Incentives to differentiate under environmental liability laws : Product customization and precautionary effort

Andreea Cosnita-Langlais
Eric Langlais

2022-20 Document de Travail/ Working Paper

EconomiX - UMR 7235 Bâtiment Maurice Allais
Université Paris Nanterre 200, Avenue de la République
92001 Nanterre Cedex

Site Web : economix.fr
Contact : secreteriat@economix.fr
Twitter : @EconomixU

Incentives to differentiate under environmental liability laws : Product customization and precautionary effort

Andreea Cosnita-Langlais*[*] and Eric Langlais*[†]

July 3, 2022

Abstract

We endogenize location/product specification choices in a spatial Cournot duopoly on the linear market, when firms' output entails an accidental harm to the environment. Under a strict liability regime, the equilibrium involves no differentiation when the expected harm is low enough. This outcome is suboptimal, and identical to the spatial pattern obtained under a no-liability regime. With larger harm, the equilibrium displays some dispersion/product differentiation, the degree of which is increasing with the level of harm towards the first best locations/product choices. Our results are robusts when allowing for firms' investment in environmental measures. Moreover, we show that vertical/care differentiation occurs whenever horizontal product differentiation arises. Finally, we show that under a negligence rule, firms always comply with the due care level, but the equilibrium involves no differentiation, either horizontal/product or vertical/care.

Keywords: Cournot competition, spatial model, strategic location, product choice, horizontal differentiation, vertical differentitation, environmental liability, strict liability, negligence.

JEL classification: L41, K21, D82

[*]EconomiX, UPL, Univ. Paris Nanterre, CNRS F92000; acosnita@parisnanterre.fr.

[†]EconomiX, UPL, Univ. Paris Nanterre, CNRS F92000; elanglais@parisnanterre.fr.

1 Introduction

It is widely acknowledged that environmental awareness has been steadily increasing throughout the general public. On the one hand, customers make more environmentally conscious purchase decisions, and are more and more willing to buy from firms that are committed to protecting the environment.[1] In response, firms react by investing in socially responsible product innovations (Iyer and Soberman 2016). On the other hand, ex ante instruments of environmental policy such as regulations, taxes[2] or permits[3], regularly face strong public rejections. Environmental liability laws usually have a better social acceptability, because they allow an ex post intervention based on the polluter-payer principle. As such, they have been recognized to have "numerous implications for firms competitiveness"[4], including product innovation and R&D activities. More generally, pricing and output decisions, as well as product specification choices will clearly be impacted by both consumers' environmental concerns and environmental laws and regulations.

Most often than not, the economic literature tackling these above-mentioned changes in the outcome of market competition focused on firms' vertical differentiation strategy (i.e. improving the quality/environmental friendliness of their products) but neglected the product customization strategic choices (i.e. exogenously assuming the products' horizontal differentiation - see Clemenz, 2010). However, within a given quality category, customizing a standard product to bring it closer to the preferences of some customers will likely further improve profits (see Conrad 2005). In this paper we intend to help

[1]See, for example, the 2015 GfK study on environmental values and ethical shopping at https://www.gfk.com/insights/environmental-values-and-ethical-shopping.

[2]Green taxes raise substantial protest on a regular basis, either from corporations or from citizens. See for example the reaction of road truck companies in France in 2014: https://www.europe1.fr/economie/Royal-enterre-encore-l-ecotaxe-les-routiers-arretent-leur-greve-683018. See more recently the so-called revolt of "yellow vests" from October 2018 to January 2020: https://en.wikipedia.org/wiki/Yellow_vests_movement.

[3]Ecological associations and the Green Party are usually strong opponents to markets for pollution permits. See for example https://npa2009.org/content/droits-%C3%A0-polluer-un-syst%C3%A8me-injuste-et-contre-productif-par-g%C3%A9rard-vaysse.

[4]Speech to the press (April 29, 2016) by French Minister of Justice, Jean-Jacques Urvoas.

fill this gap, and consequently endogenize both the horizontal and vertical differentiation choices when firms are subject to environmental liability.

The issue of horizontal differentiation in terms of environmental friendly characteristics is a concern for many different kinds of products/markets. This is the case for the provision of power energy ("green" versus "fossil" electricity), for detergents and washing powder (products with "soft" versus "aggressive" cleaning power), for (products made with) paper (bleached/wooded paper versus recycled paper), for vegetables and/or fruits (use of natural versus chemical fertilizers and pesticides), and so on. Some consumers will pay attention to the origin and surrounding conditions under which products have been manufactured and/or marketed, and will accept to be charged higher prices for "greener" products. Hence there exist profitable/sustainable market shares for firms willing to manufacture and sell such products. However, in some cases it can be very difficult if ever impossible for environmentally oriented consumers to avoid the consumption of specific products or substances, given the absence of substitutes and their extensive use in many sectors. Typically, this may occur with main materials and substances used for product conditioning (i.e. packaging, bags, bottles, cans and so on) such as aluminum and many plastics.[5] Given the weak incentives of firms, and sometimes governments' unwillingness[6] to foster the switch to different practices, it is important to assess the potential role of the civil society, and mainly the impact of the private enforcement of environmental laws

[5] Each year between 19 and 23 millions tonnes of plastic are disseminated in oceans, and it may double by 2040. The extensive use of plastic by the industry since 1950 (toys, paintings, cars and so on) as a production input as well as for packaging is due to plastic's inexpensive, durable, and very adaptable use for different purposes, implying that manufacturers choose to use plastic over other materials. Since its first industrial applications by Nylon© and Scotch© in the 30s, its general use has been supported in the long run by a ongoing stream of innovations and a dynamic R&D activity, with very diversified applications today, such as the textile or aerospace industries. As of 2020, the estimated global mass of produced plastic exceeds the biomass of all land and marine animals combined. For the WWF, "plastic pollution must be a major challenge" for environment, and specifically for the preservation of oceans - see https://www.europe1.fr/societe/one-ocean-summit-pour-wwf-la-pollution-plastique-doit-etre-un-enjeu-majeur-4092694.

[6] See for example the appeal of five environmental associations against the French State: https://www.lesechos.fr/politique-societe/politique/protection-de-la-biodiversite-les-ong-attaquent-letat-en-justice-1378020

under alternative environmental liability regimes.

In this respect, the impact of environmental liability laws on horizontal differentiation has not been addressed so far. The purpose of this paper is to provide a theoretical analysis of product specification/strategic location choices when firms compete in quantities and face environmental liability. For this we use a duopoly setting with Cournot spatial competition on the linear market (Anderson and Neven 1991): assuming a uniform consumer distribution along the line, we consider a two-stage game where firms choose first their locations or product specifications, and then compete in quantities at each location or local market on the unit line. In this standard framework we examine the role of environmental liability laws for the horizontal differentiation decision. Assuming that firms' output entails an accidental harm to the environment, we first consider the case where firms are subject to the strict liability rule (no-fault rule), and Courts award damages that correspond to the full compensation of harm. We assume that expected harm is linear w.r.t. individual output, i.e. proportional to it, and compare the spatial/product choice equilibrium that obtains under strict liability with the one under a "no liability regime", as well as to the socially optimal outcome.

We show that the properties of the spatial equilibrium are strongly related to the size of the external harm. For low levels of harm, firms choose to not differentiate their products in equilibrium, i.e. central agglomeration obtains, as is the case without liability. Therefore, the existence of environmental liability does not impact firm location/product specification choices, although it is not neutral for their output decisions, since strict liability reduces the equilibrium level of outputs as compared with no-liability regime. Moreover, the liability based product choice equilibrium is suboptimal, since the first best involves some spatial dispersion or product differentiation, with firms (products) being symmetrically located around the market center (at 1/4, 3/4). In contrast, larger levels of harm will trigger some dispersion in equilibrium: in this case, liability does matter for firms' product choices, although the equilibrium degree of horizontal differentiation is generally suboptimal. In particular, we show that the degree of differentiation/dispersion is increasing with the harm, and tends to the socially optimal one when the level of expected harm approaches the upper limit.

We then extent this basic set up in two ways. To start with, we allow firms to invest in precautionary measures. This extension is important for two main reasons. First, although several seminal papers do not distinguish between output and care as effective decisions in controlling harms when addressing the functioning of tort law (Kornhauser and Revesz 1989, Miceli and Segerson 1991), the major part of the Law & Economics literature actually agrees that the objective of liability rules is to provide firms/individuals with incentives to engage in care-taking activities.[7] Secondly, introducing thus preservation measures together with firms' location/product decisions is relevant since it allows to deal simultaneously with both vertical differentiation (preservation measures) and horizontal differentiation (product differentiation/spatial dispersion). When allowing for precautionary measures, we obtain that the resulting spatial/product differentiation equilibrium pattern is qualitatively similar to that of the base model. The (quantitative) difference consists in the extent of product differentiation in equilibrium, since the horizontal differentiation now occurs for different (threshold) values of the environmental harm. Furthermore, we show that horizontal (product) differentiation parallels vertical (safety) differentiation.

In a second extension we analyze the implications of a regime of environmental liability based on the negligence (at-fault) rule associated with a flexible standard of due care, defined as the socially optimal reply of care at any level of output. Our conclusion is that negligence always induces firms' compliance with this standard. Regarding the equilibrium spatial/location pattern, we find that central agglomeration always obtains under negligence, i.e. it occurs in equilibrium regardless of the size of environmental harm. This implies in turn that firms choose the same (location-dependant) level of care, i.e. they do not vertically differentiate under negligence. Hence, the liability regime matters for location choices/products differentiation, as well as for firms' investments in environmental preservation measures.

The rest of the paper is organized as follows. Section 2 provides a brief survey of related contributions from both IO and Law and Economics (L&E henceforth) literature.

[7]We use interchangeably the terms of care, precautionary activities, and preservation measures throughout the paper. Note however that the distinction between the notions of care and activity level may be controversial. Hence, which of the parties' precautionary measures should be included in, or, instead, left out from the determination of negligence may be complex and actually depends on Courts attitude - see for example Dari-Mattiacci (2006).

In section 3, we examine the output and location/product choice decisions when environmental harm is linear w.r.t. the individual outputs and firms face strict liability. We compare the equilibrium outcome to the one occurring in two benchmarks : the "no liability regime", and the social optimum. Section 4 deals with environmental preservation measures, together with output and location decisions. In section 5 we depart from the strict liability rule and extend the analysis of liability regimes to the negligence (at-fault) rule. Section 6 concludes.

2 Related literature

This paper displays a two-stage location-then-quantity game, and as such is a related to the vast literature on spatial competition with location/product choice.[8] Since the seminal work of Hotelling (1929), a substantial part of this literature considers a location-then-price game over a continuous space of product characteristics. The well-known outcome is that of maximum differentiation: firms choose product specifications or locations as far as possible from one another in order to soften the intense price competition, either on the linear (see d'Aspremont et al. 1979) or the circular (see Kats 1995) markets. Spatial agglomeration or, equivalently, identical product specifications can only occur in equilibrium if some other differentiation dimension is allowed and this non-spatial product heterogeneity is sufficiently large (de Palma et al. 1985). The type of competition is actually determinant for the possibility to obtain the maximum differentiation outcome: a more recent strand of the strategic location literature focused on Cournot-playing firms[9] to show that the resulting spatial or product choice equilibrium of the location-then-quantity game may involve complete agglomeration, partial clustering or even complete dispersion. Central agglomeration obtains on the linear market with uniform consumer distribution (Anderson and Neven 1991), and more generally, as long as the population density is high

[8] For a recent survey see Biscaia and Mota (2013).

[9] The alternative, Cournot competition assumption, is actually more appropriate quite often: when quantity is less flexible than price at each market point (Anderson and Neven 1991; Pal and Sarkar 2002), or when there is a substantial lag between the production and the price-setting decisions (Hamilton et al. 1994), but also because it basically replicates the outcome of a two-stage capacity-then-pricing game (Kreps and Scheinkman 1983).

enough (Gupta et al. 1997) or the production cost convex enough (Mayer 2005).[10] In contrast, Pal (1998), Matsushima (2001) and Gupta et al. (2004) showed that Cournot competitors cannot completely agglomerate on the circular market, but instead disperse, although they may cluster at several distinct locations. Pal and Sarkar (2002) shows that this partial clustering also obtains on the linear market for different firms' outlets, by assuming Cournot competition among multi-store firms. Recall that the spatial Cournot competition framework affords some empirically relevant features, such as overlapping firm areas, i.e. consumers being served by several firms simultaneously,[11] or the spatial price discrimination across the set of spatially differentiated markets/consumers.[12]

To our best knowledge our paper is the first to examine the product specification/strategic location choices when firms' outputs generate harm for the environment. The only contribution, to our knowledge, which also endogenizes product choice in a spatial framework allowing for environmental-related issues is Conrad (2005), but this paper considers price competition in a spatial duopoly where consumers are uniformly distributed and have environmental concerns. Conrad (2005) confirms the robustness of the standard result of the spatial price competition literature, i.e. the interior solution (intermediate level of differentiation, no firm being located at an extreme point of the market) does not occur in (the pure strategy) equilibrium. Instead, he finds that depending on the magnitude of the difference between the intensity of consumers' environmental concerns and firms' marginal cost, the equilibrium spatial pattern may consist in a firm being located at one extreme point of the unit line while the other taking a different, but not extreme location, or, alternatively, each firm being located at one of the two opposite extreme points, i.e. the standard maximal differentiation result.

