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Effects of development aid (grants and loans) on the economic dynamics of the recipient country

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Abstract

This paper investigates the nexus between foreign aid (in both forms: grant and loan), poverty trap, and economic development in a recipient country by using a Solow model with two new ingredients: a development loan and a fixed cost in the production process. The presence of this fixed cost generates a poverty trap. We show that foreign aid may help the country to escape from the poverty trap and converge to a stable steady-state in the long run, but only if (i) the country's characteristics, such as saving rate, initial capital, governance quality, and productivity are good enough, (ii) the fixed cost is relatively low, and (iii) loans rule is generous enough. We also show that our model with foreign aid has room for endogenous cycles, unlike the standard Solow model.

Keywords: Economic dynamics, economic growth, foreign aid, development loan, grant, poverty trap, endogenous cycle.
JEL Classification: O11, O19, O41

1 Introduction

In the context of the continual increase of foreign aid allocated by the donors in the OECD-Development Assistance Committee (DAC), evaluating aid effects is ever more necessary for determining an efficient allocation. Constituting a great source of revenue for numerous recipient countries, foreign aid composes of two main forms: grants and loans (with low-interest rates). Even though donors generally adopt a mixture of two types of aid, their determinants and effectiveness are pretty different (Collier, 2006; Cohen et al., 2007; Gaibulloev and Younas, 2018). These two transfer types both accounted for the foreign aid category

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of the OECD-DAC,[1] are two complementary instruments having usually different motives from donors. In general, loans are allocated for infrastructure and development projects with obligations for repayment in the future, and grants are for social and humanitarian purposes with free transfers. For instance, one of the recommendations of the Meltzer Commission (2000) for the development banks is to "use grants instead of loans to improve the quality of life in the poorest countries".[2] However, a development loan is a sustainable instrument for ODA (Official Development Assistance). It represents a higher share of total bilateral than multilateral aid.

Table 1: Aid flows of DAC members in grant equivalent (in 2019 US$ million). Source: OECD.

Types of aid	2018	2019	2020
ODA, bilateral total	107 749.84	108 723.10	111 507.10
ODA, bilateral grants	96 041.42	95 055.58	95 321.44
ODA, bileteral loans	9 055.78	9 812.36	12 586.06
ODA, multilateral total	43 013.39	42 959.50	45 518.85
ODA, multilateral grants	41 513.74	42 116.42	44 377.86
ODA, multilateral loans	1 530.94	843.09	1 140.98

Both theoretical and empirical studies analyze aid effectiveness by investigating the effect of the total aid or the grant component of aid. The aid effectiveness literature pays little attention to development loans while this aid component is on a rising proportion of total aid flows. As a matter of fact, the increase in ODA gross disbursements to the least developed countries since 2011 is due to a rise in ODA loans, whereas grants have remained essentially stagnant.[3] Table 1 indicates that bilateral loans in 2020 by DAC members on a grant equivalent basis increased by 28.27% compared to 2019, while multilateral loans, even with a lower amount, increased by 26.1%. For bilateral aid, as shown in Figure 1, loans made up almost two-thirds of Japan's bilateral ODA (61.2% in 2017), while France disburses nearly half of its bilateral ODA as loans.

Motivated by the fact that the literature of aid effectiveness has paid little attention to development loans, our paper aims to explore the effects of foreign aid (in both forms: grant and loan) on the economic dynamics of the recipient countries.[4]

To address this question, we introduce two new ingredients (a fixed cost and a development aid) in a growth model à la Solow. The fixed costs in developing countries may be due to the lack of essential infrastructure such as road, rail, access to electricity, etc.[5] Con-

[1]From 2018, the ODA grant-equivalent methodology is used, whereby only the amount given by lending below market rates, is counted as ODA (https://data.oecd.org/oda/net-oda.htm).

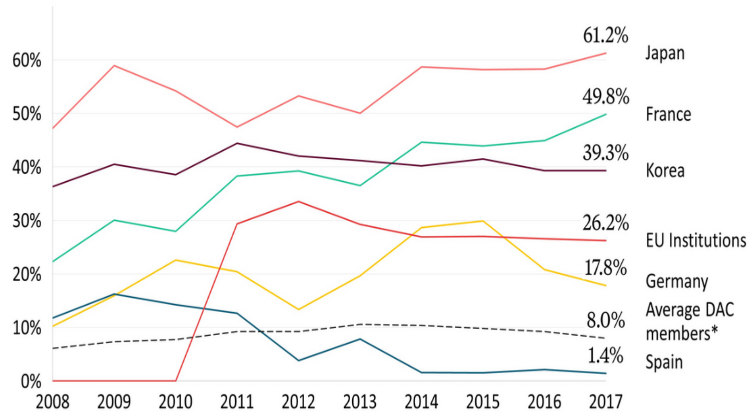
[2]The Meltzer Commission: The future of the IMF and World Bank (page 7).

[3]For more details, see "Least developed countries Report 2019"-UNCTAD.

[4]A number of studies examine the aid effectiveness in terms of economic growth, poverty reduction, income distribution, or fiscal behavior, etc. See for instance, Burnside and Dollar (2000), Guillaumont and Chauvet (2001), Ouattara (2006), Guillaumont and Wagner (2014), Morissey (2015), Maruta et al. (2020) for empirical studies and Charterjee et al. (2003), Charteerjee and Tursnovky (2007), Dalgaard (2008), Pham and Pham (2020) for theoretical growth models with foreign aid (in form of grant).

[5]See Gurara et al (2017) and references therein for more empirical evidence showing the role of essential infrastructures on economic development.

Figure 1: Share of bilateral ODA disbursed as loans, from "the OECD's new way of counting ODA loans - what's the impact?" (https://donortracker.org/. *: Average includes DAC donor countries and the EU institutions)



cerning development aid, the country can promote its investment by borrowing from foreign organizations. The gross interest rate R is non negative; when $R = 0$, the development aid is a grant. The aid rule imposes two constraints: (i) the country can access development aid only if its capital stock is not high (in the sense that it is lower than the so-called *eligibility threshold*), and (ii) the aid amount cannot exceed an exogenous limit imposed by lenders.[6] To sum up, the aid rule is represented by three exogenous variables: the eligibility threshold, the interest rate, and the borrowing limit.

This framework generates a two-dimensional non-linear dynamical system. Our contribution is to explore the global dynamics of the economy by studying this system. First, we find that in the absence of aid, the presence of the fixed cost generates a poverty trap which is a threshold such that the economy cannot overcome (resp., can converge to a steady state which is higher than this threshold) if its initial capital stock is lower (resp., higher) than this threshold.

Then, and more importantly, we investigate conditions under which development aid may enable the recipient country to escape from its poverty trap. Given a low-income country that faces a poverty trap if there is no foreign aid, we characterize different scenarios depending on the aid rule and the country's characteristics.

(i) Suppose the fixed cost is too high and the maximum amount of aid is low (so that even if the country borrows the maximum level to promote its investment, its capital stock cannot overcome the fixed cost). In that case, the country will collapse, whatever the level of productivity. Moreover, although the country is eligible to access development loans, it does not borrow at any time. This finding implies that when the maximum amount of loan is very limited, and donors want to take care of the recipient country, they should offer a grant rather than a loan because a permanent grant allows the country to avoid a collapse while loans cannot. This means that aid composition

[6] Our paper is also related to the large literature on foreign borrowing and growth. However, our first constraint of the loans rule makes our framework different from studies in this literature.

matters for the economic growth of the recipient country.[7]

(ii) If the fixed cost is not so high and the rule of aid is quite generous, the country can avoid collapse by borrowing the maximum amount to promote investment. This can happen if the productivity and the governance quality have a middle level. In this scenario, although there are two different steady states, the economy converges to a high steady state, which is also higher than that in the situation without aid. Notice that the country is dependent on the foreign aid (in the sense that it borrows at any period) because the capital stock is always lower than the eligibility threshold.

(iii) If the fixed cost is not so high, the rule of aid is quite generous, and more importantly, the productivity and the governance quality are good, then the economy converges to a steady state. Moreover, this steady state is higher than the eligibility threshold and does not depend on foreign aid. In this scenario, the country does not borrow development loans in the long run. It just needs development loans for the first stage of the development process.

(iv) Finally, we show that an endogenous cycle may appear. This scenario happens if the rate of capital depreciation is high, the productivity is low, and the rule of aid is generous (i.e., with a high borrowing limit and a low interest rate). The idea is simple: when the country has a low income, it can borrow from a development loan having a low interest rate to promote its investment, and the income goes up. When the income is higher than the eligibility threshold, the country can no longer borrow, and then the income goes down because of low productivity and a high depreciation rate. Hence, a cycle arises. In this case, the recipient country can escape from collapsing but fall in the very special middle income trap which is the two-period cycle.

It should be noted that our above analyses still apply to the case of grants because grants can be considered as a particular case of loans (with zero interest rate). To the best of our knowledge, our paper is the first one focusing on the loan component of foreign aid and its impacts on the transitional dynamics of the recipient country, in particular, the possibility of escaping the poverty trap and of endogenous cycles.

