EconomiX

Endogenous market structures, product liability, and the scope of product differentiation

Andreea Cosnita-Langlais
Eric Langlais

2022-18 Document de Travail/ Working Paper

EconomiX - UMR 7235 Bâtiment Maurice Allais
Université Paris Nanterre 200, Avenue de la République
92001 Nanterre Cedex

Site Web : economix.fr
Contact : secreteriat@economix.fr
Twitter : @EconomixU

Université Paris Nanterre

Endogenous market structures, product liability, and the scope of product differentiation

Andreea Cosnita-Langlais* **and Eric Langlais†**

June 16, 2022

Abstract

The paper considers how product liability may shape firm size, product specification choices and market structure. We introduce a spatial Cournot duopoly on the linear market, where firms make an initial decision of product differentiation, then invest in precaution, before competing in quantity. Our main results are fourfold: 1) with full coverage of the market by the duopoly, there exist two equilibria (in pure strategies): central agglomeration (which is stable for low liability costs), and dispersion (which is stable for not too large liabiliy costs); 2) for larger liability costs, a mixed market structure duopoly/monopoly sustains a unique equilibrium with product differentiation; 3) this equilibrium enables a scope of differentiation higher (smaller) than the full duopoly (the social optimum); 4) the impact of liability costs on firms size and profit is complex, since it depends on the impact on both product differentiation and market structure. Finally, we show that consumer surplus and social welfare are both higher under the mixed market structure than under the full duopoly with dispersion.

Keywords: horizontal differentiation, Cournot competition, spatial model, endogenous market structures, product liability, strict liability, negligence

JEL classification: D43, D47, K13, K23, L13, L25

*EconomiX, UPL, Univ. Paris Nanterre, CNRS F92000; acosnita@parisnanterre.fr.

†EconomiX, UPL, Univ. Paris Nanterre, CNRS F92000; elanglais@parisnanterre.fr.

1 Introduction

Are large firms better suited for implementing high levels of product safety, or does a small size provide a better protection against potentially large liability costs ? A result well established in the L&E literature is that a monopoly subject to strict liability faces the full cost of accident its own activity imposes to society, and thus provides an efficient response in terms of product safety at any level of output (Boyd 1994; Marino 1988; Spulber 1988). Nevertheless compared with negligence (coupled with an efficient standard of due-care), a monopoly may trigger too much or not enough precaution under strict liability, depending on various constraints, including the care technology (durable or non durable care - see Daughety and Reinganum 2006, 2013), whether the expected harm to consumers is linear or not in output (Marino 1991, Daughety and Reinganum 2014), whether consumers' beliefs on product failures are consistent with Bayesian updating (Daughety and Reinganum 1995, Epple and Raviv 1978) or if they account for perception biases (Polinsky and Rogerson 1983, Spence 1977, Baumann and Friehe 2021). In other words, the equilibrium outcome for a monopoly reflects the same types of constraints as those applying in an oligopoly framework.

The empirical evidence on the link between liability regimes and firms size and/or market entry goes in both directions. Ringleb and Wiggins (1990) ran estimates for hazardous sectors such as chemical, petroleum and transportation, and concluded that the shift from negligence to strict liability between 1967 and 1980 in USA has led to a large increase in the number of small firms entering these sectors. In other words, the main change w.r.t. liability is not a better hazard controlling these markets, but a shift of the riskier operations and activities from large companies towards small firms. The explanation is that large companies would serve as "deep-pocket" when plaintiffs file for recovering damages, a risk that materialized in different lawsuits cases including other economic sectors (asbestos, tobacco, medical malpractice litigations; see Viscusi 1995). For the medical sector, anecdotal but documented evidence (i.e. US Senate 2004, US House 2003) suggest that the high litigations risk and the subsequent impact on insurance rates for medical malpractice in the US led physicians to leave several medical specialties (plastic surgery, obstetrics).[1] A view also largely shared is that product liability may deter the introduction of new products and has a negative impact on product innovation, because i) Courts may be biased against new products, ii) damages awarded are uncertain, and iii) punitive damages are largely misused (see Parchomovski and Stein 2008, Viscusi 2012). Viscusi and Moore (1993) report several cases of contraceptive products, vaccines, but also fibers, for which the developers decided either to terminate research or not to market the innovative products because of a potential high risk of liability. Their empirical estimates conducted on a large set of US sectors show an inverse U-shaped relationship between product innovation and expected costs of liability.