In the L&E literature, several recent contributions examined the impact of product liability in a context of product differentiation,[13] but did not consider the firms' product

[10] Shimizu (2002) relaxed the product homogeneity assumption and still confirmed the central agglomeration result on the unit line.

[11] In contrast, Bertrand spatial competition yields exclusive sales territories for firms, i.e. consumers at each location/local market being served only by the most cost-efficient firm there.

[12] This actually replicates the flexible manufacturing production systems (see Eaton and Schmitt 1994), where the firm's basic product (its location) is customized at a cost (transport cost) to make it appropriate for a consumer.

[13] In the last decade, many contributions deal with product liability under imperfect competition more

design choices. The first to do so is Daughety and Reinganum (2006), who consider a oligopoly model allowing for both horizontal and vertical differentiation. Firms compete in quantities, and invest in a care-taking activity that impacts the marginal cost of production and reduces consumers' harm. Daughety and Reinganum (2006) show that under a strict liability regime, the relationship between, on the one hand, the equilibrium levels of care and output, and on the other hand, the degree of product differentiation, is U-shaped. They also find that the equilibrium levels of care and output are lower (higher) than their second best levels for a high (low) degree of product differentiation.[14] More recently, Baumann and Friehe (2015) extend this set-up in several ways: price in addition to quantity competition, symmetric versus heterogeneous firms, strict liability versus negligence. They focus their analysis on the determination of optimal damage multipliers, i.e. how Courts may restore first best efficient decisions in care-taking thanks to compensation schemes that provide victims with more than full-compensation for their harm. Chen and Hua (2017) use the hub-and-spokes model of spatial competition (that generalizes the Hotelling model) to show that when firms compete in prices, incomplete strict liability (partial compensation of harms) combined with firms' reputation concerns will provide incentives to invest in safety. Furthermore, they also show that the relationship between the intensity of competition and the level of product safety is non monotonic, and depends on how competition is measured (i.e. degree of product differentiation versus number of firms). Baumann, Friehe, and Rasch (2018) and Baumann and Friehe (2021) also study the role of incomplete strict liability in a spatial model of competition *à la* Hotelling, when firms have different costs for providing safety. In Baumann, Friehe, and Rasch (2018), consumers are differentiated in terms of the harm incurred, and it is shown that some degree of loss sharing between firms and consumers is always socially beneficial. In Baumann and Friehe (2021), consumers are heterogeneous in terms of misperception of their harm. This paper finds that less-than-full compensation for consumer harm im-

generally - see Daughety and Reinganum (2013) for a survey.

[14] Daughety and Reinganum (2006) show that a monopoly chooses inefficient levels of care and output. When the monopoly market is contestable, Spulber (1988) showed that any liability rule (strict liability, negligence, no liability) is a second best solution. Endres and Lüdecke (1998) examined the role of incomplete strict liability when a monopoly sellsdifferent quality goods to consumers with unobservable characteristics.

proves product allocation although it lowers safety provision by firms; nevertheless, the social benefits associated with the output effect may dominate the social costs due to inefficient product safety.[15]

The case for environmental liability[16] has been addressed in relation to technical change and firms' incentives to adopt abatement technology. Endres and Friehe (2013) look into the effects of strict liability vs negligence on the output, abatement, and investment decisions in case of a monopolistic polluter. They find that the relative performance of liability rules depends on whether the technology is exogenous (then negligence is better) or endogenous (then strict liability is better). Endres, Friehe and Rundshagen (2015) consider a duopoly, and examine how the joint use of environmental liability laws (strict liability versus negligence under different standards of care) and R&D subsidies allows firms to internalize both the double externality created by the environmental harm they generate, as well as the R&D spillover effects from their investments in environmentally friendly technologies. A previous study by Van Egteren and Smith (2002) does focus on the issue of firms' location when two competing jurisdictions aim at attracting risky activities, but without firms' strategic market interactions.

3 Model and assumptions

Consider two Cournot rival firms, denoted 1 and 2, operating on the unit linear market, where infinitely many consumers lie uniformly. The firms produce the same basic homogenous good with the same production technology characterized by constant marginal costs, normalized to zero. Firm's $i \in \{1, 2\}$ location is denoted x_i. At each consumer location

[15]The question whether full, less-than-full, or over-compensation of victim's harm is optimal has long been topical in the L&E literature, but is beyond the scope of the paper. For a general treatment see D'Antoni and Tabbach (2014). Friehe, Langlais and Schulte (2018) also discuss how consumer preferences for partial product liability and litigation costs condition the emergence of inefficient liability laws.

[16]In this brief review of the related literature we only discuss contributions with environmental liability under imperfect competition and divisible environmental harms, i.e. firms competing on the same market and inflicting divisible/perfectly separable expected harms on the environment/a third party. For the case of indivisible/joint environmental harms, see Baumann, Charreire, and Cosnita-Langlais (2020), and Charreire and Langlais (2021).

x on the unit line demand is given by $p(x) = a - Q(x)$, with $a > 0$ and where $p(x)$ and $Q(x)$ are the price and total output supplied at location x (hence $Q(x) = q_1(x) + q_2(x)$ where $q_1(x), q_2(x)$ denote individual outputs at each location x). Firms incur the same transport cost $C_i = t|x_i - x|$, linear in distance and quantity, to deliver output to consumers.[17] Consumers are assumed to have a prohibitive costly transport cost, preventing arbitrage, so firms can and will price discriminate across the set of spatially differentiated markets. t is a positive constant, and given that the transport cost parameter enters as a multiple in the profit expressions, we assume $t = 1$ w.l.o.g.[18]

At each location x the output produced by a firm generates some harm to the environment, denoted $d(q_i(x))$. Note that an alternative but formally equivalent interpretation would be that the harm is borne by some third-party victims, i.e. having no contractual relationship nor market interaction with industry firms. We will assume that the expected harm that a firm generates is proportional to its output level, i.e. $d(q_i(x)) = h \times q_i$, where $h > 0$ is a scale parameter. With "linear harm", each unit of output delivered by a firm contributes equally to expected damage. A large body of the L&E literature introduces this assumption as a basic ingredient of the analysis of tort law and liability rules.

We assume that Courts award expected damages corresponding to the harm caused by a firm; in the next (final) section, we assume they use the strict liability (negligence) rule. Hence the liability cost borne by firm i is defined as $L_i = d(q_i(x))$. To ensure that both firms serve all local markets along the unit line (i.e. total transportation costs are smaller than the willingness to pay of consumers), we assume that :

Assumption 1 : $a > 2$.

Moreover, to guarantee that SOCs regarding location decisions are met, we will make the following assumption :

[17] Recall that the Cournot spatial competition framework that we use is actually a shipping model in which product differentiation comes up as follows: the firm's basic product (its location) is customized at a cost (transport cost) to make it appropriate for particular consumers (located at x).

[18] Equivalently, let a be the transport-cost adjusted reservation price, and recall that in the product-differentiation analogy of this model, a can be interpreted as an inverse measure of the extent to which consumer tastes are strongly localized.

Assumption 2 : $h < a - 1$.

Both assumptions could be merged in a single one : $a > \sup\{1 + h, 2\}$; however, referring separately to A1 and/or A2 is more convenient in the analysis below. h does not take a given value *a priori*, and thus $h \lesseqgtr 1$ is possible without loss of generality.[19]

The timing of the game is as follows : at stage 0, Courts announce a liability regime (strict liability) to which they commit, with both firms observing the liability regime they will face in case of an accidental environmental harm; at stage 1, firms simultaneously choose a location $x \in [0, 1]$; at stage 2, they compete in quantities at every local market x; at stage 3, the liability regime is enforced.

4 Equilibrium under Strict Liability

Numerous existing environmental laws have adopted a regime based on a no-fault/strict liability rule, often with a view to targeting specific activities/industries that are recognized as highly polluting or dangerous. The Comprehensive Environmental Response, Compensation, and Liability Act adopted in 1980 in USA, as an example, has been conceived as a tool allowing to clean up uncontrolled or abandoned hazardous-waste sites existing throughout the United States, as well as accidents, spills, and other emergency releases of pollutants and contaminants into the environment; all these activities are subject to strict liability. Similarly, the Environmental Liability Directive of the European Union targets several potentially dangerous and polluting activities that are subject to strict liability all being listed in its Annex III.

Under strict liability, firms anticipate that in case their product inflicts an environmental harm, they will be legally liable for an amount corresponding to the damages

[19] Although here h is a scale parameter which captures the magnitude of the marginal impact of the output on the environmental harm (i.e. a pure "technological" effect in terms of response reaction), it could also be interpretated more broadly as capturing the failures of the judicial institutions. For example, standard issues discussed in the L&E literature are related to the disapearing defendant problem (reducing the defendent's probability of being seen as liable), or the issue of incomplete vs over-compensation of damages (damages manipulations by Courts).

awarded by the Court. Below we assume that this compensation (i.e. the liability cost) is set at the full value of the environmental harm. In what follows we identify the Subgame Perfect Nash equilibrium of the game when firms are subject to the strict liability rule.

4.1 Stage 2: the quantity choice

At stage 2, each firm chooses a level of output $q_i(x)$ that maximizes its profit given the output of the other at each location x on the unit line (i.e. Cournot competition at each local market x),[20] where the individual profit is written for $i \in \{1, 2\}$ as

$$\pi_i(x) = (a - Q(x) - C_i)\, q_i(x) - h \times q_i(x).$$

At each location x, individual output $q_i(x)$ chosen by firm $i \in \{1, 2\}$ solves the FOC:[21]

$$(a - Q(x) - C_i) - q_i(x) = h. \tag{1}$$

The LHS of this condition is the standard marginal market proceeds under Cournot competition, while the RHS corresponds to the marginal liability cost borne by firm i. Solving (1) yields the stage-2 subgame equilibrium output level, given by:

$$q_i(x) = \frac{1}{3}\left(a - h - 2C_i + C_j\right). \tag{2}$$

Condition (2) illustrates that the liability cost, h, reduces the output level at each local market, but does not impact the way in which location choices drive the output decisions: whether a liability rule exists ($h > 0$) or not ($h = 0$), a firm' decision to locate away from its opponent increases its cost of operating on any given local market, and thus reduces its output there ($\frac{dq_i(x)}{dC_i} = -2 < 0$); in contrast, its rival's decision to locate farther away provides a reciprocal strategic advantage, although of smaller size ($\frac{dq_i(x)}{dC_j} = 1 > 0$).

[20] Given that marginal costs are constant and consumer arbitrage is nonbinding, quantities set at different points by the same firm are strategically independent, therefore the stage-2 Cournot equilibrium can be characterized by a set of independent Cournot equilibria, one for each local market $x \in [0, 1]$. Remember also that in the context of our spatial analysis, firms decide on their aggregate output but also on the quantity to allocate to several submarkets (several points in space) - see Anderson and Neven (1991).

[21] The SOC is verified since $\frac{\partial^2 \Pi_i}{\partial q_i^2(x)} = -2$.

4.2 Stage 1: the location choice

At stage 1, a firm chooses its location $x_i \in [0,1]$ to maximize its total profit over the whole unit line, anticipating it will deliver its Cournot-Nash output at stage 2. Using (1) and substituting the LHS in the profit expression, the stage 2 equilibrium individual profit at each location x may be written for $i \in \{1,2\}$ as

$$\pi_i(x) = (q_i(x))^2 \,. \tag{3}$$

As a result, the introduction of a environmental harm does not entail any additional strategic interaction compared with basic Cournot competition in the sense that the individual profit at each location is the Cournot profit.