Our paper is in line with theoretical studies examining the effectiveness of foreign aid (Charterjee et al., 2003; Charteerjee and Tursnovky, 2007; Dalgaard, 2008; Pham and Pham, 2020).[8] However, these papers focus on the case of grants. By contrast, our results apply

[7] Gupta et al. (2004) suggest that aid composition matters for the relationship between the aid and the fiscal revenue. Indeed, Gupta et al. (2004) examine the effects of grants and loans on the recipient revenue effort using a data with 107 recipient countries during 1970-2000. They shows that an increase in loans is associated with higher tax effort, whereas an increase in grants is associated with weaker tax effort in recipient countries. These effects vary across countries and depend on the quality of institutions.

[8] For instance, Charterjee et al. (2003), Charteerjee and Tursnovky (2007) use continuous-time growth models to analyze the economic dynamics around the steady-state and show the contrasting effects of untied and tied foreign aid on the recipient macroeconomic performance. Dalgaard (2008) considers an OLG growth model where households live for two periods while Pham and Pham (2020) uses a discrete-time general equilibrium framework with AK technology. In both Dalgaard (2008) and Pham and Pham (2020), the capital path can be recursively described with a capital stock at period $t+1$ depending on that at period t. Pham and Pham (2020) underline, however, the role of the recipient fiscal policy and government effort in financing public investment and the efficiency of aid use.

for both grants and loans. We also note that the amount of development aid in our paper is endogenous and chosen by the recipient country but not by donors as in the above studies. In our paper, donors impose the limit of aid, the interest rate, and the eligibility threshold.

Our paper is also related to the literature on the poverty trap.[9] For instance, it is related to Le Van et al. (2016) who use an optimal growth model without foreign aid to study how the country can avoid the poverty trap. In their paper, the existence of a poverty trap is due to high fixed costs or lack of infrastructure, being unfavorable for economic development. Under mild conditions on the efficiency of the infrastructure technology and the initial level of fixed costs, Le Van et al. (2016) shows that a poverty trap can be avoided. We complement Le Van et al. (2016) by pointing out that foreign aid (in the form of loans or grants) may contribute to preventing such a poverty trap. Moreover, unlike Le Van et al. (2016) where the capital stock converges, an endogenous cycle may arise in our model due to the introduction of a development loan.

If we interpret the agent in our growth model as a micro-entrepreneur, our paper has a link with the literature on poverty dynamics (see Diwakar and Shepherp (2021) and references therein). Diwakar and Shepherp (2021) uses the q-squared method to investigate why some households' escapes from poverty are sustained while others escape only to fall back into poverty (a transitory escape) in several countries (Bangladesh, Cambodia, Ethiopia, Kenya, Nepal, the Philippines, Rwanda, Tanzania, and Uganda). Diwakar and Shepherp (2021) point out that the ability to manage risks to livelihoods and (tangible and intangible) assets makes the difference between a sustained and a temporary escape from poverty. They then argue it is critical to enable the environment (economic and social policies and norms, education and health systems, financial infrastructure, ...).

Unlike Diwakar and Shepherp (2021), we use growth models and prove that fixed costs (lack of essential infrastructure such as road, rail, electricity, ...), productivity (including human capital, quality of machines, ...), and credit markets play an essential role in getting sustained poverty escapes.

The remainder of the paper is organized as follows. Section 2 describes our framework and characterizes its economic dynamics when there is no development loan. Section 4 focuses on the endogenous choice of loans and its impacts in such an economy with a poverty trap. Section 5 concludes. Technical proofs are presented in Appendix A.

2 A Solow growth model with development aid and fixed cost

For the sake of tractability, we consider a model à la Solow.[10] The population size is constant over time and normalized to unity. At each date t, the wealth W_t (see Section 2.2) is divided

[9]For poverty trap, see Azariadis and Stachurski (2005) and Balboni et al. (2021) and references therein.

[10]Adopting a model à la Solow with exogenous saving rate allows us to examine the global dynamics and the cycles, contrary to optimal growth models à la Ramsey, where we can only explicitly compute the optimal allocation with some specific utility and production functions. Moreover, in a Ramsey model, it is difficult to study the global dynamics without the monotonicity of the capital path.

between consumption and saving.

$$c_t + S_t = W_t \tag{1a}$$
$$S_t = sW_t \tag{1b}$$
$$k_{t+1} = (1-\delta)k_t + I_t \tag{1c}$$

where c_t, S_t, I_t are consumption, saving, and investment at the period t ($t = 0, 1, \ldots, +\infty$), $s \in (0, 1)$ is the exogenous saving rate. k_t represents the physical capital stock at date t ($k_0 > 0$ is given) while $\delta \in [0, 1]$ is the capital depreciation rate.

We introduce two new ingredients to the standard Solow model: (1) fixed cost in the production function and (2) development aid.

2.1 Production function with fixed cost

We now describe the form of production function F. Following Le Van et al. (2016), we introduce a fixed cost in the production process.

Assumption 1 (production function with fixed cost)**.** *Assume that*

$$F(k_t) = \begin{cases} 0 & \text{if } k_t < b_0 \\ Af(k_t - b_0) & \text{if } k_t \geq b_0 \end{cases} \tag{2}$$

where A represents the exogenous total factor productivity, and $b_0 \geq 0$ the fixed cost. The function f is strictly increasing, concave. Moreover, $f'(0) = \infty$, $f'(\infty) = 0$, and $f(0) = 0$.

The presence of b_0 means that the economy must pay a fixed cost before production. The fixed costs in some developing countries may be due to the lack of essential infrastructure such as roads, rail, access to electricity, which are documented by empirical studies. For instance, according to Gurara et al (2017) and the World Bank's Enterprise Surveys over the period 2006-2016, there are 43% (resp., 24%) of firms in low-income developing countries that identify access to electricity (resp., transportation) as a significant constraint to their business activity (see Le Van et al. (2016), Gurara et al (2017) for more discussions).

However, it should be noticed that our model also covers the case where $b_0 = 0$, which corresponds to the standard production function.

2.2 Development aid

We consider that the total investment of the recipient country at t equals the sum of its savings and a fraction of foreign capital flow:

$$I_t = S_t + \lambda a_t \tag{3}$$

where a_t represents the foreign capital flow while $\lambda \in [0, 1]$ is an exogenous parameter. Notice that $(1 - \lambda)a_t$ measures the amount of foreign aid, which is wasted due to, for example, a corruption problem or an inefficient use.[11] So, λa_t can be interpreted as the efficient amount of aid.

The model in Chenery and Strout (1966) corresponds to the case where $\lambda = 1$ and a_t equals a fraction of the recipient country's GNP.[12]

We now describe how the amount a_t is endogenously determined.

1. First, a_t equals zero if the capital stock of the recipient country at date t exceeds an exogenous threshold b_1: $k_t \geq b_1$. This assumption may be illustrated, for instance, by the case of countries changing their income category from one period to another, such as South Korea. South Korea was a recipient during 1960-1990, after the Korean war (1950-1953), and experienced high economic growth. South Korea is now a developed country and has became one of the thirty members of the OECD-DAC since 2010. Other countries such as China and Argentina see their net ODA (in % of GNI) becoming null in recent years (2016 for Argentina and 2010 for China).

2. Second, when $k_t < b_1$, the recipient country can borrow from foreign organizations, but it cannot borrow more than a borrowing limit $\bar{a}$, which is exogenous. The amount a_t is chosen by the recipient country so that it maximizes the country's wealth in the next period. Formally, a_t equals the solution of the following problem:

$$(P_t): \quad W_{t+1} \equiv \max_{0 \leq x \leq \bar{a}} \Big\{ F(k_{t+1}) - Rx \Big\} = \max_{0 \leq x \leq \bar{a}} \Big\{ F[(1-\delta)k_t + S_t + \lambda x] - Rx \Big\}$$

   where k_t and S_t are taken as given, $R \geq 0$ is the exogenous gross interest rate imposed by the donor (when $R = 0$, we recover grants).[13] As an example, Japan applies fixed net interest rates of 0.10%, 0.15%, 0.20% or 0.25% to low-income least developed countries (i.e., income per capita below US$ 1036) for a 15, 20, 25, or 30-year repayment period, respectively.[14]

To sum up, the foreign capital flow a_t is defined by the following rule:

Assumption 2 (rule of aid)**.** *We consider the following rule for the foreing aid with the interest rate R*

$$a_t = \begin{cases} x_t & \text{if } k_t < b_1 \\ 0 & \text{if } k_t \geq b_1 \end{cases} \tag{4}$$

where x_t is the solution of the problem (P_t).

[11]We can write $a = a_u + a_i$ with $a_u = (1-\lambda)a$ and $a_i = \lambda a$, which is used to finance public expenditures. Pham and Pham (2019) endogenize a_i and a_u in a political-economic analysis in which the recipient government chooses a_i, a_u and a tax effort given the amount of a.

[12]Charterjee et al. (2003), Charteerjee and Tursnovky (2007) also assume that $\lambda = 1$ and a_t equals a fraction of the recipient country's output. See Dalgaard (2008), Carter (2014), Pham and Pham (2020) for other rules.

[13]Here, we do not focus on the case where the recipient country can lend.

[14]Interest rates are higher for the lower and upper-middle income countries. See *https://www.jica.go.jp/english/our_work/types_of_assistance/oda_loans/standard/2022_1.html*

The aid rule is represented by three exogenous parameters $(b_1, \bar{a}, R)$. Our modeling of foreign aid is related to the huge literature on the relationship between foreign borrowing and growth (see, for instance, Uribe and Schmitt-Grohé (2017), Aguiar and Amador (2021)). While the borrowing constraint $a_t \leq \bar{a}$ has been used, our constraint "$a_t = 0$ if $k_t \geq b_1$" has not been focused by the literature of international borrowing.