Alternatively, Buzby and Frenzen (1999) and Loureiro (2008) study the impact of strict liability on the food sector. The former study uses jury verdicts for 175 cases from 1988 to 1997 in the US, and concludes that compensations obtained by victims are not very large, leading food providers to face low/insufficient incentives to increase safety. The second study analyzes the effects of strict liability together with punitive damages on the reduction of food contamination episodes from 1990 to 2000, and finds a statistically significant decrease in the number of incidents. Moreover, US States allowing claims for punitive damages are less likely (from 15% to 30%) to experience food safety episodes. Galasso and Luo (2019) focus on the sector of medical implants (prostheses, heart valves, pacemakers and so on) and consider in their data (1975-2015) both upstream inputs suppliers and downstream implants makers. They conclude that product liability had a negative effect on medical implants manufacturers (measured as the decrease in the number of patents), but no significant impact on upstream polymer suppliers.

Going back to "theory", the argument that an increase in liability costs (such as, for example, the switch from negligence to strict liability) leads a firm to reduce its activity/size is supported by the theoretical analysis only as a mechanical consequence of the unilateral model of accident for monoproduct firms – but the prediction holds for competitive, oligopoly and monopoly firms as well. Considering instead the more realistic case of firms selling differentiated products and operating on different markets allows to account for the fact that a firm may have a small market share for a subset

[1] Helland and Showalter (2009) estimate the impact of malpractice reforms on the number of working hours for physicians.

of its products, but in turn may be dominant on other segments of the market. Hence, the impact of liability regimes on the firms' size needs to be revisited since total profit results in this case from the sum of profits made on the different markets. Eventually, this also points at the sustainable market structure. The sector of biomaterials is a good illustration of this since small innovative firms compete against a few large companies. Biomaterials such as metals, polymers and ceramics are used in numerous industries for very diverse applications (i.e. fabrics, sport devices, vehicles, aeronautics) but also by implants producers. Interestingly, two large companies (DuPont and Dow Chemicals), are the dominant suppliers of polymers and silicone used by the medical implants that are seen as riskier in the US.[2]

Given all this, our paper investigates how product liability regimes shape competition and the emergence of endogenous market structures, in contexts where product differentiation is a strategic decision for firms. We consider the case of a duopoly subject to product liability, where firms compete *à la* Cournot on the linear market (see Anderson and Neven 1991), and face a uniform consumer distribution along the line. We use a three-stage game where firms first decide simultaneously on their locations/product specifications, then simultaneously choose their product safety levels, and finally compete in quantities at each location/local market on the unit line. We assume that the expected harm borne by consumers as well as the cost of care are linear w.r.t. individual output, i.e. proportional to it. In this set-up, we compare the spatial/product choice equilibrium that obtains under strict liability with the socially optimal outcome.

We show in this paper that the size of the expected unit cost of accident (expected harm plus cost of care per unit of output) will determine both the (type of) spatial/product choice equilibrium and the market structure itself. More precisely, we find that for low levels of harm/cost of care, the market equilibrium involves no product differentiation, i.e. central agglomeration obtains (as is the case without liability). Equivalently, the mere existence of product liability does not necessarily impact firms location/product speci-

[2] Against the "deep-pocket" argument, Galasso and Luo (2019) report that several unexpected problems arose in US by the end of 80's with medical implants produced by Vitek (silicone breast implants, joint jaw implants). When Vitek filed for bankruptcy in 1990, plaintiffs/implants recipients switched to filing against DuPont as Vitek's supplier of polymers and silicone. DuPont won all suits, but it took ten years and cost $40 million.