Allowing for possibly distinct locations along the linear market, with $0 \leq x_1 \leq x_2 \leq 1$, total individual profit for firm i over the whole unit line writes:[22]

$$\Pi_i = \int_0^{x_1} (q_i(x))^2 \, dx + \int_{x_1}^{x_2} (q_i(x))^2 \, dx + \int_{x_2}^{1} (q_i(x))^2 \, dx. \tag{Π_G}$$

Differentiating w.r.t. x_i yields for $i \in \{1,2\}$

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{4}{3}\left(-\frac{dC_i}{dx_i}\int_0^{x_1} q_i(x)dx - \frac{dC_i}{dx_i}\int_{x_1}^{x_2} q_i(x)dx - \frac{dC_i}{dx_i}\int_{x_2}^{1} q_i(x)dx\right)$$

since (2) gives us $\frac{dq_i(x)}{dx_i} = -\frac{2}{3}\frac{dC_i}{dx_i}$. Setting $\frac{\partial \Pi_i}{\partial x_i} = 0$ provides the two best reply functions w.r.t. locations.[23] Solving the simultaneous system of FOCs gives the location outcome in equilibrium, for which we show that the following holds :

Proposition 1 *Under A1 and A2, there exists a unique stable location equilibrium which is either : a)* $x_1 = \frac{1}{2} = x_2$ *if* $h < a - \frac{3}{2}$*; the associated equilibrium output at any location* x

[22] Note that output at each location depends on transportation costs, and the latter write differently along the unit line, depending on the firms' relative locations: $C_1 = (x_1 - x), C_2 = (x_2 - x)$ for $x \in [0, x_1]$, but $C_1 = (x - x_1), C_2 = (x_2 - x)$ for $x \in [x_1, x_2]$, and finally $C_1 = (x - x_1), C_2 = (x - x_2)$ for $x \in [x_2, 1]$.

[23] Given that at each local market x a firm's profit is equal to the square of its output deliverd at that location, the expression of the FOC w.r.t. a firm's location basically states that the optimal choice satisfies the quantity-median property (see Anderson and Neven, 1991, for instance): accordingly, the quantity-median of a firm's market is the location such that the total quantity supplied by the firm to the left of that point is equal to the total quantity supplied by it to the right of that point.

is $q_i(x) = \frac{1}{3}(a - h - \left|\frac{1}{2} - x\right|)$ *for* $i \in \{1, 2\}$*; or b)* $x_1 = \frac{1}{2}(a-h) - \frac{1}{4} < x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$ *if* $h \in [a - \frac{3}{2}, a - \frac{11}{10})$*; the associated equilibrium output at any location* x *is defined for each firm as follows:*

	$q_1(x)$	$q_2(x)$
$[0, x_1]$	$\frac{1}{3}(\frac{7}{4} - \frac{1}{2}(a-h) + x)$	$\frac{1}{3}(\frac{5}{2}(a-h) - \frac{11}{4} + x)$
$[x_1, x_2]$	$\frac{1}{3}(\frac{3}{2}(a-h) + \frac{3}{4} - 3x)$	$\frac{1}{3}(\frac{3}{2}(a-h) - \frac{9}{4} + 3x)$
$[x_2, 1]$	$\frac{1}{3}(\frac{5}{2}(a-h) - \frac{7}{4} - x)$	$\frac{1}{3}(\frac{11}{4} - \frac{1}{2}(a-h) - x)$

Proof. See the Appendix. Note that since we focus on the duopoly case (i.e. neither firm holds a monopoly position at any location), then for both firms to deliver positive quantities throughout the entire set of local markets (i.e. full coverage of the market), it must be that

– for the agglomerated equilibrium pattern ($x_1 = \frac{1}{2} = x_2$): $q_i(x=0) = \frac{1}{3}\left(a - h - \frac{1}{2}\right) = q_i(x=1) > 0$ which holds under A1;

– for the dispersed equilibrium ($x_1 = \frac{1}{2}(a-h) - \frac{1}{4}, x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$) : $q_1(x = 1) = \frac{1}{3}\left(\frac{5}{2}(a-h) - \frac{11}{4}\right) = q_2(x=0) > 0$, which holds only if $h < a - \frac{11}{10}$, which is more restrictive than A2.

■

Proposition 1 states that the spatial/product choice equilibrium outcome under strict liability mainly depends on the size of the expected damage, or more precisely, it depends on the (marginal) impact of the output on the environmental harm/liability cost captured by h: small (large) expected harm fosters central agglomeration (respectively, dispersion).

The intuition is actually straightforward (see also Chamorro-Rivas, 2000, and Benassi et al., 2007), since the optimal location choice is always the outcome of a trade-off between agglomeration and dispersion incentives: on the one hand, choosing the same central location as the rival provides better, cost-minimizing access to demand throughout the set of local markets, but on the other hand, locating further apart form the rival firm dampens competition and thereby yields higher profits. The former is a market-coverage effect, which in our linear market setting fosters central agglomeration, whereas the latter is a strategic effect which pushes firms to differentiate/locate apart, so as to serve at a lower cost the local markets/demands where the rival firm delivers low quantities. With strict liability, as compared with the no-liability setting (see below), firms bear an additional cost reflecting the liability burden which is increasing (at a constant rate) with the level of the output. But the latter is itself decreasing with the distance between the firm's location and the local market supplied. Hence, when h is low enough, the result of central agglomeration is still valid because the impact of transportation cost on aggregate output is still low enough, i.e. the demand-maximization effect is dominant. In contrast, when the impact of the output on the liability cost becomes large enough, or, equivalently, the impact of transportation costs becomes higher compared with the size of the demand that can be reached, firms prefer to locate further apart so as to get closer to distant local markets and thus serve a higher demand than the rival firm at these distant locations. Note also that, in line with the intuition provided for the spatial dispersion equilibrium, $x_1 = \frac{1}{2}(a-h) - \frac{1}{4}$ is decreasing with h, whereas $x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$ is increasing with h: in other words, the extent/degree of differentiation is increasing with the level of harm.

Finally, we exclude in our analysis values of h larger than $a - \frac{11}{10}$, which is more restrictive than Assumption 2. As argued in the proof, the rationale is that we focus on the duopoly case everywhere throughout the linear unit market, i.e. both firms delivering positive outputs at every location.[24]

4.3 Two benchmarks : "no liability" and social optimum

In order to gain further insight into the implications and scope of Proposition 1, let us briefly establish the spatial outcome obtaining in two benchmark settings.

4.3.1 The "no liability" regime

The spatial equilibrium pattern under a "no liability" regime is easily inferred from the previous analysis, by using (2) and part a) of Proposition 1, and setting $h = 0$: at each

[24] Again, see also Chamorro-Rivas, 2000, and Benassi et al., 2007. A further implication of our analysis is that duopoly as a market structure is not sustainable under strict liability for the largest levels of harm (s.t. $h \in [a - \frac{11}{10}, a - 1]$). Hence the liability regime matters for the endogeneous market structure. This is beyond the scope of the paper. We develop the argument for product liability in Cosnita-Langlais and Langlais (2022).

local market, $q_i(x) = \frac{1}{3}(a - 2C_i + C_j)$, and given that $0 < a - \frac{3}{2}$, then $x_1 = \frac{1}{2} = x_2$ is the unique spatial equilibrium (see Anderson and Neven, 1991).

Thus Proposition 1 establishes that for low values of (the scale parameter of) expected harm, strict liability and no-liability have equivalent effects, i.e. both foster central agglomeration, and thus do not leads firms to differentiate their products/locations. When expected harm is low, whatever the quantity delivered at each local market, the liability regime has no bearing on location choice. In contrast, larger values of the expected harm weigh on the trade-off between competitive pressure and transportation costs, and the liability cost pushes firms to differentiate: the spatial equilibrium involves symmetrical differentiation, with firms' locations/varieties to the left and to the right of the mid-market point.

To sum up, we find that liability does not impact the spatial/differentiation decisions when the expected harm is low enough: whether firms face strict liability or no liability, it will only affect their output decisions at each local market. Hence, with firms identical in all respects at each local market, the (only) spatial equilibrium is the "standard" one, where both firms share the central location (i.e. minimum differentiation - see Anderson and Neven, 1991). In contrast, liability matters when the expected environmental harm becomes large enough, and impacts both the output decision (which is still decreasing with the harm) and firms' incentive to differentiate (which is increasing with the harm).

4.3.2 Social Optimum

Let us now consider the problem a benevolent social planner would face:[25] the planner first chooses a location for each firm, in order to maximize Social Welfare over the different markets on the whole line defined as $W_G = \int_0^1 W(x)dx$. In a second stage, he chooses a level of output at each local market x, such that given firms' locations, Social Welfare $W(x)$ at each location x is maximized.

In this set up, Social Welfare at each location is the sum of gross consumers' surplus,

[25] For preliminary results regarding the welfare analysis of location equilibrium in the Cournot spatial competition framework that we use here, see Matsumura and Shimizu (2005).

minus total transportation costs, minus the expected harm:

$$W(x) = aQ(x) - \frac{(Q(x))^2}{2} - \sum_{i=1}^{2} C_i q_i(x) - h \times Q(x). \tag{Wnc}$$

Let us start with the socially optimal output decisions:

At each location x the derivative of (SW) w.r.t. quantity delivered by firm $i \in \{1, 2\}$ is written as

$$\frac{\partial W(x)}{\partial q_i(x)} = a - Q(x) - C_i - h.$$

Hence, at any given location x, either a) $C_1 = C_2 = C$, and the solution to the stage-2 FOC, $\frac{\partial W(x)}{\partial q_i(x)} = 0$, is $q_1(x) = q_2(x) = \frac{Q(x)}{2} = \frac{a-h-C}{2}$; or b) $C_i < C_j$, and the solution is $q_i(x) = Q(x) = a - h - C_i > q_j(x) = 0$.

At stage 1, assume the planner contemplates distinct locations/product specifications for the two firms: $0 \leq x_1 \leq \frac{1}{2} \leq x_2 \leq 1$. At each local market x, the aggregate output $Q(x)$ is always delivered with the lowest transportation cost (denoted $\tilde{C}$), and thus using stage-2 FOC, Social Welfare at each x may be written as $W(x) = \frac{1}{2}(Q(x))^2 = \frac{1}{2}\left(a - h - \tilde{C}\right)^2$. Given the assumption of distinct locations, minimizing total transportation costs implies that firm 1 supplies all the local markets in $[0, x_1]$ together with those in $[x_1, \frac{x_1+x_2}{2}]$, whereas firm 2 supplies all the local markets in $[x_2, 1]$ together with those in $[\frac{x_1+x_2}{2}, x_2]$ respectively. As a result, Social Welfare over all locations is given by

$$W_G = \frac{1}{2}\int_0^{x_1} (Q(x))^2\, dx + \frac{1}{2}\int_{x_1}^{\frac{x_1+x_2}{2}} (Q(x))^2\, dx + \frac{1}{2}\int_{\frac{x_1+x_2}{2}}^{x_2} (Q(x))^2\, dx + \frac{1}{2}\int_{x_2}^{1} (Q(x))^2\, dx, \tag{WGnc}$$

where as before, $Q(x)$ depends on transportation costs at each location, with $\frac{dW(x)}{dx_i} = -Q(x)\frac{d\tilde{C}}{dx_i}$ at each location where firm i operates. In the Appendix, we show that the next result holds:

Proposition 2 *The first best location/product specification choices are* $x_1 = \frac{1}{4}$ *and* $x_2 = \frac{3}{4}$.

Comparing Propositions 1 and 2 reveals that for low levels of the environmental harm, strict liability fails to provide firms with efficient output and location incentives, both decisions being suboptimal. In contrast, the socially optimal product differentiation/location pattern corresponds to symmetric dispersion around the market center. In turn, for larger levels of expected harm, firms have incentives to differentiate under strict liability, thus some dispersion (symmetrical around the central location) occurs in equilibrium. However, although the extent of differentiation is increasing with the harm, it is nevertheless generally inefficient (for $h = a - \frac{11}{10}$, strict liability leads to $x_1 = \frac{3}{10} > \frac{1}{4}$ and $x_2 = \frac{7}{10} < \frac{3}{4}$).

5 Precautionary measures

We now extend the base model to allow for firms' investments in environmental preservation measures. To this end, we assume that the expected harm generated by firm's i output is given by $D(q_i(x)) = \theta_i(x) \times h \times q_i(x)$, where $\theta_i(x)$ is chosen by firm $i \in \{1, 2\}$ and denotes the probability of accidental harm at each local market x. Hence, a decrease in $\theta_i(x)$ here corresponds to an increase in preservation measures. Let us assume that the total cost of care borne by firm i at each location is $c \times \theta_i(x) \times q_i(x) + k(\theta_i(x))$, assuming $k'(\theta_i(x)) < 0 < k''(\theta_i(x))$; this means that apart from the harm-mitigating effect, investments in environmental preservation measures impact the (constant) marginal cost of production, and play the role of a fixed cost in the production process; w.l.o.g. we assume in what follows that $c = 0$.[26] To obtain closed-form solutions latter on in the analysis, we specify the care technology as:

Assumption 3 : for any $\theta \leq \theta_0 (< 1)$, let $k(\theta) = \frac{k}{2} (\theta - \theta_0)^2$ where $a - h > k > \frac{4}{3} h^2$.

