. From the latter expression of ϕ, α is here now defined so that $\frac{1}{2} < \alpha < 1$. The monitoring authority therefore sets its optimal monitoring strategy ϕ^* which maximizes function $H(\phi) = \frac{f}{1-\alpha}\phi - \frac{a}{2}\phi^2$ on $0 < \phi < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \frac{v}{c}(1-\alpha)$. Remind that $H(\phi) = \frac{f}{1-\alpha}\phi - \frac{a}{2}\phi^2$ is an inverted U-shaped function for $\phi \geq 0$ which reaches out its optimal state at $\phi = \frac{f}{a(1-\alpha)}$, with $H(0) = H\left(\frac{2f}{a(1-\alpha)}\right) = 0$. The monitoring authority sets $\phi^* = \frac{v}{2f}\left(\frac{1-\alpha}{\alpha}\right)$ when $0 < a < \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$ and $\phi^* = \frac{f}{a}\left(\frac{1}{1-\alpha}\right)$ when $a > \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$. $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{2f}\left(\frac{1-\alpha}{\alpha}\right)\right)$ (resp.$(p^*, \phi^*) = \left(v - \alpha\left(\frac{1}{1-\alpha}\right)^2\left(\frac{f^2}{a}\right), \frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)$) defines the solution of the public enforcement setting in subcase 1b2 when $0 < a < \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$ (resp. $a > \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$). From here, we obtain $\pi^* = \frac{v}{4\alpha}$ and $H^* = \frac{v}{2\alpha} - \frac{a}{2}\left(\frac{v}{2f}\left(\frac{1-\alpha}{\alpha}\right)\right)^2$ when $0 < a < \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$, whereas we have $\pi^* = \frac{f^2}{av}\left(\frac{1}{1-\alpha}\right)^2\left(v - \frac{\alpha f^2}{a}\left(\frac{1}{1-\alpha}\right)^2\right)$ and $H^* = \frac{f^2}{a}\left(\frac{1}{1-\alpha}\right)^2 - \frac{a}{2}\left(\frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)^2$ when $a > \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$.

Subcase 1c refers to the subcase in which $\frac{v}{f}(1-\alpha) < \phi < \frac{1}{2}\frac{v}{f}$. It results from the latter condition that α is here defined so that $\frac{1}{2} < \alpha < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the monitoring authority, we find $H(\phi) = \frac{v}{2\alpha} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1c, we show that ϕ has to be defined so that $\phi = \frac{v}{f}(1-\alpha)$ and $\frac{1}{2} < \alpha < 1$. This defines the optimal monitoring strategy of the monitoring authority. As such, the optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha)\right)$ defines the solution of the public enforcement setting in subcase 1c. From here, we obtain $\pi^* = \frac{v}{4\alpha}$ and $H^* = \frac{v}{2\alpha} - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$.

Subcase 1d refers to the subcase in which $\phi > \max\left\{\frac{v}{f}(1-\alpha), \frac{1}{2}\frac{v}{f}\right\}$. By substituting $p^* = \phi f$

into the objective function of the monitoring authority, we find $H(\phi) = \frac{v-\phi f}{\alpha} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \phi f$ with all the constraints of subcase 1d, we show that ϕ has to be defined so that $\phi = \frac{v}{f}(1-\alpha)$ and $0 < \alpha < \frac{1}{2}$. This defines the optimal monitoring strategy of the monitoring authority. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ defines the solution of the public enforcement setting in subcase 1d. From here, we obtain $\pi^* = v(1-\alpha)$ and $H^* = v - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$.