fication choices. It will nonetheless affect their output decisions, because strict liability reduces the equilibrium outputs as compared with the no-liability regime. Moreover, the liability-based product choice/location equilibrium is suboptimal, since the first best spatial pattern involves some dispersion (or product differentiation), with firms (products) being symmetrically located around the market center (at 1/4 and 3/4 respectively). In contrast, larger levels of harm/cost of care are found to support dispersion in equilibrium. In other words, this is a case where liability does matter for firms' product/location choices, although the equilibrium degree of horizontal differentiation is generally suboptimal. We find that the degree of product differentiation/spatial dispersion is increasing with the harm/cost of care, and approaches the socially optimal one as the level of expected harm/cost of care grows to its upper bound. Furthermore, we find that above some threshold for consumers harm/cost of care, the duopoly is no longer sustainable (i.e. no equilibrium exists in pure strategies): the full market coverage in a duopoly rules out situations where consumers' harm/cost of care is too large. In terms of endogenous market structures, we show that the combination of duopoly for central locations and monopoly for extreme locations (close to the market border) triggers product differentiation regardless of the level of consumers' harm/cost of care. Equivalently, with high levels of harm/cost of care (and thus, liability costs), the full market coverage and product differentiation occur in equilibrium in exchange for a higher market power for some firm. This also implies that the relationship between liability costs and firms' size is complex, depending on the scope of product differentiation and the actual market structure. With full market coverage in a duopoly, the individual output increases with liability costs in the neighborhood of local markets where each firm is located; whereas under a mixed structure duopoly/monopoly, it is increasing at least in the neighborhood of local markets where the rivals holds a monopoly position. However, we also show that this entails no trade-off between consumers' surplus and firms' profits, to the extent that the mixed market structure duopoly/monopoly supports a higher consumer's surplus, together with higher profits for firms as compared with the full duopoly (when sustainable).

The rest of the paper is organized as follows. Section 2 provides a brief review of literature. Section 3 introduces the model and assumptions, and considers the properties of the social optimum. Section 4 discusses the solutions under the strict liability rule.

We also discuss how the results extend to the negligence rule. In Section 5 we perform some numerical applications in order to compare industry profits, consumers' surplus and social welfare under the different equilibrium patterns. Section 6 concludes.

2 Literature

This review will focus on market structure, and to begin with, on product differentiation with/without product liability. Following Anderson and Neven's (1991) analysis of spatial Cournot competition on the linear market, several contributions have examined how alternative assumptions for the (transportation) costs and/or consumers' population density impact the equilibrium spatial pattern (see Gupta, Pal, and Sarkar (1997), Mayer (2000), Matsushima (2001), Shimizu (2002)). Matsumura and Shimizu (2005) discussed the conditions for the social optimum to be consistent with firms agglomeration vs dispersion on the linear market. Chamorro Rivas (2000) extended the admissible range for transportation costs leading to dispersion in the duopoly case, while Benassi, Chirco, and Scrimitore (2007) showed that, in that case, the spatial equilibrium may correspond to a mixed market structure with a duopoly at central locations and monopolies close to the market boundaries.

In the L&E literature, several papers examined the impact of product liability when products are exogenously differentiated, but to our knowledge none dealt with endogenous product design, nor with endogenous markets structures. Daughety and Reinganum (2006) consider strict liability in an oligopoly with both (exogenous) horizontal and (endogenous) vertical differentiation. They find that the relationship between, on the one hand, the equilibrium levels of care and output, and on the other hand, the degree of product differentiation, is U-shaped. Baumann and Friehe (2015) allow for both price and quantity competition to compare strict liability and negligence with (exogenously) differentiated products, and focus on the determination of optimal damage multipliers. More recently, several contributions used different spatial models to look into the outcome of liability rules, but without allowing for firms' endogenous specification choices. Chen and Hua (2017) used the hub-and-spokes model of spatial competition (which generalizes the Hotelling model) to show that when firms compete in prices, strict liability with partial compensation of harm combined with firms' reputation concerns provide incentives to invest in safety. Baumann, Friehe, and Rasch (2018), assuming consumers are heterogenous w.r.t. the harm incurred, and Baumann and Friehe (2021), assuming they heterogeneous in terms of misperception of their harm, also studied the impact of incomplete strict liability in a model *à la* Hotelling.