We modify slightly the timing of the game as follows : at stage 0, Courts announce a liability regime (strict liability) to which they commit, both firms observing the liability regime they will face in case of an accidental harm to environment; at stage 1, firms simultaneously choose a location/product specification $x_i \in (0, 1)$, and at stage 2, they choose a level of care/probability of harm $\theta_i(x)$; at stage 3, they compete in quantities at every local market x; at stage 4, the liability regime is enforced.

[26] Alternatively, h may be changed to $h + c$ in the sequel, the constant (marginal cost of production) c playing thus a role similar to h, the (constant) scale parameter capturing the impact of output on the environmental harm.

At stage 3, each firm delivers at each location x on the unit line an output $q_i(x)$ that maximizes its profit given the output of the other, where the individual profit is written for $i \in \{1,2\}$ as

$$\pi_i(x) = (a - Q(x) - C_i - h \times \theta_i(x))\, q_i(x) - k(\theta_i(x)).$$

The FOC w.r.t. output at each location x is given by

$$(a - Q(x) - C_i) - q_i(x) = h \times \theta_i(x), \tag{4}$$

which has a similar interpretation to (1), excepted that the marginal liability cost (RHS) is no longer constant. Thus the output level chosen by firm $i \in \{1,2\}$ is now set according to

$$q_i(x) = \frac{1}{3}\left(a - 2\left(h\theta_i(x) + C_i\right) + \left(h\theta_j(x) + C_j\right)\right). \tag{5}$$

At stage 2, firms choose their care activity $\theta_i(x)$ at each local market in order to maximize their total profit, anticipating they will play Cournot-Nash quantities defined by (5). Using (4), it can be verified that stage 3-equilibrium profit at each location x for $i \in \{1,2\}$ is now given by

$$\pi_i(x) = (q_i(x))^2 - k(\theta_i(x)).$$

Thus the profit derivative w.r.t. care at each location x is $\frac{\partial \pi_i}{\partial \theta_i(x)}(x) = 2q_i(x)\frac{dq_i(x)}{d\theta_i(x)} - k'(\theta_i(x))$, where, using (5), $\frac{dq_i(x)}{d\theta_i(x)} = -\frac{2}{3}h$. As a result, the FOC w.r.t. care level, i.e. $\frac{\partial \pi_i}{\partial \theta_i(x)}(x) = 0$, implies that at each local market, care is set such that[27]

$$\frac{4}{3}hq_i(x) = -k'(\theta_i(x)). \tag{6}$$

The LHS of condition (6) is the marginal benefit of care activity, while the RHS corresponds to the marginal cost of care. The meaning of (6) is that at each local market x a firm chooses a level of care that minimizes the total cost of an accidental harm at that location; the implication is that the level of care chosen is increasing in the level of output to be delivered (i.e. care and output are complement according to (6)). As a consequence,

[27] The SOC requires that $\frac{\partial^2 \pi_i}{\partial \theta_i^2(x)}(x) = \frac{8}{9}h^2 - k''(\theta_i(x)) < 0$, which holds for $k''(\theta_i(x))$ large enough.

firms choose a location-specific level of care at each local market: the level of care depends both on the characteristics of the safety technology and (through the output) on those of the market demand and transportation costs.

At stage 1, firms choose their location $x_i \in [0,1]$ in order to maximize their total profit, anticipating they will play their Cournot-Nash quantities at stage 3 and choose care activities at stage 2, as defined by (5)-(6). Assuming $0 \le x_1 \le x_2 \le 1$ and using (Π_G) and (5), total profit for firm i over the whole unit line writes as

$$\Pi_i = \left(\begin{array}{c} \int_0^{x_1} \left((q_i(x))^2 - k(\theta_i(x)) \right) dx + \int_{x_1}^{x_2} \left((q_i(x))^2 - k(\theta_i(x)) \right) dx \\ + \int_{x_2}^{1} \left((q_i(x))^2 - k(\theta_i(x)) \right) dx. \end{array} \right).$$

Differentiating w.r.t. x_i the profit at any location x yields

$$\frac{\partial \pi_i(x)}{\partial x_i} = 2q_i(x)\frac{dq_i(x)}{dx_i} - k'(\theta_i(x))\frac{d\theta_i(x)}{dx_i} = 2q_i(x)\left(\frac{dq_i(x)}{dx_i} + \frac{2}{3}h\frac{d\theta_i(x)}{dx_i}\right),$$

where the second equality results from (6). Note that by differentiating (5) we obtain that $\frac{dq_i(x)}{dx_i} + \frac{2}{3}h\frac{d\theta_i(x)}{dx_i} = -\frac{2}{3}\frac{dC_i}{dx_i}$. Therefore, after substituting in $\frac{\partial \pi_i(x)}{\partial x_i}$, the derivative of total profit w.r.t. x_i is given by

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{4}{3}\left(-\frac{dC_i}{dx_i}\int_0^{x_1} q_i(x)dx - \frac{dC_i}{dx_i}\int_{x_1}^{x_2} q_i(x)dx - \frac{dC_i}{dx_i}\int_{x_2}^{1} q_i(x)dx\right).$$

This expression is very similar to the one obtained in the case without care,[28] the only difference being that the output level depends here on care.

The next proposition provides the equilibrium outcome in terms of locations/product choices. Denote $\underline{h}$ the level of harm that satisfies the condition $\frac{4}{3}\frac{\underline{h}^2}{k}\left(a - \underline{h}\theta_0 - \frac{1}{2}\right) + \underline{h}\theta_0 = a - \frac{3}{2}$, and $\overline{h}$ the level of harm that satisfies $\frac{4}{5}\frac{\overline{h}^2}{k}\left(a - \overline{h}\theta_0 - \frac{1}{2}\right) + \overline{h}\left(\frac{4}{5} + \frac{3}{5}\theta_0\right) = a - \frac{11}{10}$; we obtain :

Proposition 3 *Under A1,A2 and A3, the unique stable location equilibrium is either : a)* $x_1 = \frac{1}{2} = x_2$ *if* $h < \underline{h}$; *or b)* $0 < x_1 = \frac{1}{2}(a - h\theta_0)\left(1 - \frac{4}{3}\frac{h^2}{k}\right) + \frac{1}{3}\frac{h^2}{k} - \frac{1}{4} < x_2 = \frac{5}{4} - \frac{1}{3}\frac{h^2}{k} - \frac{1}{2}(a - h\theta_0)\left(1 - \frac{4}{3}\frac{h^2}{k}\right) < 1$ *if* $h \in [\underline{h}, \overline{h})$.

[28] And still represents the quantity-median property.

Proof. See the Appendix, where both specific thresholds of the environmental harm, $\underline{h}$ and $\overline{h}$, are determined. Note that these different thresholds are required for reasons that are similar as before: the lower bound, $\underline{h}$, allows to distinguish the stable/unstable equilibrium, whereas the upper one, $\overline{h}$, guarantees the full market coverage by both firms. ■

According to Proposition 3, the consequences in terms of product differentiation/location choices are very similar to those obtained in Proposition 1 : minimum differentiation prevails for low levels of environmental harm, whereas dispersion obtains for higher levels. However, some new findings are now available. In case of central agglomeration, the minimum differentiation also characterizes the investment in care: there is neither horizontal differentiation, nor vertical differentiation. In turn, when firms are dispersed in equilibrium, they are also vertically differentiated: at each location x they obtain different market shares for their output (because they do not share the same location/product specification), and at the same time they undertake different levels of investment in environmental measures. Finally, note that similarly to the previous case without care, the larger the environmental harm, the larger the degree of differentiation under strict liability. Nevertheless, the highest possible degree of differentiation obtained in equilibrium falls short of the socially efficient one - hence, product differentiation in a duopoly is always suboptimal, as we show next.

The properties of the social optimum with durable precaution are as follows:

At stage 3, Social Welfare at each location is given by

$$W(x) = aQ(x) - \frac{(Q(x))^2}{2} - \sum_{i=1}^{2} (C_i + h\theta_i(x))\, q_i(x) - \sum_{i=1}^{2} k(\theta_i(x)). \qquad \text{(Wdc)}$$

Thus the derivative of $W(x)$ w.r.t. output for firm $i \in \{1,2\}$ is

$$\frac{\partial W(x)}{\partial q_i(x)} = a - Q(x) - (C_i + h\theta_i(x)) \,.$$

This implies that at any given location x, either a) $C_1 + h\theta_1(x) = C_2 + h\theta_2(x)$, and the solution to the stage-2 FOC, $\frac{\partial W(x)}{\partial q_i(x)} = 0$, is $q_i(x) = \frac{a - C_i - h\theta_i(x)}{2} > 0$ for $i \in \{1,2\}$ with $q_1(x) = \frac{Q(x)}{2} = q_2(x)$; or b) $C_i + h\theta_i(x) < C_j + h\theta_j(x)$, and the solution is $q_i(x) = Q(x) = a - C_i - h\theta_i(x) > q_j(x) = 0$.

At stage-2, the derivative of Social Welfare w.r.t. care investment for firm $i \in \{1, 2\}$ at a location where $q_i(x) > 0$ is $\frac{\partial W(x)}{\partial \theta_i(x)} = -\left(q_i(x)h + k'(\theta_i(x))\right)$. Hence, the stage-2 FOC for firm $i \in \{1, 2\}$ corresponds to a $\theta_i(x) > 0$ (otherwise $\theta_i(x) = 0$), now given by:

$$hq_i(x) = -k'(\theta_i(x)). \tag{7}$$

Note that (7) implies that at a location x where both firms are active, they deliver the same quantity $q_1(x) = q_2(x)$ and undertake the same level of care, i.e. $\theta_1(x) = \theta_2(x) = \theta(x)$, which is no longer constant across locations but now depends on the distance (i.e. the transportation cost through $q_i(x)$).

At stage 1, assume the planner contemplates distinct locations/product specifications for the two firms: $0 \leq x_1 \leq \frac{1}{2} \leq x_2 \leq 1$. At each local market x, the aggregate output $Q(x)$ is always delivered with the lowest transportation cost (denoted $\tilde{C}$), while avoiding to duplicate the cost of care. Thus, using the stage-2 FOC, Social Welfare at each x may be written as $W(x) = \frac{1}{2}\left(Q(x)\right)^2 - k(\theta(x))$ with $Q(x) = a - h\theta(x) - \tilde{C}$ and $\theta(x)$ satisfying

$$hQ(x) = -k'(\theta(x)). \tag{7'}$$

Given the potentially distinct locations, firm 1 will deliver output and exert a certain amount of care at all the local markets in $[0, x_1]$ together with those in $[x_1, \frac{x_1+x_2}{2}]$, whereas firm 2 will do the same at all the local markets in $[x_2, 1]$ together with those in $[\frac{x_1+x_2}{2}, x_2]$ respectively. As a result, Social Welfare over all locations along the unit line is now written as

$$\begin{aligned} W_G \;=\; & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(WGdc)} \\ & \int_0^{x_1} \left(\frac{1}{2}\left(Q(x)\right)^2 - k\left(\theta(x)\right)\right) dx + \int_{x_1}^{\frac{x_1+x_2}{2}} \left(\frac{1}{2}\left(Q(x)\right)^2 - k\left(\theta(x)\right)\right) dx \\ & + \int_{\frac{x_1+x_2}{2}}^{x_2} \left(\frac{1}{2}\left(Q(x)\right)^2 - k\left(\theta(x)\right)\right) dx + \int_{x_2}^{1} \left(\frac{1}{2}\left(Q(x)\right)^2 - k\left(\theta(x)\right)\right) dx. \end{aligned}$$

Differentiating W_G w.r.t. x_i at any location x where firm i operates yields

$$\frac{\partial W(x)}{\partial x_i} = Q(x)\frac{dQ(x)}{dx_i} - k'(\theta(x))\frac{d\theta(x)}{dx_i} = Q(x)\left(\frac{dQ(x)}{dx_i} + h\frac{d\theta_i(x)}{dx_i}\right),$$

where the second equality results from stage-2 FOC. From the definition of total output $Q(x)$, one obtains after differentiating w.r.t. x_i that $\frac{dQ(x)}{dx_i} = -h\frac{d\theta_i(x)}{dx_i} - \frac{dC_i}{dx_i}$. Substituting yields $\frac{\partial W(x)}{\partial x_i} = -Q(x)\frac{dC_i}{dx_i}$, which is very similar to what happens without care, except that here, the output does depend on the level of care. Again, to obtain closed-form solutions, we still consider Assumption 3. In the Appendix, we show that the next result holds:

Proposition 4 *Under A1,A2 and A3, the first best solution corresponds to a differentiated level of care at each location according to (7'), and the same locations as without care, i.e.* $x_1 = \frac{1}{4}, x_2 = \frac{3}{4}$.