2.3 Dynamical system of capital path

According to the setup above, the capital accumulation becomes:

$$k_{t+1} = (1-\delta)k_t + sW_t + \lambda a_t = \begin{cases} (1-\delta)k_t + sW_t + \lambda x_t & \text{if } k_t < b_1 \\ (1-\delta)k_t + sW_t & \text{if } k_t \geq b_1 \end{cases} \tag{5}$$

while the recipient's wealth at date $t+1$ is determined by:

$$W_{t+1} = \begin{cases} \max_{0 \leq x \leq \bar{a}} \Big\{ F[(1-\delta)k_t + S_t + \lambda x] - Rx \Big\} & \text{if } k_t < b_1 \\ F[(1-\delta)k_t + sW_t] & \text{if } k_t \geq b_1 \end{cases} \tag{6}$$

Here, we assume, by convention, that $W_0 = F(k_0)$ is exogenously given.

When there is no loan (i.e., $a_t = 0$, $\forall t$), the wealth becomes the production (i.e., $W_t = F(k_t)$, $\forall t$), and hence we recover the standard capital accumulation: $k_{t+1} = (1-\delta)k_t + sF(k_t)$.

Remark 1 (grants versus loans)**.** *When $R = 0, x_t = \bar{a}$ $\forall t$, we recover the case of grants. The foreign aid amount verifies the following rule:*

$$\textit{(Rule of grant):}\ a_t = \begin{cases} \bar{a} & \textit{if } k_t < b_1 \\ 0 & \textit{if } k_t \geq b_1 \end{cases} \tag{7}$$

and the dynamics of capital follows a one-dimensional dynamical system

$$k_{t+1} = \begin{cases} (1-\delta)k_t + sF(k_t) + \lambda\bar{a} & \textit{if } k_t < b_1 \\ (1-\delta)k_t + sF(k_t) & \textit{if } k_t \geq b_1 \end{cases} \tag{8}$$

We introduce the notion of poverty trap.

Definition 1 (collapse and poverty trap)**.**

1. *The economy collapses if* $\lim_{t\to\infty} k_t = 0$.
2. *A value $\bar{k}$ is called a poverty trap if, for any initial capital stock $k_0 \leq \bar{k}$, and we have $k_t \leq \bar{k}$ for any t high enough.*

Our formal definition of poverty trap means that a poor country ($k_0 \leq \bar{k}$) continues to be poor. It is in line with the notion of the poverty trap in Azariadis and Stachurski (2005): A poverty trap is a self-reinforcing mechanism that causes poverty to persist.

Our main objective is to explore the impact of aid rule $(b_1, \bar{a}, R)$ and the fixed cost b_0 on the economic dynamics of the recipient.

3 Preliminary results

3.1 Economic dynamics without development aid

In the absence of development loans, the following result shows the transitional dynamics and the existence of a poverty trap in this economy.

Proposition 1 (poverty trap without development aid). *In the absence of foreign capital flow, the dynamics of capital become*

$$k_{t+1} = \begin{cases} (1-\delta)k_t & \textit{if } k_t < b_0 \\ (1-\delta)k_t + sAf\big(k_t - b_0\big) & \textit{if } k_t \geq b_0 \end{cases}. \tag{9}$$

Denote $M_f \equiv \max_{k\geq 0} \big\{sAf((k-b_0)^+) - \delta k\big\}$. *Let* $b_0 > 0$.

1. *If* $M_f < 0$, *then the economy collapses, i.e.,* $\lim_{t\to\infty} k_t = 0$ *for any* $k_0 > 0$.
2. *If* $M_f > 0$, *then* $k = (1-\delta)k + sAf\big(k-b_0)^+\big)$ *has two positive solutions,* $k^0_{low} < k^0_{high}$, *and these solutions are higher than* b_0.
   (a) *If the initial capital* k_0 *is strictly lower than* k^0_{low}, *then* k_t *converges to zero.*
   (b) *If the initial capital* k_0 *is strictly higher than* k^0_{low}, *then* k_t *converges to* k^0_{high}.
3. *If* $M_f = 0$, *then* $k = (1-\delta)k + sAf\big(k-b_0)^+\big)$ *has a unique positive solution, denoted by* k_s.
   (a) *If* $k_0 < k_s$, *then* k_t *converges to zero.*
   (b) *If* $k_0 \geq k_s$, *then* k_t *converges to* k_s.

Proof. See Appendix A. □

Figure 2 illustrates the transitional dynamics of capital in different cases. First, in the absence of fixed cost, we recover the standard Solow model (see the graph on the left side of Figure 2). In this case, the capital stock converges to a unique steady-state.

Second, the presence of the fixed cost b_0 generates rich dynamics. Point 1 of Proposition 1 provides a necessary and sufficient condition under which the economy collapses, whatever its initial capital level. This scenario, corresponding to the middle graph in Figure 2, happens when $M_f < 0$, i.e., the recipient country's characteristics are not favorable for the development process: technology level A and saving rate are low, the lack of infrastructure is important (i.e., fixed costs b_0 very high), the depreciation rate of capital δ is high.

When conditions in technology and investment are good enough (given δ) so that $M_f > 0$, point 2 of Proposition 1 indicates the existence of two steady-states k^0_{low}, k^0_{high}, which are shown in the graph on the right hand side. According to Proposition 1 and Definition 1, the low steady-state k^0_{low} is considered as a poverty trap for this economy. More precisely, for all initial capital k_0 higher than k^0_{low}, the economy will be on a convergent path towards the high steady-state k^0_{high}. In contrast, the economy will collapse for all initial capital k_0 lower than k^0_{low}, even if k_0 is higher than the fixed cost b_0.

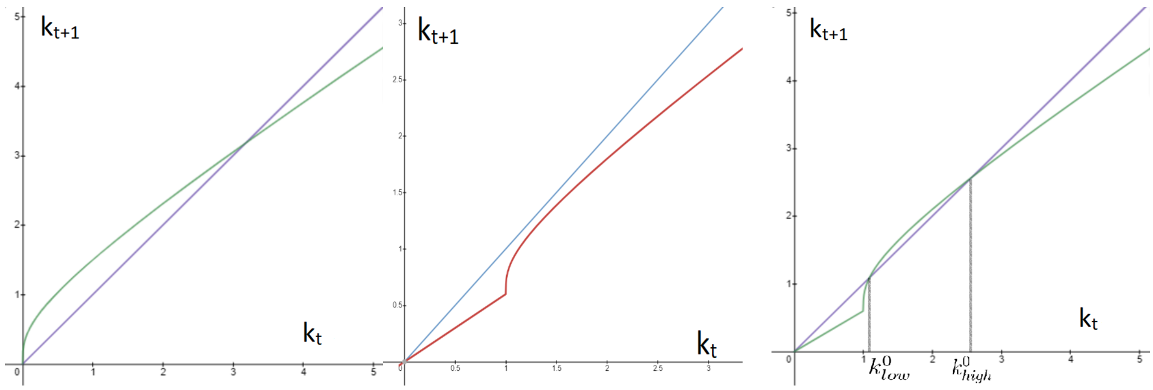


Figure 2: Dynamics of capital. LHS: without fixed cost. Middle: with fixed cost and low investment (no trivial steady-state). RHS: with fixed cost and high investment (two steady-states).

3.2 Static analysis: optimal choice of loans

In this section, we study the optimal choice of the recipient country regarding foreign loans for a given period. We obtain the following result.

Proposition 2 (optimal borrowing)**.** *Assume that the technology verifies Assumption 1. We consider the problem (P_t).*

$$(P_t): \quad \max_{0 \le x \le \bar{a}} \left\{ Af\Big(\big[(1-\delta)k_t + S_t + \lambda x - b_0\big]^+ \Big) - Rx \right\}$$

The optimal borrowed amount x_t is determined as follows:

1. $x_t = 0$ *if and only if one of the two following conditions holds:*
   - *(a)* $(1-\delta)k_t + S_t + \lambda\bar{a} \le b_0$.
   - *(b)* $(1-\delta)k_t + S_t \ge b_0$ *and* $\lambda A f'\big[(1-\delta)k_t + S_t - b_0\big] < R$.
2. $x_t = \bar{a}$ *if and only if* $(1-\delta)k_t + S_t + \lambda\bar{a} > b_0$ *and* $\lambda A f'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] \ge R$.
3. $x_t = a_t^*$, *where a_t^* is uniquely determined by* $(1-\delta)k_t + S_t + \lambda a_t^* - b_0 > 0$ *and* $\lambda A f'\big((1-\delta)k_t + S_t + \lambda a_t^* - b_0\big) = R$, *if and only if one of two following conditions holds:*
   - *(a)* $(1-\delta)k_t + S_t \ge b_0$ *and* $\lambda A f'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] < R < \lambda A f'\big[(1-\delta)k_t + S_t - b_0\big]$.
   - *(b)* $(1-\delta)k_t + S_t < b_0 < (1-\delta)k_t + S_t + \lambda\bar{a}$ *and* $\lambda A f'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] < R$.

Proof. See Appendix A. □

Proposition 2 provides several insights. First, given the loans rule $(b_1, \bar{a}, R)$, a country does not necessarily choose the maximum amount $\bar{a}$ proposed by the donors. Indeed, point 1 shows us that the country does not borrow if either (i) the fixed costs are too high so that even if the country borrows the maximum level, the capital stock cannot overcome the fixed costs and/or (ii) the TFP A and the governance quality are low given the repayment rate R.