Putting all these results together we identify four candidate solutions of the public enforcement setting, regardless of the nature of the subcases. These depend on α and a: (i) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)$ when $\frac{1}{2} < \alpha < 1$, (ii) $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ when $0 < \alpha < \frac{1}{2}$, (iii) $(p^*, \phi^*) = \left(v - \alpha\left(\frac{1}{1-\alpha}\right)^2\left(\frac{f^2}{a}\right), \frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)$ when $0 < \alpha < \frac{1}{2}$ and $a > \frac{f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$or when $\frac{1}{2} < \alpha < 1$ and $a > \frac{2\alpha f^2}{v}\left(\frac{1}{1-\alpha}\right)^2$, and (iv) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha)\right)$ when $\frac{1}{2} < \alpha < 1$. As the monitoring authority plays first, it also indirectly shapes the strategy of the legal seller. Indeed, the monitoring authority selects its strategy so that the outcome of the public enforcement setting provides the best level $H^*$for her objective function $H(p, \phi)$. Let us thus focus on levels of H for the four candidate solutions of the public enforcement setting, that is, (i) $H_1 = \frac{v}{2\alpha} - \frac{a}{2}\left(\frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)^2$, (ii) $H_2 = v - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$, (iii) $H_3 = \frac{f^2}{a}\left(\frac{1}{1-\alpha}\right)^2 - \frac{a}{2}\left(\frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)^2$ and (iv) $H_4 = \frac{v}{2\alpha} - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$. One can show that $H_3 > H_2$ and that $H_3 > H_1 > H_4$. Reminding that we have assumed cost parameter a to be high (i.e., $a > \frac{4f^2}{v}$), the solution of the public enforcement setting is therefore given by $(p^*_{PU}, \phi^*_{PU}) = \left(v - \alpha\left(\frac{1}{1-\alpha}\right)^2\left(\frac{f^2}{a}\right), \frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)$. The profit of the legal seller is $\pi^*_{PU} = \frac{f^2}{av}\left(\frac{1}{1-\alpha}\right)^2\left(v - \frac{\alpha f^2}{a}\left(\frac{1}{1-\alpha}\right)^2\right)$ and the surplus of her customers is $CS^*_{B,PU} = \frac{\alpha f^4}{a^2 v(1-\alpha)^4}$.

Proof of proposition 2. In the public enforcement setting, optimal values for price and monitoring level are given by $(p^*_{PU}, \phi^*_{PU}) = \left(v - \alpha\left(\frac{1}{1-\alpha}\right)^2\left(\frac{f^2}{a}\right), \frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)$. The sign of $\frac{\partial p^*_{PU}}{\partial \alpha}$(resp. $\frac{\partial \phi^*_{PU}}{\partial \alpha}$) shows how a higher level of efficiency for the illegal non-monitored channel affects the optimal value for price (resp. monitoring level). $\frac{\partial p^*_{PU}}{\partial \alpha} = \frac{\partial\left(v - \alpha_A\left(\frac{1}{1-\alpha_A}\right)^2\left(\frac{f^2}{a}\right)\right)}{\partial \alpha} = -\frac{f^2(1+\alpha)}{a(1-\alpha)^3} < 0$. In a similar fashion, $\frac{\partial \phi^*_{PU}}{\partial \alpha} = \frac{\partial\left(\frac{f}{a}\left(\frac{1}{1-\alpha}\right)\right)}{\partial \alpha} = \frac{f}{a(1-\alpha)^2} > 0$.

7.3 Private enforcement setting

Proof of proposition 3. Consider once again the market to be fully-covered and fully-shared by the three channels, that is, $0 < \phi < \frac{p}{f}$ and $\frac{v-p}{v-p+\phi f} < \alpha < \frac{v-\phi f}{v}$.

At the second stage of the private enforcement setting, the legal seller still maximizes their profit function w.r.t p. However note that she now also has to take monitoring costs into account in her profit-maximizing program. Since monitoring costs do not here depend from p, the results are here the same compared to that of the public enforcement setting. Indeed, we eventually have to consider subcases 1a, 1b, 1c and 1d, which were previously identified in the proof of proposition 1.

We now move to the first stage of the private enforcement game. Remind here that the legal seller is also in charge of monitoring channel M. Here, she maximizes her objective function w.r.t. monitoring level ϕ.This function is here defined by the difference between her revenues and the cost she has to invest to control channel M. Here, as we come from the second stage of the game, p is given as an expression of ϕ. To identify the optimal strategy (p^*, ϕ^*) of the legal seller, we successively deal with subcases 1a, 1b, 1c and 1d.