Proposition 4 establishes that the socially efficient locations are still independent from the care decision, the same degree of firm dispersion being always socially efficient. In contrast, the socially optimal level of durable care does depend on the optimal location (through the output level, and thus the transportation costs): hence, some degree of vertical differentiation is always socially optimal.

Returning to Proposition 3, several implications are worth mentioning w.r.t. strict liability. The comparison of (6) and (7') suggests that the equilibrium level of care now departs from the first best level at any location, and furthermore, it may be smaller than the socially optimal one. Duopoly firms may invest too much in care, since this choice is driven by two opposing forces. First, at each location the marginal benefit of care per unit of output is larger under strict liability than at the social optimum, all else equal – this tends to raise the equilibrium level of care above the socially optimal one for any given level of output. Second, the marginal benefit of care under strict liability depends on the individual level of output, whereas at the social optimum, it depends on the aggregate output – this in turns tends to make the market equilibrium level of care fall below the socially optimal level.[29] A further third reason explains the difference at each location between the market-set level of care and the socially optimal one: this is related to the firms' location decisions, which are always suboptimal. The distortion in output and care, which stems from the fact that transportation costs are not minimized, is the third factor behind the difference between the equilibrium and the optimal levels of care. The net effect of these three forces is thus generally hard to sign.

[29] This ambiguous effect of strict liability when care is durable has been also obtained for a duopoly with a homogeneous good - see Charreire and Langlais 2021.

6 Equilibrium under Negligence

We now examine the case of a negligence (at-fault) rule, which covers the activities out of scope for existing liability laws such as the CERCLA, or the Environmental Liability Directive of the European Union, which only address strict liability.

An injurer subject to a negligence rule is only liable for damages if she acted negligently, in the sense that her decision falls short of some predetermined behavioral standard of care; according to the L&E literature, a natural behavioral standard is provided by the first best level of care.

The timing of the game is as before, except that at stage 0, Courts enforce the negligence rule, using for each local market x a standard of care $\hat{\theta}_i(x)$ set according to the first best efficient level of care defined by (7); this corresponds to a flexible standard of care, i.e. firms are not considered negligent as soon as their investment in care is the best reply to the output delivered to the local market.[30]

At stage 3, each firm chooses a level of output $q_i(x)$ that maximizes its profit given the output of the competitor at each location x. The individual profit at x writes for $i \in \{1, 2\}$ as

$$\pi_i(x; \mu_i) = (a - Q(x) - C_i - \mu_i \times h \times \theta_i(x))\, q_i(x) - k(\theta_i(x)),$$

with $\begin{cases} \mu_i = 0 & if\ \theta_i(x) \leq \hat{\theta}_i(x) \\ \mu_i = 1 & otherwise \end{cases}$, meaning that abiding by the standard of care allows a firm to avoid the liability cost (i.e. a firm faces a "no liability" rule, but at a cost $k(\theta_i(x))$), while departing from it makes a firm face the same consequences as under strict liability. Thus, a firm's profit under the negligence rule is either the profit obtained under a "no liability" rule (up to the cost of efficient care activities when it adheres to the standard) or, alternatively the profit made when facing the strict liability rule (as it deviates from the standard), all else equal.

[30] The due care depends on the quantity to be delivered at a local market. This captures the fact that Courts may not focus on a specific level of care, but instead, in order to establish whether a firm is liable or not in case of environmental harm, they proceed with a negligence test so as to assess whether the firm/defendant adopted an efficient rule of behavior, in the sense that it was the offender's best reponse, given the circumstances.

To see this, note that the FOC w.r.t. output at each location x are similar to (5) (up to the multiplicative factor μ_i), and solving for $q_i(x)$ for $i \neq j \in \{1,2\}$, we obtain

$$q_i(x) = \frac{1}{3}\left(a - 2\left(\mu_i h\theta_i(x) + C_i\right) + \left(\mu_j h\theta_j(x) + C_j\right)\right). \tag{8}$$

This means that abiding by the standard rather than not provides a strategic advantage which materializes into a higher output. However, this advantage is dampened (respectively, enhanced) when the rival also complies with (respectively, deviates from) his own standard.

Furthermore, at stage 2, complying with the standard may be costly since for any given level of output, the first best level of care leads to a different cost incurred as compared with the profit-maximizing care level (see before): thus, whether a firm is better off at each local market by complying with $\hat{\theta}_i(x)$ which satisfies (7) or, on the contrary, deviating towards $\theta_i(x)$ which satisfies (6), eventually depends on the sign of the difference $\pi_i(x;\mu_i = 0) - \pi_i(x;\mu_i = 1)$. Actually, we show that the output effect dominates the cost of care effect, and therefore the next result holds:

Proposition 5 *At each location, a firm's profit is higher when complying with the flexible standard instead of not complying ($\pi_i(x;\mu_i = 0) > \pi_i(x;\mu_i = 1)$ for $i \in \{1,2\}$).*

Proof. The individual profit for firm $i \in \{1,2\}$ at any location x may be written as $\pi_i(x;\mu_i) = (q_i(x;\mu_i))^2 - k(\theta_i(x;\mu_i))$ (still using the stage-3 FOC). Differentiating w.r.t. μ_i yields $\frac{d\pi_i}{d\mu_i}(x;\mu_i) = 2q_i(x)\frac{dq_i(x)}{d\mu_i} - k'(\theta_i(x))\frac{d\theta_i(x)}{d\mu_i}$. Note that the stage-2 choice of care $\theta_i(x)$ for firm $i \in \{1,2\}$ is set according to

$$h\left(1 + \frac{1}{3}\mu_i\right) q_i(x) = -k'\left(\theta_i(x)\right), \tag{9}$$

which encompasses both (6) (when $\mu_i = 1$) and (7) (when $\mu_i = 0$). Differentiating (8)-(9) w.r.t. μ_i and solving, we obtain

$$\frac{dq_i(x)}{d\mu_i} = \left(-\frac{2}{3}h\right)\frac{k''\left(\theta_i(x)\right)\theta_i(x) - \frac{1}{3}h\mu_i q_i(x)}{k''(\theta_i(x)) - \frac{2}{3}h^2\left(1 + \frac{1}{3}\mu_i\right)\mu_i}, \tag{10a}$$

$$\frac{d\theta_i(x)}{d\mu_i} = \left(-\frac{1}{3}h\right)\frac{q_i(x) - 2h\left(1 + \frac{1}{3}\mu_i\right)\theta_i(x)}{k''(\theta_i(x)) - \frac{2}{3}h^2\left(1 + \frac{1}{3}\mu_i\right)\mu_i}. \tag{10b}$$

Now differentiating $\pi_i(x;\mu_i)$ w.r.t. μ_i, we can write

$$\begin{aligned}\frac{d\pi_i}{d\mu_i}(x;\mu_i) &= q_i(x)\left(2\frac{dq_i(x)}{d\mu_i}+h\left(1+\frac{1}{3}\mu_i\right)\frac{d\theta_i(x)}{d\mu_i}\right)\\ &= \left(-\frac{1}{3}\frac{h^2}{\Lambda}\right)(q_i(x))^2(1-\mu_i)+\left(-\frac{4}{3}\frac{h}{\Lambda}\right)q_i(x)\theta_i(x)\left(k''(\theta_i(x))-\frac{1}{2}\left(1+\frac{1}{3}\mu_i\right)^2h^2\right).\end{aligned}$$

The first line is obtained using (9), while the second line is obtained using (10a)-(10b), where $\Lambda \equiv k''(\theta_i(x))-\frac{2}{3}h^2\left(1+\frac{1}{3}\mu_i\right)\mu_i>0$ by the SOC. It is obvious that $1-\mu_i\geq 0$ and $k''(\theta_i(x))-\frac{1}{2}h^2\left(1+\frac{1}{3}\mu_i\right)^2>0$. Thus $\frac{d\pi_i}{d\mu_i}(x;\mu_i)<0$.

■

In other words, at stage 2, it is sequentially rational for firms to comply with the flexible standard of care (7).

Finally, at stage 1, firms choose their location $x_i\in[0,1]$ in order to maximize their total profit, anticipating they will both comply with their care standard at stage 2 and then play their Cournot-Nash quantities at stage 3, defined respectively by (7)-(8). The next proposition states the equilibrium outcome in terms of locations/product choices:

Proposition 6 *Under A1,A2 and A3, the unique location equilibrium is* $x_1=\frac{1}{2}=x_2$.

Proof. See the Appendix. The intuition of the result is that since both firms comply with their standard of care at stage 2, then the marginal impact of location choices on their profits is driven by the adjustment of output with the change in the transportation costs, as it is the case under the "no liability" regime.

■

To fully grasp the result displayed in Proposition 6, remember that negligence guarantees that firms adhere to their flexible standard of care at each location. But given the up-front cost associated with the investment in environmental preservation, and given that firms thereby avoid liability whatever the level of output delivered at each location, they ultimately sell at each local market the same Cournot quantity as under a "no liability" rule. As a result, the negligence rule prevents firms' dispersion, and instead, the equilibrium spatial pattern corresponds to the "no liability" outcome (i.e. central agglomeration/no differentiation: $x_1=\frac{1}{2}=x_2$).

Thus, similarly to the case of strict liability, negligence also triggers suboptimal product specification choices. However, for large enough levels of environmental harm, the negligence rules entails more distortions in the scope of product differentiation than strict liability. As a result, the comparison of equilibrium output and precautionary effort between the two liability regimes is not straightforward: it depends on the various parameters (a, h, $c(.)$), and also on the distance to the local market where the product is sold. To see this, consider a level of environment harm low enough such that under both liability rules the equilibrium is characterized by central agglomeration. Then comparing (5) and (8) (by setting $\mu_i = 0 = \mu_j$) indicates that for any given level of care, the output under strict liability is smaller than under negligence (the marginal liability cost is higher under strict liability); in turn, comparing (6) and (7) shows that for any given level of output, the level of care under strict liability is larger than under negligence (the marginal benefit of care is larger under strict liability) – the net effect being thus ambiguous. Therefore, for levels of environmental harm that are large enough for spatial dispersion to occur under strict liability, the comparison is complicated by the influence of distance and transport costs.

7 Conclusion

The main objective of this paper is to shed some light on the issue of the endogenous product specification (or, equivalently, strategic location choices) when firms' outputs generate harm for the environment; more generally, the results extend to any situation involving third parties (victims having no economic nor contractual relationships with firms/the injurers), including cases where accidents lead to casualties. We examine the impact on the equilibrium spatial/product differentiation pattern of the size of the external harm and of the type of environmental precautionary measures that firms may undertake, under alternative liability regimes. The next table summarizes our central results, according to which strict liability and negligence lead to quite different outcomes in terms of both horizontal and vertical differentiation, as long as the environmental harm is large enough:

	Horizontal differentiation (Product customization)		Vertical differentiation (Precautionary effort)
Strict Liability	no	low harm	no
	yes	high harm	yes
Negligence	no	low harm	no
	no	high harm	no

Under strict liability, firms will vertically differentiate whenever it is individually profitable to horizontally differentiate – which occurs for large enough levels of environmental harm. In contrast, negligence has a chilling effect on both horizontal and vertical differentiation for any size of the environmental harm.

Our results are obtained under few assumptions (uniform density of consumers, linear demand, production and transportation costs) that may easily be relaxed without altering our qualitative conclusions. Moreover, our assumption regarding the cost of preservation measures is general enough, i.e. with a mixed nature durable/non durable, to encompass a large variety of situations.

References

[1] Anderson S.P. and Neven D.J. 1991. Cournot competition yields spatial agglomeration, *International Economic Review*, 32, 793-808.

[2] d'Aspremont C., Gabszewicz J. and Thisse, J.F. 1979. On Hotelling's stability in competition, Econometrica, 47, 1145-1150.

[3] Baumann F., Charreire M., and Cosnita-Langlais A. 2020. Market collusion with joint harm and liability sharing. *International Review of Law and Economics*, 62, 105885.