Second, the country borrows the maximum level if the TFP and the governance quality are relatively high, and the capital stock augmented by loans exceeds the fixed costs so that the problem related to the lack of infrastructure is solved and the production process (function (2)) takes place. In other words, we can state that given the loans rule $(b_1, \bar{a}, R)$, the recipient's choice of loans strongly depends on lack of infrastructure, its institutional and macroeconomic performance (TFP, capital productivity, and governance quality), as well as on domestic investment rate and depreciation of capital.

4 Main results: global dynamics

This section focuses on the global dynamics of the economy in the presence of development aid.

4.1 The country is eligible for development aid but does not borrow

First, we provide conditions under which the recipient country is eligible for development loan but it chooses a null amount.

Proposition 3 (no borrowing)**.** *Let Assumptions 2 and 1 be satisfied.*

1. *(High fixed cost and limited loans). Assume that $R > 0$, $\lambda\bar{a} < \delta b_0$, and $(1-\delta)k_0 + sW_0 + \lambda\bar{a} \leq b_0$. Then, we have that:*
   - *(a) The country never borrows: $a_t = 0$, $\forall t \geq 0$.*
   - *(b) $k_t < b_0$, $k_{t+1} = (1-\delta)k_t$, $\forall t \geq 1$, and hence k_t converges to zero.*
2. *(No fixed cost and high interest rate) Assume that $b_0 = 0$. Then the equation $k = (1-\delta)k + sAf(k)$ has a unique positive solution, denoted by k^*.*

   Assume that $sAf'(k^*) < R$ and $sAf'\big((1-\delta)k_0 + sAf(k_0)\big) < R$. Then, we have that:
   - *(a) The country never borrows: $a_t = 0$, $\forall t \geq 0$.*
   - *(b) $k_{t+1} = (1-\delta)k_t + sAf(k_t)$, $\forall t \geq 1$, and hence k_t converges to k^*.*

Proof. See Appendix A. □

According to Proposition 3, there are two situations where the recipient country does not borrow. The first scenario happens when the fixed costs b_0 and capital depreciation δ are relatively high, and other macroeconomic indicators (saving rate, TFP, governance quality) are bad. Meanwhile, the maximum loan allocated by the lenders could be relatively low (in the sense that $\lambda\bar{a} < \delta b_0$ and $(1-\delta)k_0 + sW_0 + \lambda\bar{a} \leq b_0$). In this case, it is better for the country to not borrow because even though the country borrows the maximum amount $\bar{a}$, its capital stock is never enough to overcome the fixed costs b_0, which implies that the capital stock decreasingly converges to zero. The second scenario indicates that the country never borrows even there is not fixed cost (i.e., $b_0 = 0$). This happens when the interest rate R is high.

Proposition 3 leads to an important implication: if the donors take care of the recipient countries' development, then they should sets a low interest rate or/and offer a grant (which corresponds to the case $R = 0$) rather than a loan (i.e., $R > 0$) because a permanent grant helps the country to prevent a collapse.[15] Our point is in line with the practical policies of the French Development Agency (AFD) - a development bank and an implementing agency. Indeed, the AFD differentiates recipient countries based on their income level: it provides loans to emerging economies and grants to low-income countries.[16]

Our insight helps us to explain why there are no loans (or very low amounts of loans) but only free grants in some very low-income countries such as Burundi, Central African Republic, Malawi, Afghanistan since 2000 as shown in Figure 3.[17]

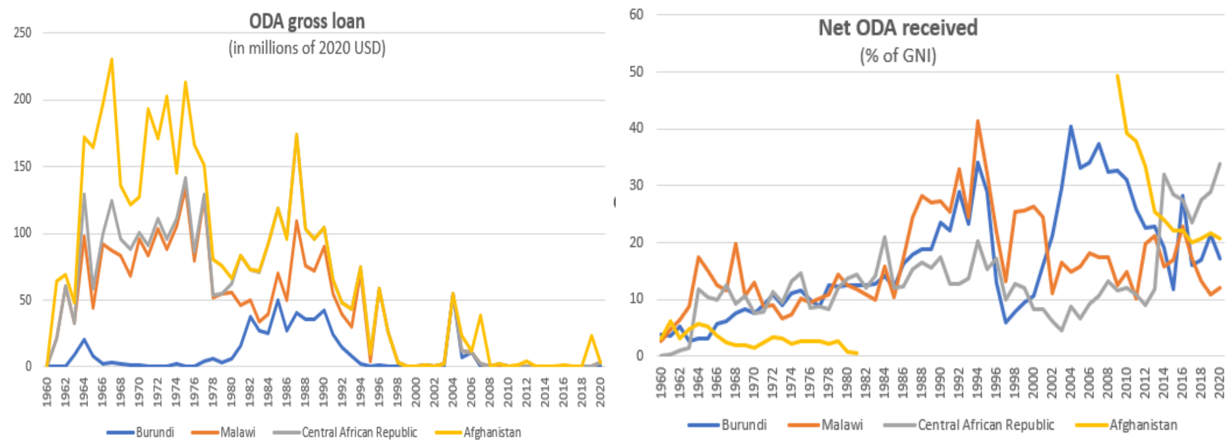


Figure 3: Evolution of ODA gross loan and net ODA received during the period 1960-2020.

Actually, these countries are grant-dependent. For decades, their received foreign aid (in the form of grants) is always a considerable ratio in their GNI as shown in Figure 3. Notice that these recipients are in the group of the world's poorest countries. For instance, according to the database of the World Bank, the GNI per capital in 2015 of Burundi, Central African Republic, Malawi, Afghanistan is 305.5, 367.5, 382.1, 561 (2015 USD). Foreign grants improve their domestic revenues and investment but cannot allow them to escape from poverty.

4.2 Development aid and global dynamics

We now wonder whether the country should borrow and what would happens along the transitional dynamics. First, we look at the steady-states.

[15]Indeed, in the case of grant ($R = 0$, $a_t = \bar{a}\ \forall t$), our model becomes (7), (8). According to (8), we can see that the capital stock cannot converge to zero whatever the level of fixed cost b_0. In other words, a collapse is avoided thanks to permanent grants.

[16]AFD disburses 95% of France's ODA loans (2017) (The OECD's new way of counting ODA loans-what's the impact?-https://donortracker.org/)

[17]Source: https://stats.oecd.org/. Data is downloaded on November 10, 2022.

Definition 2. *A steady-state of this economy in the presence of aid is a couple (k, a) such that a sequence of capital stock and foreign capital flow $(k_{t+1}, a_t)_{t\geq 0}$ determined by $k_t = k, a_t = a$, $\forall t \geq 0$ satisfies the dynamical system (5) and (6).*

We focus here on the steady-state with $a = \bar{a} > 0$ because the steady-state with $a = 0$ has been already described in Proposition 1.

Lemma 1. *Denote $H(k) \equiv (1-\delta)k + s\big(Af(k-b_0) - R\bar{a}\big) + \lambda\bar{a}$.*

1. *(k, a) is a steady state with $a = \bar{a}$ if and only if*

$$k = H(k) \tag{10}$$
$$b_0 < k < b_1 \tag{11}$$
$$\lambda A f'(k-b_0) \geq R \tag{12}$$

2. *The equation $k = H(k)$ has at most two solutions on $[b_0, \infty)$. By consequence, there are at most two steady states with $a = \bar{a}$.*

3. *If $(\lambda - sR)\bar{a} + \max_{k\geq b_0}\big\{sAf((k-b_0)^+) - \delta k\big\} > 0$ and $(\lambda - sR)\bar{a} - \delta b_0 < 0$, then the equation $k = H(k)$ has two solutions (denoted by k^a_{low}, k^a_{high}, with $k^a_{low} < k^a_{high}$) on $[b_0, \infty)$. Moreover,*

   (a) For $k \in [k^a_{low}, k^a_{high}]$, we have $H(k) \geq k$.

   (b) For $k \in [b_0, k^a_{low}]$ or $[k^a_{high}, \infty)$, we have $H(k) \leq k$.

Notice that the existence of the steady-state $(k, \bar{a})$ requires that $b_0 < k < b_1$. Condition $k > b_0$ ensures a positive output while $k < b_1$ implies that this country is always eligible for receiving foreign aid at the steady-state. In addition, condition $\lambda A f'(k - b_0) \geq R$ means that the marginal productivity of loan is higher or equal to its marginal cost, justifying a positive loan amount.

Look at point 3 of Lemma 1. Observe that k^a_{low}, k^a_{high} depend on $s, A, \delta, b_0, \lambda$ and the loans rule $(b_1, \bar{a}, R)$. Moreover, k^a_{low} is decreasing but k^a_{high} is increasing in $\bar{a}$. Observe also that:

$$k^a_{low} < k^0_{low} < k^0_{high} < k^a_{high}, \tag{13}$$

where recall that k^0_{low} and k^0_{high}, defined in Proposition 1, correspond to the two steady-states in the absence of foreign aid. Indeed, when $\bar{a} = 0$, k^a_{low} becomes k^0_{low} while k^a_{high} becomes k^0_{high}.