Subcase 1a refers to the subcase in which $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha)$. It results from the latter condition that α is here defined so that $\frac{1}{2} < \alpha < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{v}{4\alpha} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1a, we show that ϕ has to be defined so that $0 < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha) < \frac{1}{2}\frac{v}{f}$. The legal seller therefore sets her optimal monitoring strategy ϕ^* which maximizes function $\pi(\phi) = \frac{v}{2\alpha} - \frac{a}{2}\phi^2$ on $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \phi < \frac{v}{f}(1-\alpha)$. This leads her to define $\phi^* = \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$. The optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)$ defines the solution of the public enforcement setting in subcase 1a. Here, we obtain $\pi^* = \frac{v}{4\alpha} - \frac{a}{2}\left(\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)\right)^2$.

Subcase 1b refers to the subcase in which $\phi < \min\left\{\frac{v}{f}(1-\alpha), \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)\right\}$. By substituting $p^*(\phi) = v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{\phi f}{1-\alpha} - \frac{\phi^2 f^2 \alpha}{v(1-\alpha)^2} - \frac{a}{2}\phi^2$. One can show that $\pi(\phi) = \frac{\phi f}{1-\alpha} - \frac{\phi^2 f^2 \alpha}{v(1-\alpha)^2} - \frac{a}{2}\phi^2$ is an inverted U-shaped function for $\phi \geq 0$ which reaches out its optimal state at $\phi = \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}$, with $\pi(0) = \pi\left(\frac{2fv(1-\alpha)^2}{2\alpha f^2 - av(1-\alpha)^2}\right) = 0$. When combining expression $p^* = v - \left(\frac{\alpha}{1-\alpha}\right)\phi f$ with all the constraints of subcase 1b, we show that ϕ has to be defined so that $0 < \phi < \frac{v}{f}(1-\alpha)$, $0 < \phi < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$ and $0 < \phi < \frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$. Note that $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$ and that $\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) > \frac{v}{f}(1-\alpha)$. Consequently, one can show that ϕ has to be here defined so that $0 < \phi < \frac{v}{f}(1-\alpha) < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$ (subcase 1b1') or $0 < \phi < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) < \frac{v}{f}(1-\alpha) < \frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$ (subcase 1b2'). Let us first consider subcase 1b1'. From the expression of ϕ, α is here defined so that $0 < \alpha < \frac{1}{2}$. As ϕ is defined so that $0 < \phi < \frac{v}{f}(1-\alpha)$, we show that the legal seller sets $\phi^* = \frac{v}{f}(1-\alpha)$ when $\frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2} > \frac{v}{f}(1-\alpha)$ and $\phi^* = \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}$ when $\frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2} < \frac{v}{f}(1-\alpha)$. The optimal strategy $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ (resp. $(p^*, \phi^*) = \left(v\left(\frac{\alpha f^2 + av(1-\alpha)^2}{2\alpha f^2 + av(1-\alpha)^2}\right), \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}\right)$) defines the solution of the public enforcement setting in subcase 1b1' when $0 < a < \frac{f^2(1-2\alpha)}{v(1-\alpha)^2}$ (resp. $a > \frac{f^2(1-2\alpha)}{v(1-\alpha)^2}$). From here, we obtain $\pi^* = v(1-\alpha) - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$ when $0 < a < \frac{f^2(1-2\alpha)}{v(1-\alpha)^2}$, whereas we have $\pi^* = \frac{vf^2}{2\left(2f^2\alpha + av(1-\alpha)^2\right)}$ when $a > \frac{f^2(1-2\alpha)}{v(1-\alpha)^2}$. Let us secondly consider subcase 1b2'. From the expression of ϕ, α is now defined so that $\frac{1}{2} < \alpha < 1$. As ϕ is defined so that $0 < \phi < \frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)$ and as it can be shown that $\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right) > \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}$ for any value of a ($a > 0$), we find that the legal seller sets $\phi^* = \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}$. The optimal strategy $(p^*, \phi^*) = \left(v\left(\frac{\alpha f^2 + av(1-\alpha)^2}{2\alpha f^2 + av(1-\alpha)^2}\right), \frac{fv(1-\alpha)}{2\alpha f^2 + av(1-\alpha)^2}\right)$ defines the solution of the private enforcement setting in subcase 1b2'. From here, we obtain $\pi^* = \frac{vf^2}{2\left(2f^2\alpha + av(1-\alpha)^2\right)}$.