[4] Baumann F. and Friehe T. 2015. Optimal damage multipliers in oligopolistic markets, *Journal of Institutional and Theoretical Economics*, Vol. 171 (4), 622-640.

[5] Baumann F. and Friehe T. 2021. Products liability, consumer misperceptions, and the allocation of consumers to firms, *Economics Letters*, vol. 198, issue C.

[6] Baumann F., Friehe T., and Rasch A. 2018. Product liability in markets for vertically differentiated products, *American Law & Economics Review,* 20, 46-81.

[7] Benassi, C., Chirco, A., and Scrimitore, M. 2007. Spatial Discrimination with Quantity Competition and High Transportation Costs: a Note, *Economics Bulletin*, 12(1), 1-7.

[8] Biscaia R. and Mota I. 2013. Models of spatial competition: A critical review, *Papers in Regional Science*, 92(4), 851-871.

[9] Charreire M. and Langlais E. 2021. Should environment be a concern for competition policy when firms face environment liability ?, *International Review of Law and Economics*, 67, 1-16.

[10] Chamorro-Rivas J.M. 2000. Spatial dispersion in Cournot competition, *Spanish Economic Review*, 2, 145-152.

[11] Chen Y. and Hua X. 2017. Competition, product safety, and product liability, *Journal of Law, Economics, and Organization*, 33(2): 237-267.

[12] Clemenz G. 2010. Eco-Labeling and Horizontal Product Differentiation, *Environmental and Resource Economics* 45: 481-497.

[13] Conrad K. 2005. Price competition and product differentiation when consumers care for the environment, *Environmental and Resource Economics*, 31, 1-9.

[14] Cosnita-Langlais A. and Langlais E. 2022. Endogenous market structures, product liability, and the scope of product differentiation, *EconomiX Working Papers Series.*

[15] D'Antoni M. and Tabbach A. 2014. Inadequate compensation and multiple equilibria, *International Review of Law and Economics*, 38, 33-47.

[16] Dari-Mattiacci G. 2006. On the optimal scope of negligence, *Review of Law & Economics.*

[17] Das S. 2016. Certification Under Oligopolistic Competition, *Environmental and Resource Economics*, 65, 251–271.

[18] Daughety A.F. and Reinganum J.F. 2006. Markets, torts and social inefficiency, J.H., *RAND Journal of Economics,* 37, 300-323.

[19] Daughety A.F. and Reinganum J.F. 2013. Economic Analysis of Products Liability: Theory. In: Arlen, J.H., *Research Handbook on the Economics of Torts*, Edward Elgar.

[20] de Palma A., Ginsburgh V., Papageorgiou Y.Y., and Thisse J.-F. 1985. The principle of minimum differentiation holds under sufficient heterogeneity, *Econometrica*, 53 (4), 767-781.

[21] Eaton C.B. and Schmitt N. 1994. Flexible manufacturing and market structure, *American Economic Review*, 84, 875-888.

[22] Endres A. and Lüdecke A. 1998. Incomplete strict liability: effects on product differentiation and information provision. *International Review of Law and Economics*, 18, 511-28.

[23] Endres A. and Friehe T. 2013. The monopolistic polluter under environmental liability law: incentives for abatement and R&D, *Social Choice and Welfare*, 753–770.

[24] Endres A., Friehe T., and Rundshagen B. 2015. "It's All in the Mix!"- Internalizing externalities with R&D subsidies and environmental liability, *Social Choice and Welfare*, 44(1), 151-178.

[25] Gupta B., Lai F.C., Pal D., Sarkar J. and Yu C.M. 2004. Where to locate in a circular city, *International Journal of Industrial Organization*, 22, 759-782.

[26] Gupta B., Pal D., Sarkar J. 1997. Spatial Cournot competition and agglomeration in a model of location choice. *Regional Science and Urban Economics*, 27 (3), 261–282.

[27] Iyer G. and Soberman D.A. 2016. Social Responsibility and Product Innovation, *Marketing Science*, 35(5), 727-742.

[28] Kats A. 1995. More on Hotelling's stability in competition, *International Journal of Industrial Organization*, 13, 89-93

[29] Kornhauser L.A. and Revesz R.L. 1989. Sharing Damages among Multiple Tortfeasors. *Yale Law Journal*, 98, 831-884.

[30] Kreps D. and Scheinkman J. 1983. Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, *The Bell Journal of Economics*, 14(2), 326-337.

[31] Matsushima N. 2001. Cournot competition and spatial agglomeration revisited, *Economics Letters*, 73, 175-177

[32] Matsumura T. and Shimizu D. 2005. Spatial Cournot competition and economic welfare: a note. *Regional Science and Urban Economics*, 35 (6), 658-670.

[33] Mayer T. 2000. Spatial Cournot competition and heterogeneous production costs across locations, *Regional Science and Urban Economics*, 20, 325-352.

[34] Miceli J.T. and Segerson K. 1991. Joint liability in torts: Marginal and infra-marginal efficiency. *International Review of Law and Economics*, 11(3), 235-249.

[35] Pal D. 1998. Does Cournot competition yield spatial agglomeration?, *Economics Letters*, 60, 49-53.

[36] Pal D. and Sarkar J. 2002. Spatial competition among multi-store firms", *International Journal of Industrial Organization*, 20, 163-190.

[37] Shimizu D. 2002. Product differentiation in spatial Cournot markets, *Economics Letters*, 76, 317-322

[38] Spulber D. 1988. Products Liability and Monopoly in a Contestable Market, *Economica*, 55, 333-341.

[39] Van Egteren H. and Smith R.T. 2002. Environmental Regulations Under Simple Negligence or Strict Liability, *Environmental and Resource Economics*, 21(4), 367-394.

8 Appendix

In the proofs of Propositions 1, 3 and 4 below, we show that any candidate to a SPNE outcome in terms of location is a solution to the system of firms' best reply functions denoted, $x_2 = f_{F1}(x_1)$ for Firm 1, and$x_2 = g_{F2}(x_1)$ for Firm 2 respectively, which have the next general expression:

$$\begin{cases} x_2 = f_{F1}(x_1) \Leftrightarrow A + \alpha\left(x_2 - x_1\right)^2 - \beta x_1 - \alpha x_2 = 0 \\ x_2 = g_{F2}(x_1) \Leftrightarrow A - \alpha\left(x_2 - x_1\right)^2 - \alpha x_1 - \beta x_2 = 0 \end{cases}. \tag{A}$$

The precise parameters $A > 0$, $\alpha > 0$ and $\beta > 0$ (under appropriate assumptions) are specified in the different proofs below. In case of multiplicity of candidates, we select as the unique SPNE the candidate passing the stability test, i.e. the one that satisfies the condition $|f'_{F1}(x_1)| > |g'_{F2}(x_1)|$, which is written as

$$\left|\frac{2\alpha\left(x_2 - x_1\right) + \beta}{2\alpha\left(x_2 - x_1\right) - \alpha}\right| > \left|\frac{2\alpha\left(x_2 - x_1\right) - \alpha}{2\alpha\left(x_2 - x_1\right) + \beta}\right|. \tag{B}$$

Proof of Proposition 1. The (subgame perfect) equilibrium outcome in terms of location choices may be characterized as follows. Consider a couple x_1, x_2, such that $0 \leq x_1 \leq x_2 \leq 1$. Using (Π_G) and (2), total profit for firm 1 over the whole unit line writes

$$\Pi_1 = \frac{1}{9}\left(\begin{array}{c} \int_0^{x_1}\left(a - h + x_2 - 2x_1 + x\right)^2 dx + \int_{x_1}^{x_2}\left(a - h + x_2 + 2x_1 - 3x\right)^2 dx \\ + \int_{x_2}^{1}\left(a - h + 2x_1 - x_2 - x\right)^2 dx \end{array}\right).$$

The derivative w.r.t. x_1 writes:

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{9}\left(\left(a - h - \frac{1}{2}\right) + \left(x_2 - x_1\right)^2 - 2\left(a - h - 1\right)x_1 - x_2\right).$$

The SOC is satisfied since under Assumption 2, for any $0 \leq x_1 \leq x_2 \leq 1$ we obtain:

$$\frac{\partial^2 \Pi_1}{\partial x_1^2} = \frac{4}{9}\left(-2\left(a - h - 1\right) - 2(x_2 - x_1)\right) < 0$$

Hence firm 1' best reply function may be written in implicit form (A) (setting $\frac{\partial \Pi_1}{\partial x_1} = 0$), with $A = \left(a - h - \frac{1}{2}\right)$, $\alpha = 1$ and $\beta = 2\left(a - h - 1\right)$.

Similarly, total profit for firm 2 over the whole unit line writes as

$$\Pi_2 = \frac{1}{9}\left(\begin{array}{c} \int_0^{x_1} (a-h+x_1-2x_2+x)^2\,dx + \int_{x_1}^{x_2} (a-h-x_1-2x_2+3x)^2\,dx \\ + \int_{x_2}^{1} (a-h+2x_2-x_1-x)^2\,dx \end{array}\right).$$

The derivative w.r.t. x_2 is:

$$\frac{\partial \Pi_2}{\partial x_2} = \frac{4}{9}\left(\left(a-h-\frac{1}{2}\right) - (x_2-x_1)^2 - 2(a-h-1)\,x_2 - x_1\right).$$

Again the SOC is satisfied since we have $\frac{\partial^2\Pi_2}{\partial x_2^2} = \frac{\partial^2\Pi_1}{\partial x_1^2}$. Hence firm 2' best reply function may be written using the implicit form (A) (setting $\frac{\partial\Pi_2}{\partial x_2}=0$), again with $A = a-h-\frac{1}{2}$, $\alpha = 1$ and $\beta = 2(a-h-1)$.

Candidates to a SPNE.

a) It is straightforward that $x_1 = \frac{1}{2} = x_2$ is a natural solution to the system (A) (i.e. it solves $\frac{\partial\Pi_1}{\partial x_1} = 0$).

b) Let us investigate whether a asymmetric solution such as $x_2 - x_1 = \epsilon > 0$ exists. Solving (A) for ϵ gives $\epsilon = \frac{3}{2} - (a-h) > 0$ only if $h > a - \frac{3}{2}$, and thus substituting in (A) and solving for x_2, x_1 yields $x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$, and $x_1 = \frac{1}{2}(a-h) - \frac{1}{4}$.

Stability of SPNE. Given Assumption 2, it can be seen that :

– for $x_1 = \frac{1}{2} = x_2$ the stability condition (B) becomes : $2(a-h-1) > \frac{1}{2(a-h-1)}$. It can be verified after rearranging that it is equivalent to $\left(a-h-\frac{3}{2}\right)\left(a-h-\frac{1}{2}\right) > 0$, which holds if and only if $h < a-\frac{3}{2}$ (and thus for $h > a - \frac{3}{2}$, it cannot be the solution).

– for $x_1 = \frac{1}{2}(a-h) - \frac{1}{4}, x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$, the stability condition (B) is now written $\frac{1}{2(a-h-1)} > 2(a-h-1) \Leftrightarrow \left(a-h-\frac{3}{2}\right)\left(a-h-\frac{1}{2}\right) < 0$, which holds if and only if $h > a - \frac{3}{2}$ (and thus for $h < a - \frac{3}{2}$, it cannot be the solution).

Full market coverage. Finally, for each equilibrium outcome in terms of location, substituting in (2) yields the equilibrium output at each location x. Our analysis holds under the assumption of full coverage of the market by both firms, i.e. both firms sell positive outputs at any local market, and neither firm holds a monopoly position at some location:

– for $x_1 = \frac{1}{2} = x_2$, it is easy to verify that $q_1(x=1) = \frac{1}{3}\left(a-h-\frac{1}{2}\right) = q_2(x=0) > 0$ under Assumption 1.

– for $x_1 = \frac{1}{2}(a-h) - \frac{1}{4}, x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$, it can be verified that $q_1(x=1) = \frac{1}{3}\left(\frac{5}{2}(a-h) - \frac{11}{4}\right) = q_2(x=0) > 0$ only if $\frac{5}{2}(a-h) - \frac{11}{4} > 0 \Leftrightarrow h < a - \frac{11}{10}$, which is more restrictive than Assumption 2.