Let k^{bind} be defined by $\lambda A f'(k^{bind} - b_0) = R$ and $k^{bind} > b_0$ (when $R = 0$, we define $k^{bind} = \infty$). Condition $\lambda A f'(k_t - b_0) \geq R$ becomes $k_t \leq k^{bind}$ as the function f' is decreasing.

We are now ready to state our result.

Proposition 4 (avoiding a collapse with foreign aid at every period)**.** *Let Assumptions 1 and 2 be satisfied. Assume that $(\lambda - sR)\bar{a} + \max_{k\geq b_0}\big\{sAf((k-b_0)^+) - \delta k\big\} > 0$ and $(\lambda - sR)\bar{a} - \delta b_0 < 0$. Assume also that $0 < b_0 < k^a_{low} < k^a_{high} < b_1 < k^{bind}$.*

1. *If* $(1-\delta)k_0 + sW_0 < k^0_{low}$ *and there is no aid, then* k_t *converges to zero.*
2. *If*

$$k^a_{low} < (1-\delta)k_0 + sW_0 + \lambda\bar{a} < k^a_{high} \tag{14}$$

then the country always borrows the maximum level ($a_t = \bar{a}$, $\forall t \geq 0$)*, and* k_t *increasingly converges to* k^a_{high} *(which is lower than the eligibility threshold* b_1*).*

Proof. See Appendix A. □

Conditions $(\lambda - sR)\bar{a} - \delta b_0 < 0$ and $(\lambda - sR)\bar{a} + \max_{k\geq b_0}\{sAf((k-b_0)^+) - \delta k\} > 0$ (as stated in Lemma 1) guarantee that equation $k = H(k) \equiv (1-\delta)k + s\big(Af(k-b_0) - Rx\big) + \lambda x$ has two solutions k^a_{low} and k^a_{high}. Condition $k^a_{high} < b_1$ implies that the country is always eligible for receiving loans in the long run while condition $b_1 < k^{bind}$ ensures that the gross interest rate R is quite low.

Proposition 4 leads to an important implication: Consider a poor country in the sense that $(1-\delta)k_0 + sW_0 < k^0_{low}$. In this case, point 1 indicates that the country collapses if there is no foreign aid. However, point 2 shows that when there is foreign aid with a low interest rate and a middle value of efficient amount of aid $\lambda\bar{a}$ (in the sense that condition (14) holds), the country will borrow the maximum amount of loan which improves this country's investment.[18] This allows the country to overcome the fixed cost, to escape from the poverty trap and to converge to the high steady state k^a_{high}.

In Proposition 4, the recipient country borrows at every period. We now provide conditions under which the recipient country only borrows at the first stage of its development process.

Proposition 5 (Escaping poverty trap with foreign aid only at the first period)**.** *Let Assumptions 1 and 2 be satisfied. Assume that* $\max_{k\geq 0} sAf((k-b_0)^+) - \delta k > 0$ *and that*

$$k_0 < b_1 \tag{15a}$$
$$\lambda Af'\big((1-\delta)k_0 + sW_0 + \lambda\bar{a} - b_0\big) \geq R \tag{15b}$$
$$(1-\delta)k_0 + sW_0 + \lambda\bar{a} > \max(k_0, b_1, b_0, k^0_{low}) \tag{15c}$$
$$s\Big(Af\big((1-\delta)k_0 + sW_0 + \lambda\bar{a} - b_0\big) - R\bar{a}\Big) > \delta\big((1-\delta)k_0 + sW_0 + \lambda\bar{a}\big). \tag{15d}$$

Then k_t *converges to* k^0_{high} *which is higher than the eligibility threshold* b_1*. Moreover, the foreign aid at date 0 is* $a_0 = \bar{a}$ *but becomes zero from date* 1*:* $a_t = 0$ $\forall t \geq 1$*.*

Proposition 5 shows that foreign aid (in form of loan or grant) combined with a good macroeconomic and institutional performance can stimulate a strong dynamics of capital, which enables a recipient country to escape from the poverty trap.[19] The country needs to

[18] We can check that conditions in point 2 of Proposition 4 are satisfied for a quite large class of parameters, for example, $F(k) = A((k-b_0)^+)^\alpha$, $A = 1$, $\alpha = 0.3$, $b_0 = 0.5$, $s = 0.3$, $\delta = 0.3$, $\lambda = 0.8$, $k_0 = 1$, $W_0 = A(k_0 - b_0)^\alpha$, and the aid rule is given by $b_1 = 2.5$, $R = 1.01$, $\bar{a} = 0.2$.

[19] We can check that conditions in Proposition 5 are satisfied for a quite large class of parameters, for example, $F(k) = A((k-b_0)^+)^\alpha$, $A = 5$, $\alpha = 0.3$, $b_0 = 0.1$, $s = 0.2$, $\delta = 0.1$, $\lambda = 0.9$, $k_0 = 0.15$, $W_0 = A(k_0 - b_0)^\alpha$, and the aid rule is given by $b_1 = 0.75$, $R = 1.01$, $\bar{a} = 1$.

borrow, but only in the first period. A strong dynamics of capital in a recipient country is not only due to a high loan amount but also to its high level of TFP A, high quality of governance λ, and low fixed costs b_0, given other variables.[20] In other words, if the TFP A, the governance quality λ and the maximum level of loans $\bar{a}$ are relatively high, and the interest rate R is low enough at a given period, then the country can overcome its fixed costs b_0, and the poverty trap can be avoided. A permanent development loan is unnecessary as the capital stock converges autonomously to a high steady-state due to strong domestic savings. Besides, the country is no longer eligible for a concessional loan in the following periods because its capital stock exceeds the critical threshold b_1.

South Korea constitutes an interesting illustration of this result. Indeed, after the Korean war in the 1950s, foreign economic assistance was essential to South Korea's economy. About US$ 3 billion of grant aid was received before 1968, forming an average of 60% of the country's investment. Between 1964 and 1974, loans at concessionary interest rates averaged around 6.5% of all foreign borrowing. South Korea's economy overgrew and could not benefit from the concessional loans, but the country became increasingly integrated into the international capital market. From the late 1960s to the mid-1980s, its capital investment was financed from the competitive international market and increasingly from domestic savings. South Korea has became one of the thirty members of the OECD-DAC since 2010. For instance, according to the OECD's database, the amount of ODA provided by South Korea in 2020 is 2.3 billions of 2020 USD.

4.3 Endogenous cycles

Since the dynamical system (5-6) is non-monotonic and non-continuous, we may expect a room for endogenous fluctuation. The following result formalizes our idea.

Proposition 6 (endogenous fluctuation and cycle). *Assume that $F(k) = A(k - b_0)^+$, where $b_0 \geq 0$. Denote*

$$\gamma_1 \equiv \frac{\bar{a}\big[\lambda - sR(1-\delta)\big] - sAb_0(1-\delta)}{1-(1-\delta)(1-\delta+sA)}, \quad \gamma_0 \equiv \frac{\bar{a}\Big(\lambda(sA+1-\delta) - sR\Big) - sAb_0}{1-(1-\delta)(1-\delta+sA)} \tag{16}$$

Assume that $\gamma_1 > b_0 > b_1 > \gamma_0 > 0$, $1 > (1-\delta)(1-\delta+sA)$, $\lambda A > R$, $\lambda\bar{a} > b_0$.[21]

1. *If the initial capital stock k_0 is in the interval $(0, b_1)$, then we have, for any $t \geq 0$,*

$$a_{2t} = \bar{a}, \;\; a_{2t+1} = 0 \tag{17a}$$

$$k_{2t+1} > b_0 > b_1 > k_{2t} \tag{17b}$$

$$k_{2t+1} = (1-\delta)k_{2t} + \lambda\bar{a} \tag{17c}$$

$$k_{2t+2} = (1-\delta)k_{2t+1} + s\Big(A(k_{2t+1} - b_0) - R\bar{a}\Big) \tag{17d}$$

$$\lim_{t\to\infty} k_{2t+1} = \gamma_1 > b_0 > b_1 > \gamma_0 = \lim_{t\to\infty} k_{2t}. \tag{17e}$$

[20] We do not introduce in our framework variables representing macroeconomic performance (debt policy and management, trade and financial sector, etc.), which can explain an economy's dynamics. These factors are assumed to be exogenous and may be included in TFP A and the fixed costs b_0.

[21] We can easily check that these conditions are are satisfied for many parameters.

2. If $k_0 > b_0$ and $(1-\delta)k_0 + sW_0 < b_1$, then we have, for any $t \geq 0$,

$$a_{2t+1} = \bar{a}, \;\; a_{2t} = 0 \tag{18a}$$

$$k_{2t} > b_0 > b_1 > k_{2t+1} \tag{18b}$$

$$k_{2t+2} = (1-\delta)k_{2t+1} + \lambda\bar{a} \tag{18c}$$

$$k_{2t+1} = (1-\delta)k_{2t} + s\Big(A(k_{2t} - b_0) - R\bar{a}\Big) \tag{18d}$$

$$\lim_{t\to\infty} k_{2t} = \gamma_1 > b_0 > b_1 > \gamma_0 = \lim_{t\to\infty} k_{2t+1}. \tag{18e}$$

Proof. See Appendix A. □

The key factors generating a two-period cycle and an endogenous fluctuation in Proposition 6 are: (i) a high depreciation rate δ, (ii) a low productivity A, and (iii) a generous rule of loans (a high borrowing limit $\bar{a}$ and a low interest rate R) and the use of foreign loan (parameter λ) is quite good. Let us explain the mechanism of the cycle. At the initial period, the country's capital stock k_0 is lower than the threshold b_1. So, the country can borrow. Moreover, it borrows the maximum amount $\bar{a}$ because the interest rate R is low (i.e., $R < \lambda A$). This choice, in turn, implies that the capital stock at the first period k_1 becomes higher than the threshold b_1, which prevents the country from borrowing. Since the depreciation rate is high and the productivity is low, the capital stock in the second period k_2 goes down. Therefore, a two-period cycle arises.