The literature addressing endogenous market structures usually focused on the link between consumers preferences and firms' incentives to enter the market (see Etro (2014) for a survey, or Bertoletti and Etro (2016) for a synthetic model), a major area of application being the international trade. This approach goes back to Sutton's (1991) analysis of sunk costs and market structures. Horstmann and Markusen (1992) considered different technological specifications when firms compete for exports, and found the duopoly or the multi-plant monopoly to be particular cases of the international equilibrium, while Mathieu (1995), and Marjit and Mukherjee (2015) extended the analysis to exogenously differentiated products.

3 The basic set-up

3.1 Model and assumptions

Consider two Cournot rival firms, denoted 1 and 2, operating on the unit linear market, where infinitely many consumers lie uniformly. The firms produce the same basic good with the same production technology characterized by constant marginal costs, normalized to zero. Firm $i \in \{1, 2\}$ located at $x_i \in [0, 1]$ incurs a constant unit transport cost $C_i = t|x_i - x|$ in order to deliver output to consumers located at $x \in [0, 1]$ – hence total transportation costs, $C_i \times q_i(x)$, are linear in distance and quantity. t is a positive constant, and given that the transport cost parameter enters as a multiple in the profit expressions, we assume $t = 1$ w.l.o.g. Consumers are assumed to have a prohibitive costly transport cost, preventing arbitrage, so given that firms ship the product to consumers' locations, they can and will price discriminate across the set of spatially differentiated markets.[3] In this shipping model of spatial competition,[4] product differentiation comes up as follows: the firm's basic product (its location) is customized at a cost (transport cost) to make it appropriate for a particular consumer (located at x).

At each location x the firm's $i \in \{1, 2\}$ output generates some harm to consumers, denoted $d(q_i(x)) = \theta_i(x) \times h \times q_i(x)$, where $h > 0$ is a scale parameter and θ_i denotes the frequency of accidental harm at each local market x. The total cost of care borne by firm i at each location is defined as $c(\theta_i(x)) \times q_i(x)$ with $c'(\theta_i(x)) < 0 < c''(\theta_i(x))$, meaning that it comes as the marginal cost of the output.[5] We assume that when product liability exists (whatever the liability rule, either strict liability or negligence), Courts award expected damages corresponding to the harm entailed by a firm. Hence the liability cost borne by firm i is defined as $L_i = d(q_i(x))$.

Finally, demand at each consumer location x on the unit line is given by

$$p(x) = a - Q(x) - (1 - \lambda)\theta_i(x) \times h,$$

[3]This assumption captures the fact that transport costs are far more important for firms than for consumers - in real life, this is what justifies distribution networks. It also basically defines the shipping model of spatial competition, which happens to be consistent with the flexible manufacturing production systems (see Eaton and Schmitt 1994), where the firm's basic product (its location) is customized at a cost (transport cost) to make it appropriate for a consumer.

[4]In order to justify the choice of Cournot over Bertrand to model spatial competition, several arguments may be put forward. From a theoretical point of view, recall that Kreps and Scheinkman (1983) showed in a non-spatial framework that Cournot equilibrium is equivalent to the outcome of a two-stage game where firms first set capacities before competing in prices. This implies that Cournot competition is the appropriate modeling choice when fixing pricing is relatively easier than modifying capacities or adjusting quantities. Furthermore, in a spatial context, firms decide both on their aggregate output as well as on the quantities allocated to the different local markets. Hence, the Cournot assumption is reasonable when both total output and its spatial allocation are relatively inflexible. Typically, such a rigid distribution of the firm's sales is consistent with firms shipping output from a production plant to various outlets/consumer locations. Also, and still form the theoretical viewpoint, Cournot spatial models exhibit attractive features in their predictions: whereas Bertrand competition results in exclusive sales territories for firms (i.e. consumers at each location being served only by the most cost-efficient firm there), Cournot competition leads to intra-industry trade and market overlapping (see Phlips (1983) and McBride (1983)). Finally, recall also that the predictions of the Cournot spatial model in terms of delivered prices were validated by Greenhut et al. (1980) in a representative sample of industries.