■

Proof of Proposition 2. It is straightforward that the derivative of (WGnc) w.r.t. to x_1 is

$$\begin{aligned}\frac{\partial W_G}{\partial x_1} &= -\int_0^{x_1}((a-h)-(x_1-x))\,dx+\int_{x_1}^{\frac{x_1+x_2}{2}}(a-h-(x-x_1))\,dx \\ &= -2(a-h)x_1+(a-h+x_1)\left(\frac{x_1+x_2}{2}\right)-\frac{1}{2}\left(\frac{x_1+x_2}{2}\right)^2,\end{aligned}$$

and the derivative of (WGnc) w.r.t. to x_2 is

$$\begin{aligned}\frac{\partial W_G}{\partial x_2} &= -\int_{\frac{x_1+x_2}{2}}^{x_2}((a-h)-(x_2-x))\,dx+\int_{x_2}^{1}(a-h-(x-x_2))\,dx \\ &= \left(a-h-\frac{1}{2}\right)-2\left(a-h-\frac{1}{2}\right)x_2+(a-h-x_2)\left(\frac{x_1+x_2}{2}\right)+\frac{1}{2}\left(\frac{x_1+x_2}{2}\right)^2.\end{aligned}$$

Note that since the derivatives are not identical, then $x_1=\frac{1}{2}=x_2$ cannot be the solution to the system $\frac{\partial W_G}{\partial x_1}=0, \frac{\partial W_G}{\partial x_2}=0$. On the other hand, both firms being identical in all respect except maybe their location, the location equilibrium is necessarily symmetric, i.e. $x_1+x_2=1$. As a result, both FOCs can be simplified to $\frac{\partial W_G}{\partial x_1}=2\left(a-h-\frac{1}{4}\right)\left(\frac{1}{4}-x_1\right)=0$ and $\frac{\partial W_G}{\partial x2}=2\left(a-h-\frac{1}{4}\right)\left(\frac{3}{4}-x_2\right)=0$ respectively. Hence the solution is $x_1=\frac{1}{4}, x_2=\frac{3}{4}$.

■

Proof of Proposition 3. Assuming $0\leq x_1\leq x_2\leq 1$ and using (Π_G) and (4), total profit for firm 1 over the whole unit line writes:

$$\Pi_1=\begin{pmatrix}\int_0^{x_1}\left((q_1(x))^2-k(\theta_1(x))\right)dx+\int_{x_1}^{x_2}\left((q_1(x))^2-k(\theta_1(x))\right)dx \\ +\int_{x_2}^{1}\left((q_1(x))^2-k(\theta_1(x))\right)dx\end{pmatrix}.$$

Differentiating w.r.t. x_1 yields, at any x,

$$\frac{\partial\pi_1(x)}{\partial x_1}=2q_1(x)\frac{dq_1(x)}{dx_1}-k'(\theta_1(x))\frac{d\theta_1(x)}{dx_1}=2q_1(x)\left(\frac{dq_1(x)}{dx_1}+\frac{2}{3}h\frac{d\theta_1(x)}{dx_1}\right),$$

where the second equality results from (6). Differentiating (5) we obtain that $\frac{dq_1(x)}{dx_1}+\frac{2}{3}h\frac{d\theta_1(x)}{dx_1}=-\frac{2}{3}\frac{dC_1}{dx_1}$. Therefore we can write equivalently

$$\frac{\partial\pi_1(x)}{\partial x_1}=-\frac{4}{3}q_1(x)\times\frac{dC_1}{dx_1},$$

and finally

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{3}\left(-\int_0^{x_1} q_1(x)dx + \int_{x_1}^{x_2} q_1(x)dx + \int_{x_2}^{1} q_1(x)dx\right),$$

which is very similar to the expression obtained for the proof of Proposition 1. More precisely, it can be verified using (5) that

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{3}\left(\begin{array}{c} \left(\frac{\partial \Pi_1}{\partial x_1}\right)_{no\ care} - \frac{1}{3}\int_0^{x_1} h(1-2\theta_1(x)+\theta_2(x))dx \\ +\frac{1}{3}\int_{x_1}^{x_2} h(1-2\theta_1(x)+\theta_2(x))dx + \frac{1}{3}\int_{x_2}^{1} h(1-2\theta_1(x)+\theta_2(x))dx \end{array}\right).$$

Now plunging Assumption 3 into (6), we can write that at each local market x the levels of care chosen by the firms are set such that

$$\begin{aligned} -k\left(\theta_1(x)-\theta_0\right) &= \frac{4}{3}hq_1(x), \text{and} \\ -k\left(\theta_2(x)-\theta_0\right) &= \frac{4}{3}hq_2(x) \text{ respectively.} \end{aligned}$$

Solving with (6), we obtain

$$\begin{aligned} \theta_1(x) &= v - wC_1 - rC_2, \\ \theta_2(x) &= v - rC_1 - wC_2, \end{aligned}$$

where we denote

$$v = \frac{k\theta_0 - \frac{4}{9}ha}{k-\frac{4}{9}h^2}\ ;\ w = \frac{\frac{8}{9}h\left(\frac{2}{3}h^2-k\right)}{\left(k-\frac{4}{9}h^2\right)\left(k-\frac{4}{3}h^2\right)}\ ;\ r = \frac{\frac{4}{9}h\times k}{\left(k-\frac{4}{9}h^2\right)\left(k-\frac{4}{3}h^2\right)}.$$

Substituting, then integrating by part, we obtain

$$\begin{aligned} \frac{\partial \Pi_1}{\partial x_1} &= \frac{4}{3}\left(\frac{\partial \Pi_1}{\partial x_1}\right)_{no\ care} - \frac{4}{9}h\left(\left(1-v+(2w-r)x_1+(2r-w)x_2\right)x_1 - \frac{w+r}{2}x_1^2\right) \\ &+\frac{4}{9}h\left(\begin{array}{c} \left(1-v-(2w-r)x_1+(2r-w)x_2\right)x_2 + \frac{3}{2}\left(w-r\right)x_2^2 \\ -\left(1-v-(2w-r)x_1+(2r-w)x_2\right)x_1 - \frac{3}{2}\left(w-r\right)x_1^2 \end{array}\right) \\ &+\frac{4}{9}h\left(\begin{array}{c} \left(1-v-(2w-r)x_1-(2r-w)x_2\right)+\frac{w+r}{2} \\ -\left(1-v-(2w-r)x_1-(2r-w)x_2\right)x_2 - \frac{w+r}{2}x_2^2 \end{array}\right). \end{aligned}$$

Finally, collecting the different terms and developing, it can be verified that

$$\begin{aligned}\frac{\partial \Pi_1}{\partial x_1} &= \underbrace{\frac{4}{9}\left(\left(a-h-\frac{1}{2}\right)+\left(x_2-x_1\right)^2-2\left(a-h-1\right)x_1-x_2\right)}_{\text{term without care}} \\ &\quad +\frac{4}{9}h\left(\left(1-v+\frac{w+r}{2}\right)+2\left(r-\frac{w}{2}\right)\left(x_2-x_1\right)^2-2\left(1-v+w-\frac{r}{2}\right)x_1-2\left(r-\frac{w}{2}\right)x_2\right).\end{aligned}$$

The SOC requires now that for any $0 \leq x_1 \leq x_2 \leq 1$ it must be that $\frac{\partial^2 \Pi_1}{\partial x_1^2} < 0$, where

$$\frac{\partial^2 \Pi_1}{\partial x_1^2} = \frac{4}{9}\underbrace{\left(-2\left(a-h-1\right)-2(x_2-x_1)\right)}_{<0,\ under\ Ass.2}-\frac{4}{9}h\left(\underbrace{2\left(r-\frac{w}{2}\right)\left(x_2-x_1\right)}_{>0\ under\ Ass.3}+2\left(1-v+w-\frac{r}{2}\right)\right).$$

As a result, given Assumption 2, a sufficient condition for $\frac{\partial^2 \Pi_1}{\partial x_1^2} < 0$ to hold is that $1-v+w-\frac{r}{2} \geq 0 \Leftrightarrow a-h \geq k$, which corresponds to Assumption 3. Hence, collecting the different terms, firm's 1 best reply function may be written as in (A) (setting $\frac{\partial \Pi_1}{\partial x_1} = 0$), and denoting $A = \left(a-h-\frac{1}{2}\right)+h\left(1-v+\frac{w+r}{2}\right)$, $\alpha = 1+2h\left(r-\frac{w}{2}\right)$ and $\beta = 2\left((a-h-1)+h\left(1-v+w-\frac{r}{2}\right)\right)$.

By the same token, it can be shown that $\frac{\partial \pi_2(x)}{\partial x_2} = -\frac{4}{3}q_2(x) \times \frac{dC_2}{dx_1}$, and thus

$$\begin{aligned}\frac{\partial \Pi_2}{\partial x_2} &= \frac{4}{3}\left(\begin{array}{c}\left(\frac{\partial \Pi_2}{\partial x_2}\right)_{no\ care}-\frac{1}{3}\int_0^{x_1}h(1-2\theta_2(x)+\theta_1(x))dx \\ -\frac{1}{3}\int_{x_1}^{x_2}h(1-2\theta_2(x)+\theta_1(x))dx+\frac{1}{3}\int_{x_2}^{1}h(1-2\theta_2(x)+\theta_1(x))dx\end{array}\right) \\ &= \frac{4}{3}\left(\frac{\partial \Pi_2}{\partial x_2}\right)_{no\ care}-\frac{4}{9}h\left(\left(1-v+(2w-r)x_2+(2r-w)x_1\right)x_1-\frac{w+r}{2}x_1^2\right) \\ &\quad -\frac{4}{9}h\left(\begin{array}{c}\left(1-v+(2w-r)x_2-(2r-w)x_1\right)x_2-\frac{3}{2}\left(w-r\right)x_2^2 \\ -\left(1-v+(2w-r)x_2-(2r-w)x_1\right)x_1+\frac{3}{2}\left(w-r\right)x_1^2\end{array}\right) \\ &\quad +\frac{4}{9}h\left(\begin{array}{c}\left(1-v-(2w-r)x_2-(2r-w)x_1\right)+\frac{w+r}{2} \\ -\left(1-v-(2w-r)x_2-(2r-w)x_1\right)x_2-\frac{w+r}{2}x_2^2\end{array}\right).\end{aligned}$$

Finally, after developing and rearranging, it can be verified that

$$\begin{aligned}\frac{\partial \Pi_2}{\partial x_2} &= \underbrace{\frac{4}{9}\left(\left(a-h-\frac{1}{2}\right)-\left(x_2-x_1\right)^2-2\left(a-h-1\right)x_2-x_1\right)}_{\text{term without care}} \\ &\quad +\frac{4}{9}h\left(\left(1-v+\frac{w+r}{2}\right)-2\left(r-\frac{w}{2}\right)\left(x_2-x_1\right)^2-2\left(r-\frac{w}{2}\right)x_1-2\left(1-v+w-\frac{r}{2}\right)x_2\right).\end{aligned}$$

The SOC is also satisfied since it can be verified that $\frac{\partial^2 \Pi_2}{\partial x_2^2} = \frac{\partial^2 \Pi_1}{\partial x_1^2}$. Hence firm's 2 best reply function may be written (setting $\frac{\partial \Pi_2}{\partial x_2} = 0$) as in (A), now denoting $A = \left(a - h - \frac{1}{2}\right) + h\left(1 - v + \frac{w+r}{2}\right)$, $\alpha = 1 + 2h\left(r - \frac{w}{2}\right)$ and $\beta = 2\left((a - h - 1) + h\left(1 - v + w - \frac{r}{2}\right)\right)$.