Figure 4 illustrates our mechanism of fluctuations.[22] Here, we simulate by setting the thresholds $b_0 = 0.85, b_1 = 0.75$, the saving rate $s = 0.3$, the productivity $A = 1.15$, the interest rate $R = 1.01$, the borrowing limit $\bar{a} = 1.2$, and the parameter $\lambda = 0.9$. The initial capital $k_0 = 0.6$ while the depreciation rate is high: $\delta = 0.5$.[23] In this case, we can compute that $\gamma_0 = 0.44$ while $\gamma_1 = 1.30$.

Proposition 6's point 1 leads to an interesting implication showing the role of foreign aid. Let us consider a country who does not receive aid and whose characteristics satisfy: $b_0 > 0$, $1 > (1-\delta)(1-\delta+sA)$. In this case, the dynamics of capital is $k_{t+1} = (1-\delta)k_t + sA(k_t - b_0)^+$. Given $sA > \delta$, we can check that $\lim_{t\to\infty} k_t = 0$ if $k_0 < \frac{sAb_0}{sA-\delta}$ and $\lim_{t\to\infty} k_t = \infty$ if $k_0 > \frac{sAb_0}{sA-\delta}$. So, the presence of fixed cost b_0 generates a poverty trap $\frac{sAb_0}{sA-\delta}$. However, if this country receives foreign aid which satisfies conditions in Proposition 6, then a country can escape from collapsing but fall in the very special middle income trap which is the two-period cycle.

Recall that the dynamics of capital is given by the system (17c-17d), which is neither continuous nor increasing, as shown in Figure (4). This is the main source of the cycle in our model. In the literature, endogenous fluctuations have several sources, such as complex dynamics, sunspot equilibria, and learning (see Guesnerie and Woodford (1993) for an excellent survey of the literature on endogenous cycles). Our cycle concerns the first one, which suggests that a (simple) system can have very complex dynamics.

[22] In the general case, however, we have a two-dimensional dynamical system (see (5) and (6)), and hence, we cannot use Figure (2) to represent the dynamics of capital.

[23] High depreciation rates are observed in some developing countries. For instance, according to Bu (2006), for machinery and equipment in Ghana, the depreciation rate exceeds 0.5 for half of the firms during 1992–1993.

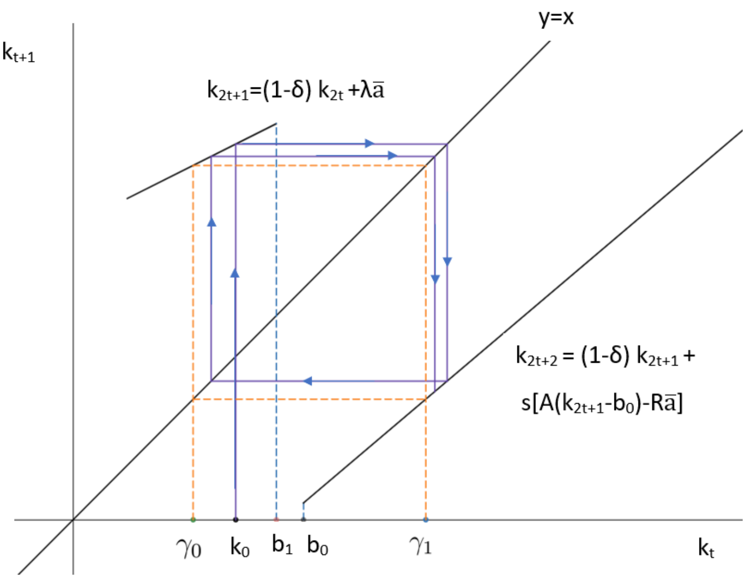


Figure 4: Dynamics of capital. For $k_0 \in (0, b_1)$, we have $\lim_{t\to\infty} k_{2t+1} = \gamma_1$, $\lim_{t\to\infty} k_{2t} = \gamma_0$.

Classically, in two-sector optimal growth models, two-period cycles endogenously arise when the factor intensity of the capital good sector is larger than the factor intensity of the consumption sector (see, for instance, Le Van and Dana (2003)). Proposition 6 contributes to the literature by showing that introducing a development aid in a Solow model can generate a two-period cycle. The cycle we obtain comes from, on the one hand, the rule of aid given by (4), which is a source of non-monotonicity of our dynamical system, and on the other hand, the fundamentals of the economy, such as its initial endowment, its TFP and the quality of its governance.[24]

Remark 2 (poverty dynamics)**.** Proposition 6 is related to an important notion in the literature of poverty dynamics: *transitory poverty escape*, i.e., a situation where people used to live in poverty, succeeded in escaping poverty, and then subsequently fell back into poverty (see Diwakar and Shepherp (2021) and references therein). For instance, according to Mariotti and Diwakar (2016), in rural Ethiopia, between 1997 and 2000, 15% of all households experienced a transitory poverty escape.

A two-period cycle in Proposition 6 can be interpreted as the situation where people succeed in escaping poverty but then falls back into it. By the way, Proposition 6 provides a theoretical explanation for the presence of transitory poverty escapes. To sum up, our results show that the rule of loans (credit market), fixed costs, productivity (including human capital, quality of machines, etc.), and governance quality are important factors for escaping poverty.

Remark 3 (endogenous cycle when there is no fixed cost)**.** When there is no fixed cost, we can also have an endogenous cycle. Indeed, let $b_0 = 0$ and consider an AK technology: $F(k) = Ak$. Denote $B \equiv 1 - \delta + sA$. Let the productivity and saving rate be low and the

[24]See Walde (2005) and references therein for endogenous growth cycles.

depreciation rate is high in the sense that $B < 1$. Then, let the interest rate R be low and the use of foreign loan (parameter λ) is quite good in the sense that $\lambda > BsR$, $\lambda B > sR$, $\lambda A > R$. Denote

$$\sigma_0 \equiv \frac{(\lambda B - sR)\bar{a}}{1 - B^2}, \quad \sigma_1 \equiv \frac{(\lambda - BsR)\bar{a}}{1 - B^2}. \tag{19}$$

Observe that $\sigma_1 > \sigma_0$. Assume that $\sigma_0 < b_1 < \sigma_1$. We prove in Appendix A that if $k_0 \in \left(\frac{b_1 - \lambda\bar{a}}{B}, \sigma_0\right)$,[25] then we have, for any $t \geq 0$,

$$a_{2t} = \bar{a},\ a_{2t+1} = 0, \quad k_{2t+1} > b_1 > k_{2t} \tag{20a}$$

$$k_{2t+1} = Bk_{2t} + \lambda\bar{a}, \quad k_{2t+2} = Bk_{2t+1} - sR\bar{a} \tag{20b}$$

$$\lim_{t\to\infty} k_{2t+1} = \sigma_1 > b_1 > \sigma_0 = \lim_{t\to\infty} k_{2t}. \tag{20c}$$

The insight of this two-period cycle and fluctuation is similar to that in Proposition 6.

5 Conclusion

We have studied the nexus between the poverty trap, economic dynamics, and international aid by introducing development loans and fixed costs in a growth model à la Solow. Our tractable framework generates several insights on the possibility of escaping the poverty trap. We indicate that a country may have a poverty trap due to the high fixed cost. In such a case, development loans or grants may be helpful for an aid recipient's development process. However, a country may choose a null loan amount as its capital dynamics cannot allow it to escape from its poverty trap. In such a situation, a collapse may be avoided if the donors offer grants without obligation for repayment.

Our results also indicate that whether or not the country can overcome the poverty trap depends not only on the rule of loans but also, and mainly, on its capacity (such as the TFP, the depreciation rate, the saving rate, and the governance quality). Moreover, under the presence of development loans, an endogenous cycle may arise.

A Appendix: Formal proofs

Proof of Proposition 1. Point 1 is obvious. Let us prove point 2 (the proof of point 3 is similar). Note that the function $sAf((k-b_0)^+) - \delta k$ is concave on $[0, \infty)$ and strictly concave on $[b_0, \infty]$. Moreover, $\lim_{k\to b_0} sAf((k-b_0)^+) - \delta k = -\delta b_0 < 0$ and $\lim_{k\to\infty} sAf((k-b_0)^+) - \delta k < 0$.

(2a) If $k_0 < k^0_{low}$, using the induction argument, we can prove that $k_t < k_{t+1}\ \forall t$. So, k_t converges to zero.

(2b) If $k_0 > k^0_{low}$, we can see that $k_0 \leq (\geq) k^0_{high}$, then k_t increasingly (decreasingly) converges to k^0_{high}.

[25]Observe that $\frac{b_1 - \lambda\bar{a}}{B} < \sigma_0$ is equivalent to $b_1 < \sigma_1$. So, conditions $\frac{b_1 - \lambda\bar{a}}{B} < k_0 < \sigma_0 < b_1 < \sigma_1$ are non-empty.