[5]Remember that here investing in precautionary measures means reducing $\theta_i(x)$.

with $a > 0$ and where $p(x)$ and $Q(x)$ are the price and total output supplied at location x (with $Q(x) = q_1(x) + q_2(x)$, where $q_1(x), q_2(x)$ denote individual outputs at each location x). The parameter λ captures the fact that consumers are compensated for the harm incurred ($\lambda = 1$) only if a liability regime exists, otherwise ($\lambda = 0$), they bear the full burden of the harm.

In order to guarantee full market coverage by both firms absent any liability costs (i.e. total transportation costs are smaller than the willingness to pay of consumers), we assume that:

Assumption 1 : $a > 2$.

Below we start our analysis by characterizing the social optimum.

3.2 Social optimum

In this set up, Social Welfare at each location is the sum of gross consumers' surplus, minus total transportation costs and the costs of care, minus the expected harm:

$$W(x) = aQ(x) - \frac{(Q(x))^2}{2} - \sum_{i=1}^{2} (C_i + c(\theta_i(x))) \times q_i(x) - h \times \sum_{i=1}^{2} \theta_i(x) \times q_i(x). \quad \text{(SW)}$$

For the consistency of comparison between the different configurations we examine in the paper, we assume that the planner chooses in a first stage a location for each firm in order to maximize Social Welfare over the different locations, defined by $W_G = \int_0^1 W(x)dx$. In a second stage, he chooses the level of care for each firm at each location x; finally at stage 3, the planner chooses a level of output delivered by each firm at each location x. We solve this game backwards.

Considering the last stage, the derivative w.r.t. output for firm $i \in \{1, 2\}$ is

$$\frac{\partial W(x)}{\partial q_i(x)} = a - Q(x) - C_i - c(\theta_i(x)) - h\theta_i(x).$$

Hence at any given location x, either a) $C_1 + c(\theta_1(x)) + h\theta_1(x) = C_2 + c(\theta_2(x)) + h\theta_2(x)$, and the solution to the stage-2 FOC, $\frac{\partial W(x)}{\partial q_i(x)} = 0$, is $q_i(x) = \frac{a - C_i - c(\theta_i(x)) - h\theta_i(x)}{2} > 0$ for

$i \in \{1,2\}$ with $q_1(x) = \frac{Q(x)}{2} = q_2(x)$; or b) $C_i < C_j$, and the solution is $q_i(x) = Q(x) = a - C_i - c(\theta_i(x)) - h\theta_i(x) > q_j(x) = 0$. It is easy to verify that the SOCs are met.

At stage-2 the derivative of Social Welfare w.r.t. care investment for firm $i \in \{1,2\}$ at a location where $q_i(x) > 0$ is $\frac{\partial W(x)}{\partial \theta_i(x)} = -q_i(x)\left(h + c'(\theta_i(x))\right)$. Hence at a location where $q_i(x) > 0$, the stage-2 FOC $\frac{\partial W(x)}{\partial \theta_i(x)} = 0$ is given by

$$-c'(\theta_i(x)) = h, \tag{1}$$

which means that the planner chooses a level of care that minimizes the total cost of an accidental harm per unit of output at this location. The SOCs are also met. Condition (1) implies that the first best level of care is independent from the level of output to be delivered, and transportation costs: indeed, the socially optimal level of care, denoted $\theta \equiv c'^{-1}(h)$, is constant across locations, identical for both firms, and depends only on the characteristics of the safety technology.