Subcase 1c refers to the subcase in which $\frac{v}{f}(1-\alpha) < \phi < \frac{1}{2}\frac{v}{f}$. It results from the latter condition that α is here defined so that $\frac{1}{2} < \alpha < 1$. By substituting $p^* = \frac{v}{2}$ into the objective function of the legal seller, we find $\pi(\phi) = \frac{v}{4\alpha} - \frac{a}{2}\phi^2$, which is a decreasing function with $\phi \geq 0$. When combining expression $p^* = \frac{v}{2}$ with all the constraints of subcase 1c, one can show that ϕ has to be defined so that $\phi = \frac{v}{f}(1-\alpha)$ and $\frac{1}{2} < \alpha < 1$. This defines the optimal monitoring strategy of the legal seller. As such, the optimal strategy $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha)\right)$ defines the solution of the private enforcement setting in subcase 1c. From here, we obtain $\pi^* = \frac{v}{4\alpha} - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$.

Subcase 1d refers to the subcase in which $\phi > \max\left\{\frac{v}{f}(1-\alpha), \frac{1}{2}\frac{v}{f}\right\}$. By substituting $p^* = \phi f$ into the objective function of the legal seller, we find $\pi(\phi) = \left(\frac{f}{\alpha}\right)\phi - \left(\frac{2f^2+av\alpha}{2v\alpha}\right)\phi^2$. One can show that $\pi(\phi) = \left(\frac{f}{\alpha}\right)\phi - \left(\frac{2f^2+av\alpha}{2v\alpha}\right)\phi^2$ is an inverted U-shaped function for $\phi \geq 0$ which reaches out its optimal state at $\phi = \frac{fv}{2f^2+av\alpha}$, with $\pi(0) = \pi\left(\frac{2fv}{2f^2+av\alpha}\right) = 0$. When combining expression $p^* = \phi f$ with all the constraints of subcase 1d, we eventually show that ϕ has to be defined so that $\phi = \frac{v}{f}(1-\alpha)$ and $0 < \alpha < \frac{1}{2}$. This defines the optimal monitoring strategy of the legal seller. As such, the optimal strategy $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ defines the solution of the private enforcement setting in subcase 1d. From here, we obtain $\pi^* = v(1-\alpha) - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$.