□

Proof of Proposition 2. Observe that when $(1-\delta)k_t+S_t+\lambda x > b_0$, the objective function $Af((1-\delta)k_t + S_t + \lambda x_t - b_0) - Rx_t$ has the derivative $\lambda Af'((1-\delta)k_t + S_t + \lambda x_t - b_0) - R$ which is decreasing in x_t. We now consider all possible cases:

1. If $(1-\delta)k_t + S_t + \lambda\bar{a} \leq b_0$ then $x_t = 0$. We obtain case 1.a in Proposition 2
2. If $(1-\delta)k_t + S_t \geq b_0$, there are three sub-cases:
   (a) If $\lambda Af'\big[(1-\delta)k_t + S_t - b_0\big] \leq R$, then we have $x_t = 0$. We obtain case 1.b in Proposition 2
   (b) If $\lambda Af'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] \geq R$, then we have $x_t = \bar{a}$. We obtain case 2 in Proposition 2
   (c) If $\lambda Af'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] < R < \lambda Af'\big[(1-\delta)k_t + S_t - b_0\big]$, then we have $x_t = a_t^*$ where a_t^* is uniquely determined by $\lambda Af'\big([(1-\delta)k_t+S_t+\lambda a_t^*-b_0]^+\big) = R$. We obtain case 3.a in Proposition 2
3. If $(1-\delta)k_t + S_t < b_0 < (1-\delta)k_t + S_t + \lambda\bar{a}$, then there are two sub-cases:
   (a) If $\lambda Af'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] \geq R$, then we have $x_t = \bar{a}$. We obtain case 2 in Proposition 2
   (b) If $\lambda Af'\big[(1-\delta)k_t + S_t + \lambda\bar{a} - b_0\big] < R$, then we have $x_t = a_t^*$ where a_t^* is uniquely determined by $(1-\delta)k_t+S_t+\lambda a_t^*-b_0 > 0$ and $\lambda Af'\big((1-\delta)k_t+S_t+\lambda a_t^*-b_0\big) = R$. We obtain case 3.b in Proposition 2

□

Proof of Proposition 3. Point 1. We will prove that $k_t < b_0$, $a_t = 0$ $\forall t$. Indeed, we have

$$k_1 = (1-\delta)k_0 + sW_0 + \lambda a_0 \leq (1-\delta)k_0 + sW_0 + \lambda\bar{a} \leq b_0. \tag{21}$$

Since $k_1 \leq b_0$ and $R > 0$, we have $a_0 = 0$. So, our claim holds at the initial date. Suppose now that $k_t < b_0$, $a_{t-1} = 0$. Consider date $t+1$. We have

$$k_{t+1} = (1-\delta)k_t + s\Big(Af((k_t - b_0)^+) - Ra_{t-1}\Big) + \lambda a_t \tag{22}$$

$$= (1-\delta)k_t + \lambda a_t \leq (1-\delta)b_0 + \lambda\bar{a} \leq b_0 \tag{23}$$

where the last inequality follows from $\lambda\bar{a} \leq \delta b_0$. Since $k_{t+1} \leq b_0$ and $R > 0$, we have $a_t = 0$. Therefore, our claim is true. By consequence, we have $k_{t+1} = (1-\delta)k_t, \forall t \geq 1$, and, hence, k_t converges to zero.

Point 2. Since f is strictly concave, we observe that $k > (1-\delta)k + sAf(k)$, $\forall k < k^*$ and $k < (1-\delta)k + sAf(k)$, $\forall k > k^*$.

We consider three cases:

1. Case 1: $k_0 < k^*$. By assumption, $sAf'\big((1-\delta)k_0+sW_0\big) = sAf'\big((1-\delta)k_0+sAf(k_0)\big) < R$. So, Proposition 2's point 1 implies that $x_0 = 0$ and, hence, $a_0 = 0$. By consequence, $k_1 = (1-\delta)k_0 + sW_0 + \lambda a_0 = (1-\delta)k_0 + sAf(k_0)$.

   Since $k_0 < k^*$, we have $k_1 = (1-\delta)k_0 + sAf(k_0) > k_0$ and $k_1 = (1-\delta)k_0 + sAf(k_0) < (1-\delta)k^* + sAf(k^*) = k^*$.

   The wealth at date 1 is $W_1 = Af(k_1)$. We have that

   $$sAf'\big((1-\delta)k_1+sW_1\big) = sAf'\big((1-\delta)k_1+sAf(k_1)\big) < sAf'\big((1-\delta)k_0+sAf(k_0)\big) < R.$$

   Consequently, Proposition 2's point 1 implies that $x_1 = 0$ and hence $a_1 = 0$. By induction, we obtain that $k_t < k^*, a_t = 0$, and $k_{t+1} = (1-\delta)k_t + sAf(k_t), \forall t$. This implies that k_t converges to k^*.

2. Case 2: $k_0 > k^*$. By using a similar argument and the assumption $sAf'(k^*) < R$, we can prove that $k_t > k^*, a_t = 0$, and $k_{t+1} = (1-\delta)k_t + sAf(k_t), \forall t$.

3. Case 3: $k_0 = k^*$. It is easy to prove that $k_t = k^*, a_t = 0, \forall t$.

□

Proof of Proposition 4. We need an intermediate step.

Lemma 2. *Assume that $b_0 < k^a_{low} < k^a_{high} < b_1$, $k_t \in [k^a_{low}, k^a_{high}]$ and*

$$k_{t+1} = (1-\delta)k_t + s\big(Af(k_t - b_0) - R\bar{a}\big) + \lambda\bar{a} \tag{24a}$$

then $k_{t+1} \geq k_t$ and $k_{t+1} \in [k^a_{low}, k^a_{high}]$.

Proof. According to Lemma 1, for all $k_t \in [k^a_{low}, k^a_{high}]$, we have $k_{t+1} = H(k_t) \geq k_t$. Moreover, we have $k_{t+1} \in [k^a_{low}, k^a_{high}]$ because

$$k^a_{low} = H(k^a_{low}) \leq H(k_t) = k_{t+1} \leq H(k^a_{high}) = k^a_{high}. \tag{25}$$

□

According to point 2 of Proposition 2, conditions $b_0 < (1-\delta)k_0 + sW_0 + \lambda\bar{a} < k^{bind}$ imply that $a_0 = \bar{a}$. Hence, $k_1 = (1-\delta)k_0 + sW_0 + \lambda\bar{a}$. Therefore, $k_1 \in [k^a_{low}, k^a_{high}]$ because $k^a_{low} \leq (1-\delta)k_0 + sW_0 + \lambda\bar{a} \leq k^a_{high}$ (condition (14)).

Since $a_0 = \bar{a}$, we have $W_1 = Af(k_1 - b_0) - R\bar{a}$ and hence

$$k_2 = (1-\delta)k_1 + sW_1 + \lambda a_1 = (1-\delta)k_1 + s\big(Af(k_1 - b_0) - R\bar{a}\big) + \lambda a_1 \tag{26}$$

We have

$$(1-\delta)k_1 + s\big(Af(k_1 - b_0) - R\bar{a}\big) + \lambda\bar{a} = H(k_1) \geq k^a_{low} > b_0 \tag{27}$$

$$(1-\delta)k_1 + s\big(Af(k_1 - b_0) - R\bar{a}\big) + \lambda\bar{a} = H(k_1) \leq k^a_{high} < k^{bind} \tag{28}$$

According to point 2 of Proposition 2, these conditions imply that $a_1 = \bar{a}$, and hence $k_2 = H(k_1)$.

By induction and using Lemma 2, we have $a_t = \bar{a}\ \forall t \geq 0$, and $k_t = H(k_{t-1}) \in [k^a_{low}, k^a_{high}]$, $\forall t \geq 1$. According to Lemma 2, it is easy to see that k_t increasingly converges to k^a_{high}.

□

Proof of Proposition 5. Since $k_0 < b_1$, we have

$$a_0 = \arg\max_{x\in[0,\bar{a}]} Af\Big(\big((1-\delta)k_0 + sW_0 + \lambda x - b_0\big)^+\Big) - Rx. \tag{29}$$

By assumptions (15b) and (15c), we have $(1-\delta)k_0 + sW_0 + \lambda\bar{a} - b_0 > 0$ and $\lambda Af'\big((1-\delta)k_0 + sW_0 + \lambda\bar{a} - b_0\big) \geq R$. By consequence, Proposition 2 implies that $a_0 = \bar{a}$. Therefore, the capital stock at date 1 is $k_1 = (1-\delta)k_0 + sW_0 + \lambda\bar{a}$.

Condition (15c) implies that $k_1 > b_1$, and hence $a_1 = 0$. Therefore, we have

$$k_2 = (1-\delta)k_1 + s\Big(Af\big((1-\delta)k_0 + sW_0 + \lambda a_0 - b_0\big) - Ra_0\Big) + \lambda a_1 \tag{30}$$

$$= (1-\delta)k_1 + s\Big(Af\big((1-\delta)k_0 + sW_0 + \lambda\bar{a} - b_0\big) - R\bar{a}\Big) \tag{31}$$

$$> (1-\delta)k_1 + \delta\big((1-\delta)k_0 + sW_0 + \lambda\bar{a}\big) = k_1 \tag{32}$$

The last inequality follows from (15d). So, $k_2 > k_1$. By induction, we have $k_{t+1} > k_t \ \forall t \geq 1$. Hence, $k_t > b_1$, $\forall t \geq 1$, which implies that $a_t = 0 \ \forall t \geq 1$.