This implies that it is not socially efficient to have $q_1(x) > 0$ and $q_2(x) > 0$ at any given location: it is socially inefficient to have more than one firm operate at any local market x, so as to avoid the duplication of both transportation and precaution costs. This guarantees that the aggregate output at each location x, written as $Q(x) = a - h\theta - c(\theta) - \tilde{C}$, is obtained at the lowest total cost. To ensure positive quantities at any location, we make the following basic assumption :

Assumption 2 : $\theta h + c(\theta) < a - 1$.

The rationale is that h represents here a scale parameter that captures the magnitude of the impact of the output on consumers' harm (i.e. a pure "technological" effect, corresponding to complex mechanisms in terms of response reaction); hence h does not take a given value *a priori*, $h \lesseqgtr 1$ being possible w.l.o.g.

Finally assume the planner contemplates different locations/product specifications for the two firms assuming w.l.o.g. that $0 \leq x_1 \leq \frac{1}{2} \leq x_2 \leq 1$, where x_1 (x_2) is the location of firm 1 (firm 2). Given the distinct locations, firm 1 delivers output and care at all the local markets in $[0, x_1]$ together with those in $[x_1, \frac{x_1+x_2}{2}]$, whereas firm 2 delivers output and care at all the local markets in $[x_2, 1]$ together with those in $[\frac{x_1+x_2}{2}, x_2]$, respectively. As a result, using the stage-2 FOC, Social Welfare may thus be written as $W(x) = \frac{1}{2}\left(Q(x)\right)^2 = \frac{1}{2}\left(a - h\theta - c(\theta) - \tilde{C}\right)^2$ at each x.

Therefore, Social Welfare from all locations/local markets along the unit line is now written as

$$W_G = \int_0^{x_1} \frac{1}{2}\left(Q(x)\right)^2 dx + \int_{x_1}^{\frac{x_1+x_2}{2}} \frac{1}{2}\left(Q(x)\right)^2 dx + \int_{\frac{x_1+x_2}{2}}^{x_2} \frac{1}{2}\left(Q(x)\right)^2 dx + \int_{x_2}^{1} \frac{1}{2}\left(Q(x)\right)^2 dx. \tag{WG}$$

Note that at each location x the output depends on transportation costs, and the latter write differently along the unit line, depending on their relative locations: $C_1 = (x_1 - x), C_2 = (x_2 - x)$ for $x \in [0, x_1]$, but $C_1 = (x - x_1), C_2 = (x_2 - x)$ for $x \in [x_1, x_2]$ and finally $C_1 = (x - x_1), C_2 = (x - x_2)$ for $x \in [x_2, 1]$. Differentiating $W(x)$ w.r.t. x_i, and using (1) yields

$$\frac{\partial W(x)}{\partial x_i} = Q(x)\frac{dQ(x)}{dx_i} = -Q(x)\frac{dC_i}{dx_i} \tag{2}$$

since at each x we have that $Q(x) = a - C_i - h\theta(x)$. As a result, the total derivative of (WG) w.r.t. to x_1 and x_2 respectively comes down to:

$$\begin{aligned}
\frac{\partial W_G}{\partial x_1} &= -\int_0^{x_1} Q(x)dx + \int_{x_1}^{\frac{x_1+x_2}{2}} Q(x)dx, \\
\frac{\partial W_G}{\partial x_2} &= -\int_{\frac{x_1+x_2}{2}}^{x_2} Q(x)dx + \int_{x_2}^{1} Q(x)dx.
\end{aligned}$$

The socially optimal outcome in terms of locations can be characterized as follows :

Proposition 1 *(Socially optimal locations)*
The first best solution is $x_1 = \frac{1}{4}, x_2 = \frac{3}{4}$.