Putting all these results together we identify four candidate solutions of the private enforcement setting, regardless of the nature of the subcases we have taken into account, which depend on α and a: (i) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{1}{2}\frac{v(1-\alpha)}{\alpha f}\right)$ when $\frac{1}{2} < \alpha < 1$, (ii) $(p^*, \phi^*) = \left(v(1-\alpha), \frac{v}{f}(1-\alpha)\right)$ when $0 < \alpha < \frac{1}{2}$, (iii) $(p^*, \phi^*) = \left(v\left(\frac{\alpha f^2+av(1-\alpha)^2}{2\alpha f^2+av(1-\alpha)^2}\right), \frac{fv(1-\alpha)}{2\alpha f^2+av(1-\alpha)^2}\right)$ when $0 < \alpha < \frac{1}{2}$ and $a > \frac{f^2(1-2\alpha)}{v(1-\alpha)^2}$ or when $\frac{1}{2} < \alpha < 1$, and (iv) $(p^*, \phi^*) = \left(\frac{v}{2}, \frac{v}{f}(1-\alpha)\right)$ when $\frac{1}{2} < \alpha < 1$. As opposed to the public enforcement setting, the legal seller plays at both first and second stages of the private enforcement setting game. The legal seller selects her strategy so that the outcome of the private enforcement setting provides the best level $\pi^*$for her objective function $\pi(p, \phi)$. Let us thus focus on levels of π for the four candidate solutions of the private enforcement setting, that is, (i) $\pi_1' = \frac{v}{4\alpha} - \frac{a}{2}\left(\frac{1}{2}\frac{v}{f}\left(\frac{1-\alpha}{\alpha}\right)\right)^2$, (ii) $\pi_2' = v(1-\alpha) - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$, (iii) $\pi_3' = \frac{vf^2}{2\left(2f^2\alpha+av(1-\alpha)^2\right)}$ and (iv) $\pi_4' = \frac{v}{4\alpha} - \frac{a}{2}\left(\frac{v}{f}(1-\alpha)\right)^2$. One can show that $\pi_3' > \pi_2'$ and that $\pi_3' > \pi_1' > \pi_4'$. Reminding that we have assumed cost parameter a to be high (i.e., $a > \frac{4f^2}{v}$), the solution of the private enforcement setting is therefore given by $(p_{PR}^*, \phi_{PR}^*) = \left(v\left(\frac{\alpha f^2+av(1-\alpha)^2}{2\alpha f^2+av(1-\alpha)^2}\right), \frac{fv(1-\alpha)}{2\alpha f^2+av(1-\alpha)^2}\right)$. The profit of the legal seller amounts is $\pi^* = \frac{vf^2}{2\left(2f^2\alpha+av(1-\alpha)^2\right)}$ and the surplus of her users is $CS_{B,PR}^* = \frac{v\alpha f^4}{\left(av(1-\alpha)^2+2\alpha f^2\right)^2}$.

Proof of proposition 4. In the private enforcement setting, optimal values for price and monitoring level are given by $(p_{PR}^*, \phi_{PR}^*) = \left(v\left(\frac{\alpha f^2+av(1-\alpha)^2}{2\alpha f^2+av(1-\alpha)^2}\right), \frac{fv(1-\alpha)}{2\alpha f^2+av(1-\alpha)^2}\right)$. As in the proof of proposition 2, the sign of $\frac{\partial p_{PR}^*}{\partial \alpha}$(resp. $\frac{\partial \phi_{PR}^*}{\partial \alpha}$) shows how a higher level of efficiency for the illegal non-monitored channel affects the optimal value for price (resp. monitoring level).

$\frac{\partial p^*_{PR}}{\partial\alpha} = \frac{\partial\left(v\left(\frac{\alpha f^2+av(1-\alpha)^2}{2\alpha f^2+av(1-\alpha)^2}\right)\right)}{\partial\alpha} = \frac{-af^2v^2(1-\alpha)(1+\alpha)}{(2f^2\alpha+av\alpha^2-2av\alpha+av)^2} < 0$. Similarly, $\frac{\partial\phi^*_{PR}}{\partial\alpha} = \frac{\partial\left(\frac{fv(1-\alpha)}{2\alpha f^2+av(1-\alpha)^2}\right)}{\partial\alpha} = \frac{-fv\left(2f^2-av\alpha^2+2av\alpha-av\right)}{(2f^2\alpha+av\alpha^2-2av\alpha+av)^2}$. The sign of $-\left(2f^2-av\alpha^2+2av\alpha-av\right)$ gives that of $\frac{\partial\phi^*_{PR}}{\partial\alpha}$. Define $h(\alpha) = av\alpha^2-2av\alpha+av-2f^2$, where h is a U-shaped function w.r.t. α. $h(\alpha)=0$ admits two solutions, namely $\alpha_1 = 1-\sqrt{\frac{2f^2}{av}}$ and $\alpha_2 = 1+\sqrt{\frac{2f^2}{av}}$. Note that $\alpha_2 > 1 > \alpha_1$. It is also possible to show that $\alpha_1 > 0$. Indeed, $\alpha_1 > 0$ if and only if $1 > \sqrt{\frac{2f^2}{av}}$, i.e., $a > \frac{2f^2}{v}$, which holds as we here focus on values for a defined so that $a > \frac{4f^2}{v}$. Consequently, the sign of $\frac{\partial\phi^*_{PR}}{\partial\alpha}$ is positive if $0 < \alpha < 1-\sqrt{\frac{2f^2}{av}}$, negative otherwise.