Since k_t is increasing and bounded, it converges to a steady state. Since $k_t > k^0_{low}$, we have $\lim_{t\to\infty} k_t = k^0_{high}$.

□

Proof of Proposition 6. First of all, since $F(k) = A(k - b_0)^+$ and $\lambda A > R$, according to Proposition 2, we can easily compute x_t as follows

$$x_t = \arg\max_{0\leq x\leq\bar{a}} \Big\{A[(1-\delta)k_t + sW_t + \lambda x - b_0]^+ - Rx\Big\} = \begin{cases} \bar{a} & \text{if } (1-\delta)k_t + sW_t + \lambda\bar{a} > b_0 \\ 0 & \text{if } (1-\delta)k_t + sW_t + \lambda\bar{a} \leq b_0 \end{cases}$$

Since we are assuming that $\lambda A > R$, we have $x_t = \bar{a} \ \forall t \geq 0$.

1. Let $k_0 \in (0, b_1)$. Since $k_0 < b_1$ and $b_1 < b_0$, we have $k_0 < b_0$ which implies that $W_0 = 0$. Again, by $k_0 < b_1$, we have $a_0 = x_0 = \bar{a}$. Thus,

$$k_1 = (1-\delta)k_0 + sW_0 + a_0 = (1-\delta)k_0 + \lambda\bar{a}$$

Observe that $k_1 \geq \lambda\bar{a} > b_0 > b_1$. This implies that $a_1 = 0$, and hence

$$k_2 = (1-\delta)k_1 + sW_1 + a_1 = (1-\delta)k_1 + s\Big(A(k_1 - b_0) - R\bar{a}\Big)$$

$$\begin{aligned} \text{and } k_2 - \gamma_0 &= (1-\delta)k_1 + s\Big(A(k_1 - b_0) - R\bar{a}\Big) - \gamma_0 \\ &= (1-\delta+sA)((1-\delta)k_0 + \lambda\bar{a}) - sAb_0 - sR\bar{a} - \gamma_0 \\ &= (1-\delta+sA)(1-\delta)(k_0 - \gamma_0) \end{aligned}$$

From this, we have

$$k_2 - b_1 = (1-\delta+sA)(1-\delta)(k_0 - \gamma_0) - (b_1 - \gamma_0) < 0$$

because $(1-\delta+sA)(1-\delta) < 1$, $k_0 - \gamma_0 < b_1 - \gamma_0$, and $b_1 - \gamma_0 > 0$. It means that $k_2 < b_1$. This condition implies that $a_2 = x_2 = \bar{a}$ and $k_2 < b_0$. Thus, we have $W_2 = 0$ and

$$\begin{aligned} k_3 &= (1-\delta)k_2 + sW_2 + a_2 = (1-\delta)k_2 + \lambda\bar{a} \\ &= (1-\delta)\Big((1-\delta)k_1 + s\Big(A(k_1 - b_0) - R\bar{a}\Big)\Big) + \lambda\bar{a} \\ &= (1-\delta)(1-\delta+sA)k_1 - (1-\delta)sAb_0 - (1-\delta)sR\bar{a} + \lambda\bar{a} \\ \text{and } k_3 - \gamma_1 &= (1-\delta)(1-\delta+sA)(k_1 - \gamma_1). \end{aligned}$$

By induction, we obtain conditions (17a-17d), and hence

$$\begin{aligned} k_{2t+2} - \gamma_0 &= (1-\delta)(1-\delta+sA)(k_{2t} - \gamma_0) \\ k_{2t+1} - \gamma_1 &= (1-\delta)(1-\delta+sA)(k_{2t-1} - \gamma_1) \end{aligned}$$

By consequence, we obtain the convergence.

2. Let $k_0 > b_0$ and $(1-\delta)k_0 + sW_0 < b_1$.

   Observe that $k_0 > b_1$ because $b_0 > b_1$. So, the country is not eligible for the development loan. This implies that $a_0 = 0$, and hence

$$k_1 = (1-\delta)k_0 + sW_0 + a_0 = (1-\delta)k_0 + sW_0 = (1-\delta)k_0 + s\Big(A(k_0 - b_0) - R\bar{a}\Big).$$

   Since $(1-\delta)k_0 + sW_0 < b_1$, we have $k_1 < b_1$. So, the country is now eligible for the development loan. This condition implies that $a_1 = x_1 = \bar{a}$ and $k_1 < b_0$. Thus, we have $W_1 = 0$ and

$$\begin{aligned} k_2 &= (1-\delta)k_1 + sW_1 + a_1 = (1-\delta)k_1 + \lambda\bar{a} \\ &= (1-\delta)\Big((1-\delta)k_0 + s\Big(A(k_0 - b_0) - R\bar{a}\Big)\Big) + \lambda\bar{a} \\ &= (1-\delta)(1-\delta+sA)k_0 - (1-\delta)sAb_0 - (1-\delta)sR\bar{a} + \lambda\bar{a} \\ \text{and } k_2 - \gamma_1 &= (1-\delta)(1-\delta+sA)(k_0 - \gamma_1). \end{aligned}$$

   Since $k_2 = (1-\delta)k_1 + \lambda\bar{a} > \lambda\bar{a} > b_0$, we have $a_2 = 0$, and hence,

$$\begin{aligned} k_3 &= (1-\delta)k_2 + sW_2 + a_2 = (1-\delta)k_2 + sW_2 = (1-\delta)k_2 + s\Big(A(k_2 - b_0) - R\bar{a}\Big) \\ &= (1-\delta+sA)k_2 - sAb_0 - sR\bar{a} \\ &= (1-\delta+sA)\Big((1-\delta)k_1 + \lambda\bar{a}\Big) - sAb_0 - sR\bar{a} \\ &= (1-\delta+sA)(1-\delta)k_1 + (1-\delta+sA)\lambda\bar{a} - sAb_0 - sR\bar{a} \\ k_3 - \gamma_0 &= (1-\delta+sA)(1-\delta)(k_1 - \gamma_0) \end{aligned}$$

   Observe that

$$\begin{aligned} k_3 - b_1 &= (1-\delta+sA)(1-\delta)(k_1 - \gamma_0) + \gamma_0 - b_1 \\ &\leq (1-\delta+sA)(1-\delta)(b_1 - \gamma_0) - (b_1 - \gamma_0) = \Big((1-\delta+sA)(1-\delta) - 1\Big)(b_1 - \gamma_0) < 0 \end{aligned}$$

because $(1-\delta+sA)(1-\delta) < 1$ and $b_1 > \gamma_0$.

By induction, we get (18a-18d) and hence

$$k_{2t+2} - \gamma_0 = (1-\delta)(1-\delta+sA)(k_{2t} - \gamma_1),\ k_{2t+1} - \gamma_1 = (1-\delta)(1-\delta+sA)(k_{2t-1} - \gamma_0)$$

By consequence, we obtain the convergence.

□

Proof of Remark 3. First, observe that $\sigma_1 = B\sigma_0 + \lambda\bar{a}$ and $\sigma_0 = B\sigma_1 - \bar{a}sR$.

Since $\lambda A > R$ and $k_0 < b_1$, we have $a_2 = x_0 = \bar{a}$. Then, $k_1 = (1-\delta)k_0 + sAk_0 + \lambda\bar{a} = Bk_0 + \lambda\bar{a} < \sigma_1$, where the last inequality is our assumption $k_0 < \sigma_0$ and the fact that $\sigma_1 = \sigma_0 B + \lambda\bar{a}$.

By assumption $Bk_0 > b_1 - \lambda\bar{a}$, we have $k_1 > b_1$ which implies that $a_1 = 0$. Thus,

$$\begin{aligned} k_2 &= (1-\delta)k_1 + s(Ak_1 - R\bar{a}) = Bk_1 - sR\bar{a} = B(Bk_0 + \lambda\bar{a}) - sR\bar{a} \\ &= B^2k_0 + \bar{a}(\lambda B - sR) < b_1 \end{aligned}$$

By assumption $k_0 < \sigma_0$, we can check that $k_0 < k_2 < \sigma_0$.

We now look at k_3. Since $k_2 < \sigma_0$, we have $k_2 < b_1$. So, $a_2 = x_2$. Since $\lambda A > R$, we have $x_2 = \bar{a}$, and, hence, $a_2 = \bar{a}$. Therefore,

$$k_3 = (1-\delta)k_2 + sAk_2 + \lambda\bar{a} = Bk_2 + \lambda\bar{a} < B\sigma_0 + \lambda\bar{a} = \sigma_1.$$

$k_3 = Bk_2 + \lambda\bar{a} = B^2k_1 + \bar{a}(\lambda - BsR) > k_1$ because $k_1 < \frac{\bar{a}(\lambda - BsR)}{1-B^2} = \sigma_1$.

To sum up, we get that, $a_{2t} = a_0 = \bar{a},\ a_{2t-1} = 0$, and

$$\sigma_1 > k_{2t+1} > k_{2t-1} > \cdots > k_1 > b_1 > \sigma_0 > k_{2t} > \cdots > k_0$$

for any $t \geq 1$. From this, we can easily prove that $\lim_{t\to\infty} k_{2t+1} = \sigma_1$ and $\lim_{t\to\infty} k_{2t} = \sigma_0$ □

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