Proof. The derivative of (WG) w.r.t. to x_1 is

$$\begin{aligned}
\frac{\partial W_G}{\partial x_1} &= -\int_0^{x_1} \left(a - h\theta - c(\theta) - (x_1 - x)\right) dx + \int_{x_1}^{\frac{x_1+x_2}{2}} \left(a - h\theta - c(\theta) - (x - x_1)\right) dx \\
&= -2(a - h\theta - c(\theta))x_1 + (a - h\theta - c(\theta) + x_1)\left(\frac{x_1+x_2}{2}\right) - \frac{1}{2}\left(\frac{x_1+x_2}{2}\right)^2
\end{aligned}$$

and the derivative of (WG) w.r.t. to x_2 is

$$\begin{aligned}\frac{\partial W_G}{\partial x_2} &= -\int_{\frac{x_1+x_2}{2}}^{x_2}(a-h\theta-c(\theta)-(x_2-x))\,dx+\int_{x_2}^{1}(a-h\theta-c(\theta)-(x-x_2))\,dx\\ &= \left(a-h\theta-c(\theta)-\frac{1}{2}\right)-2\left(a-h\theta-c(\theta)-\frac{1}{2}\right)x_2\\ &\quad +(a-h\theta-c(\theta)-x_2)\left(\frac{x_1+x_2}{2}\right)+\frac{1}{2}\left(\frac{x_1+x_2}{2}\right)^2.\end{aligned}$$

Note that since these derivatives are not symmetric, then $x_1=\frac{1}{2}=x_2$ cannot be the solution to the system $\frac{\partial W_G}{\partial x_1}=0, \frac{\partial W_G}{\partial x_2}=0$. Moreover, both firms being identical in all respect except possibly their locations, the optimal location pattern has to be symmetric in the sense that firms must be equidistant from the market center at optimum, i.e. $x_1+x_2=1$. As a result, both FOCs can be simplified to $\frac{\partial W_G}{\partial x_1}=2\left(a-h\theta-c(\theta)-\frac{1}{4}\right)\left(\frac{1}{4}-x_1\right)=0$ and $\frac{\partial W_G}{\partial x2}=2\left(a-h\theta-c(\theta)-\frac{1}{4}\right)\left(\frac{3}{4}-x_2\right)=0$ respectively. Hence the solution is $x_1=\frac{1}{4}, x_2=\frac{3}{4}$. SOCs are satisfied for $x_1=\frac{1}{4}, x_2=\frac{3}{4}$ since

$$\frac{\partial^2 W_G}{\partial x_1^2}\frac{\partial^2 W_G}{\partial x_2^2}-\frac{\partial^2 W_G}{\partial x_1 \partial x_2}=\frac{1}{2}\left(\frac{9}{2}\left(a-h\theta-c(\theta)-\frac{1}{4}\right)^2-\left(a-h\theta-c(\theta)-\frac{1}{2}\right)^2\right)>0$$

under Assumption 2. It can be verified that no other solution (consistent with the SOCs, or the restrictions $0\leq x_1\leq\frac{1}{2}\leq x_2\leq 1$) exists.

■

The intuition of this result is that the social planner aims at minimizing transportation costs and expected accident costs along the unit linear market. This implies that at each location/local market x, it is socially wasteful to locate both firms: instead, a single one allows the provision of the efficient output level for the lowest transportation cost. Given that the socially efficient care level is identical (constant) across all locations, its cost is irrelevant for the determination of the optimal distance between both firms/their products; this distance only depends on transportation costs, as in the standard/no-accident spatial model (see Matsumura and Shimizu 2005).

4 Duopoly equilibria under product liability

We now consider the market equilibrium under strict liability.[6]

We use the following timing for the game : at stage 0, Courts announce a strict liability regime with full compensation of harm ($\lambda = 1$)[7] to which they commit at stage 4, and both firms observe the liability regime for accidental harm inflicted on consumers; at stage 1, firms simultaneously choose a location $x_i \in [0,1]$; at stage 2, they choose a level of care/probability of harm $\theta_i(x)$; at stage 3, they compete in quantities at every local market x; at stage 4, the liability regime is enforced. Below we solve this game by backward induction.

At stage 3, each firm chooses a level of output $q_i(x)$ that maximizes its profit given the rival's output at each location x on the unit line.