7.4 Equilibrium comparison

Proof of proposition 5. Define $\Delta\pi^* = \pi^*_{PR} - \pi^*_{PU}$. Expliciting $\Delta\pi^*$, we find $\Delta\pi^* = \frac{-k(\alpha)}{2a^2v(\alpha-1)^4\left(2f^2\alpha+av(1-\alpha)^2\right)}$, with $k(\alpha) \equiv a^2v^2\alpha^4-4a^2v^2\alpha^3+6a^2v^2\alpha^2-4a^2v^2\alpha+a^2v^2+2af^2v\alpha^3-4af^2v\alpha^2+2af^2v\alpha-4f^4\alpha^2$. The sign of $\Delta\pi^*$ is of that $-k(\alpha)$. Complex calculations and deep numerical analyzes show that there exists a value $\widehat{\alpha}$ $(0 < \widehat{\alpha} < 1)$ so that k takes positive values if $0 < \alpha < \widehat{\alpha}$, negative values otherwise. The sign of $\Delta\pi^*$ is therefore negative for values of α defined so that $0 < \alpha < \widehat{\alpha}$, positive otherwise.

Proof of proposition 6. Define $\Delta p^* = p^*_{PR} - p^*_{PU}$, $\Delta\phi^* = \phi^*_{PR} - \phi^*_{PU}$ and $\Delta m^*_B = m^*_{B,PR} - m^*_{B,PU}$. Expliciting Δp^*, we find $\Delta p^* = \frac{2\alpha^2f^4}{a(1-\alpha)^2(2f^2\alpha+av\alpha^2-2av\alpha+av)}$, which sign is that of $2f^2\alpha+av\alpha^2-2av\alpha+av$. As some manipulations yield $2f^2\alpha+av\alpha^2-2av\alpha+av = 2f^2\alpha+av(1-\alpha)^2 > 0$, we show that $p^*_{PR} > p^*_{PU}$. We proceed the same way to analyze the sign of $\Delta\phi^*$. We eventually show that $\Delta\phi^* = \frac{2\alpha f^3}{-a(1-\alpha)(2f^2\alpha+av\alpha^2-2av\alpha+av)} < 0$, that is, $\phi^*_{PR} < \phi^*_{PU}$. Dealing with the market share of the legal seller, simple calculations lead to $\Delta m^*_B = -\frac{2f^4\alpha}{av(1-\alpha)^2(2f^2\alpha+av\alpha^2-2av\alpha+av)} = -\frac{2f^4\alpha}{av(1-\alpha)^2\left(2f^2\alpha+av(1-\alpha)^2\right)} < 0$, that is, $m^*_{B,PR} < m^*_{B,PU}$. This defines the impact of the regime enforcement setting on optimal pricing and monitoring strategies, as well as on the ensuing market share of the legal seller.

Proof of proposition 7. Define $\Delta CS^*_B = CS^*_{B,PR} - CS^*_{B,PU}$. Expliciting ΔCS^*_B, we find $\Delta CS^*_B = \frac{-4\alpha^2f^6\left(f^2\alpha+av(\alpha-1)^2\right)}{a^2v(1-\alpha_A)^4\left(2f^2\alpha_A+av(1-\alpha_A)^2\right)^2} < 0$ for any value of v, a, f and α. Define now $\Delta LW^* = LW^*_{PR} - LW^*_{PU}$. Expliciting ΔLW^*, we find that $\Delta LW^* = -\frac{2f^6\alpha^2}{a(1-\alpha)^2\left(2f^2\alpha+av(1-\alpha)^2\right)^2}$. This expression is negative for any value of v, a, f and α.