Candidates to a SPNE.

a) Let us check whether $x_1 = \frac{1}{2} = x_2$ is a solution to the system (A). By construction, as shown in the proof of Proposition 1, $x_1 = \frac{1}{2} = x_2$ satisfies $\left(\frac{\partial \Pi_1}{\partial x_1}\right)_{no\ care} = 0$, implying that $\left(\frac{\partial \Pi_1}{\partial x_1}\right)_{x_1 = \frac{1}{2} = x_2}$ simplifies to

$$\left(\frac{\partial \Pi_1}{\partial x_1}\right)_{x_1 = \frac{1}{2} = x_2} = \frac{4}{9}h \begin{pmatrix} \left(1 - v + \frac{w+r}{2}\right) + (2r - w)(x_2 - x_1)^2 \\ -2\left(1 - v + w - \frac{r}{2}\right)x_1 - (2r - w)x_2 \end{pmatrix}_{x_1 = \frac{1}{2} = x_2}.$$

It can be checked that the RHS is also equal to 0. Thus $x_1 = \frac{1}{2} = x_2$ is a solution to (A) (i.e to the FOC $\frac{\partial \Pi_1}{\partial x_1} = 0$).

b) It can be shown that an asymmetric solution such as $x_2 - x_1 = \epsilon > 0$ also exists under ad hoc conditions. Solving (A) for ϵ yields $\epsilon = \frac{1}{2}\left(1 - \frac{\beta}{\alpha}\right)$ or

$$\epsilon = \frac{4}{3}\frac{h^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right) - \left(a - h\theta_0 - \frac{3}{2}\right) = 1 - \left(a - h\theta_0 - \frac{1}{2}\right)\left(1 - \frac{4}{3}\frac{h^2}{k}\right),$$

such that on the one hand $\epsilon < 1$ is satisfied, and on the other hand $\epsilon > 0$ if $\alpha > \beta$, or, equivalently rearranging, if $\frac{4}{3}\frac{h^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right) + h\theta_0 > a - \frac{3}{2}$. Note that the LHS of this inequality is increasing in h, while the RHS does not depend on h; thus defining $\underline{h}$ the threshold that satisfies the condition $\frac{4}{3}\frac{\underline{h}^2}{k}\left(a - \underline{h}\theta_0 - \frac{1}{2}\right) + h\theta_0 = a - \frac{3}{2}$, we obtain that for any $h > (<)\underline{h}$ we have that $\epsilon > (<)0$. Substituting in (A) and solving for x_2, x_1, gives $x_1 = \frac{1}{2}(1 - \epsilon)$ and $x_2 = \frac{1}{2}(1 + \epsilon)$, or

$$x_1 = \frac{1}{2}\left(a - h\theta_0 - \frac{1}{2}\right)\left(1 - \frac{4}{3}\frac{h^2}{k}\right) = \frac{1}{2}(a - h\theta_0)\left(1 - \frac{4}{3}\frac{h^2}{k}\right) + \frac{1}{3}\frac{h^2}{k} - \frac{1}{4}$$

and

$$x_2 = 1 - \frac{1}{2}\left(a - h\theta_0 - \frac{1}{2}\right)\left(1 - \frac{4}{3}\frac{h^2}{k}\right) = \frac{5}{4} - \frac{1}{3}\frac{h^2}{k} - \frac{1}{2}(a - h\theta_0)\left(1 - \frac{4}{3}\frac{h^2}{k}\right).$$

As shown in Proposition 1, $\hat{h} = a - \frac{3}{2}$ is the threshold such that for any $h > (<)\hat{h}$ then the unique SPNE when firms do not invest in care corresponds to the agglomeration (dispersion, respectively); hence it comes $\underline{h} < \hat{h}$.

Stability of SPNE. Given (B), we obtain that

– for $x_1 = \frac{1}{2} = x_2$, the condition becomes $\frac{\beta}{\alpha} > \frac{\alpha}{-\beta} \Leftrightarrow \left(\beta + \frac{\alpha}{2}\right)\left(\beta - \frac{\alpha}{2}\right) > 0$ such that if $\beta > \frac{\alpha}{2}$ then the candidate $x_1 = \frac{1}{2} = x_2$ is a stable equilibrium, and is thus the unique SPNE (i.e. the asymmetric solution cannot be the SPNE).

– for $x_1 = \frac{1}{2}(1-\epsilon), x_2 = \frac{1}{2}(1+\epsilon)$, the condition is now $\frac{\alpha}{\beta} > \frac{\beta}{\alpha} \Leftrightarrow (\alpha+\beta)(\alpha-\beta) > 0$ such that if $\beta < \alpha$ then the candidate $x_1 = \frac{1}{2}(1-\epsilon), x_2 = \frac{1}{2}(1+\epsilon)$ is the unique SPNE (i.e. the symmetric solution cannot be the SPNE).

Full market coverage. Finally, for each equilibrium location outcome, substituting in (5) yields the equilibrium output at each local market x. Both firms sell positive outputs at any local market if:

– for $x_1 = \frac{1}{2} = x_2$, then $q_1(x=1) = \frac{1}{3}\left(a - h\theta(x) - \frac{1}{2}\right) = q_2(x=0) > 0$ holds, where $\theta(x)$ satisfies (6). It is easy to verify that $\frac{1}{3}\left(a - h\theta(x) - \frac{1}{2}\right) > \frac{1}{3}\left(a - h - \frac{1}{2}\right)$; hence $q_1(x=1) > 0$ holds under Assumption 1.

– for $x_1 = \frac{1}{2}(a-h) - \frac{1}{4}, x_2 = \frac{5}{4} - \frac{1}{2}(a-h)$, then $q_1(x=1) > 0, q_2(x=0) > 0$. Note that

$$\begin{aligned} q_1(x &= 1) = \frac{1}{3}\left(a - 2\left(h\theta_1(x) + C_1\right) + \left(h\theta_2(x) + C_2\right)\right) > \frac{1}{3}\left(a - 2h - 2C_1 + C_2\right), \\ q_2(x &= 0) = \frac{1}{3}\left(a - 2\left(h\theta_2(x) + C_2\right) + \left(h\theta_1(x) + C_1\right)\right) > \frac{1}{3}\left(a - 2h - 2C_2 + C_1\right), \end{aligned}$$

with $-2C_1 + C_2 = -2C_2 + C_1 = -\frac{11}{4} + \frac{h^2}{k} + \frac{3}{2}(a - h\theta_0)\left(1 - \frac{4}{3}\frac{h^2}{k}\right)$. Hence we have $a - 2h - 2C_1 + C_2 > 0$ if equivalently $\frac{5}{2}a - \frac{11}{4} - 2h\left(1 + \frac{3}{4}\theta_0\right) - 2\frac{h^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right) > 0$, which is also written as

$$\begin{aligned} \frac{5}{2}a - \frac{11}{4} &> 2h\left(1 + \frac{3}{4}\theta_0\right) + 2\frac{h^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right) \\ &\mathit{or} \\ a - \frac{11}{10} &> h\left(\frac{4}{5} + \frac{3}{5}\theta_0\right) + \frac{4}{5}\frac{h^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right), \end{aligned}$$

where both bracketed terms on the RHS are positive under Assumption 2, and increasing in h. Define the threshold $\overline{h}$ such that $a - \frac{11}{10} = \overline{h}\left(\frac{4}{5} + \frac{3}{5}\theta_0\right) + \frac{4}{5}\frac{\overline{h}^2}{k}\left(a - h\theta_0 - \frac{1}{2}\right)$. As a result, for any $h < \overline{h}$, we have that both $q_1(x=1) > 0$ and $q_2(x=0) > 0$ are satisfied. Hence the threshold condition may be either more or, in contrast, less restrictive as compared with Proposition 1, since it depends on several parameters θ_0, a, k.

$\blacksquare$

Proof of Proposition 4. Let us consider a location where firm $i \in \{1, 2\}$ operates; differentiating $W(x) = \frac{1}{2}(Q(x))^2 - k(\theta(x))$ w.r.t. x_i yields

$$\frac{\partial W(x)}{\partial x_i} = Q(x)\frac{dQ(x)}{dx_i} - k'(\theta(x))\frac{d\theta(x)}{dx_i} = Q(x)\left(\frac{dQ(x)}{dx_i} + h\frac{d\theta_i(x)}{dx_i}\right),$$

where the equality results from stage-2 FOC. By definition of total output $Q(x)$, one obtains after differentiating w.r.t. x_i that $\frac{dQ(x)}{dx_i} = -h\frac{d\theta(x)}{dx_i} - \frac{dC_i}{dx_i}$. Substituting in $\frac{\partial W(x)}{\partial x_i}$, we can write equivalently

$$\frac{\partial W(x)}{\partial x_i} = -Q(x)\times\frac{dC_i}{dx_i} = -\left(a - h\theta(x) - C_i\right)\times\frac{dC_i}{dx_i} = -\left((a-h-C_i) + h(1-\theta(x))\right)\times\frac{dC_i}{dx_i}.$$

Thus, both derivatives of W_G w.r.t. x_1, x_2 respectively write

$$\begin{aligned}\frac{\partial W_G}{\partial x_1} &= \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} - h\int_0^{x_1}(1-\theta(x))dx + h\int_{x_1}^{\frac{x_1+x_2}{2}}(1-\theta(x))dx \\ &\text{and} \\ \frac{\partial W_G}{\partial x_2} &= \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} - h\int_{\frac{x_1+x_2}{2}}^{x_2}(1-\theta(x))dx + h\int_{x_2}^{1}(1-\theta(x))dx.\end{aligned}$$

Using Assumption 3, and solving the stage-2 FOC, it can be verified that

$$\theta(x) = \delta + \eta C_i,$$

where we denote $\delta = \frac{k\theta_0 - ah}{k-h^2}$ and $\eta = \frac{h}{k-h^2}$. Substituting, it comes that

$$\begin{aligned}\frac{\partial W_G}{\partial x_1} &= \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} - h\int_0^{x_1}(1-\delta-\eta(x_1-x))dx + h\int_{x_1}^{\frac{x_1+x_2}{2}}(1-\delta-\eta(x-x_1))dx \\ &= \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} + h\left\{-2(1-\delta)x_1 + (1-\delta+\eta x_1)\left(\frac{x_1+x_2}{2}\right) - \frac{\eta}{2}\left(\frac{x_1+x_2}{2}\right)^2\right\}, \\ \frac{\partial W_G}{\partial x_2} &= \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} - h\int_{\frac{x_1+x_2}{2}}^{x_2}(1-\delta-\eta(x_2-x))dx + h\int_{x_2}^{1}(1-\delta-\eta(x-x_2))dx \\ &= \left(\frac{\partial W_G}{\partial x_2}\right)_{no\ care} + h\left\{\begin{array}{c}\left(1-\delta-\frac{\eta}{2}\right) - 2\left(1-\delta-\frac{\eta}{2}\right)x_2 + (1-\delta-\eta x_2)\left(\frac{x_1+x_2}{2}\right) \\ +\frac{\eta}{2}\left(\frac{x_1+x_2}{2}\right)^2\end{array}\right\}.\end{aligned}$$

A socially optimal location pattern x_1, x_2 is defined as a solution to the following system of FOCs: $\frac{\partial W_G}{\partial x_1} = 0$, $\frac{\partial W_G}{\partial x_2} = 0$. In can be seen that :

– Given that these two conditions are not symmetric, then $x_1 = \frac{1}{2} = x_2$ cannot be the first best outcome.

– Again, firms being identical, the location equilibrium is symmetric, i.e. $x_1 + x_2 = 1$, and we have that $\frac{\partial W_G}{\partial x_1} = \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} + 2h\left(1 - \delta - \frac{\eta}{4}\right)\left(\frac{1}{4} - x_1\right) = 0$ and $\frac{\partial W_G}{\partial x2} = \left(\frac{\partial W_G}{\partial x_1}\right)_{no\ care} + 2h\left(1 - \delta - \frac{\eta}{4}\right)\left(\frac{3}{4} - x_2\right) = 0$; hence (using the proof of Proposition 2) $x_1 = \frac{1}{4}, x_2 = \frac{3}{4}$ is still first best efficient under durable care.

■

Proof of Proposition 6. At each location for firm 1 for example, the derivative of profit w.r.t. x_1 yields

$$\frac{\partial \pi_1(x)}{\partial x_1} = 2q_1(x)\frac{dq_1(x)}{dx_1} - k'(\theta_1(x))\frac{d\theta_1(x)}{dx_1} = q_1(x)\left(2\frac{dq_1(x)}{dx_1} + h\frac{d\theta_1(x)}{dx_1}\right)$$

Now differentiating (8) we obtain that $\frac{dq_1(x)}{dx_1} = -\frac{2}{3}\frac{dC_1}{dx_1}$, and plugging Assumption 3 in (9) yields $\frac{d\theta_1(x)}{dx_1} = -\frac{h}{k}\frac{dq_1(x)}{dx_1}$ As a consequence, we can write

$$\frac{\partial \pi_1(x)}{\partial x_1} = -\frac{2}{3}\left(2 - \frac{h^2}{k}\right) q_1(x) \times \frac{dC_1}{dx_1},$$

with $q_1(x) = \frac{1}{3}(a - 2C_1 + C_2)$, which is the same output level at each location as under a "no liability" rule. Hence, the derivative of total profit w.r.t. x_1 is

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{3}\left(2 - \frac{h^2}{k}\right)\left(-\int_0^{x_1} q_1(x)dx + \int_{x_1}^{x_2} q_1(x)dx + \int_{x_2}^{1} q_1(x)dx\right),$$

such that the best reply function for firm 1 is identical to the one obtained under a "no liability" regime (see the proof of Proposition 1, with $h = 0$); by symmetry, this also holds for firm 2. Hence the negligence rule with durable precautionary measures yields the same SPNE outcome in terms of location as the no liability regime.

■