Given that marginal costs are constant and consumer arbitrage is nonbinding, quantities set at different points by the same firm are strategically independent, therefore the third stage Cournot equilibrium can be characterized by a set of independent Cournot equilibria, one for each local market $x \in [0,1]$. Remember also that in the context of our spatial analysis, firms decide on their aggregate output but also on the quantity to allocate to several submarkets (several points in space) - see Anderson and Neven (1991).

Under strict liability, the individual profit at x is written for $i \in \{1,2\}$ as

$$\pi_i(x) = (a - Q(x) - C_i - c(\theta_i(x)) - h \times \theta_i(x))\, q_i(x).$$

The FOC at each location x implies that $q_i(x)$ solves

$$(a - Q(x) - C_i - c(\theta_i(x))) - q_i(x) = h \times \theta_i(x). \tag{3}$$

[6] It is worth stressing here that the different results of this section extend to the negligence rule with minor modifications. The reason is that under negligence, firms face a market demand equal to $p(x) = a - Q(x) - \theta_i(x) \times h$ when they abide by any due-care standard θ^*. Thus whenever θ^* is set at the efficient level, firm meet the due-care standard. Hence Propositions 2, 3 and 4 below, which are established for strict liability, will also hold for negligence.

[7] This is for convenience, since the paper focuses on firms' location/product specification choice. But extending the analysis to cases where $\lambda \neq 1$ is obvious, since a firm's profit is independent of λ as long as consumers have no assessment bias (see Polinsky and Rogerson (1984) for an analysis where consumers have a biased estimate of the expected harm).

The meaning of condition (3) is standard : the LHS is the marginal market proceeds under imperfect competition, while the RHS is the marginal cost of production (reflecting the liability cost in this set up). Solving (3) yields $q_i(x)$ (for $i \in \{1,2\}$), i.e. the stage-3 subgame equilibrium output level, which is defined as

$$q_i(x) = \begin{cases} 0 & \text{if } q_j(x) = q_j^m(x) \\ q_i^d(x) & \text{if } q_j(x) < q_j^m(x) \\ q_i^m(x) & \text{if } q_j(x) = 0 \end{cases}, \tag{4}$$

i.e. at a local market x, the output depends on whether the market is a duopoly or a monopoly, with

$$\begin{array}{l} q_i^d(x) = \frac{1}{3}\left(a - 2\left(h\theta_i(x) + C_i + c(\theta_i(x))\right)\right) + \left(h\theta_j(x) + C_j + c(\theta_j(x))\right)) \\ \text{and} \\ q_i^m(x) = \frac{1}{2}\left(a - \left(h\theta_i(x) + C_i + c(\theta_i(x))\right)\right). \end{array} \tag{5}$$

At stage 2, firms choose their care activities $\theta_i(x)$ at each local market in order to maximize their total profit, anticipating they will play either their Cournot-Nash or the monopoly quantities defined in (5). Using (3), the stage-3 equilibrium profit at each location x where the firm is active can be written as $\pi_i(x) = (q_i(x))^2$, for $i \in \{1,2\}$. The profit derivative w.r.t. care at each x where the firm is active is thus $\frac{\partial \pi_i}{\partial \theta_i(x)}(x) = 2q_i(x)\frac{dq_i(x)}{d\theta_i(x)}$. Using (5) we have that

$$\frac{dq_i(x)}{d\theta_i(x)} = \begin{cases} -\frac{2}{3}\left(h + c(\theta_i(x))\right) & \text{if } q_j(x) > 0, \\ -\frac{1}{2}\left(h + c(\theta_i(x))\right) & \text{if } q_j(x) = 0. \end{cases}$$

As a consequence, the FOC w.r.t. care, $\frac{\partial \pi_i}{\partial \theta_i(x)}(x) = 0$, has the same expression whether the firm is a monopoly or a duopolist, namely

$$-c'(\theta_i(x)) = h, \tag{1}$$

thus still leading to $\theta \equiv c'^{-1}(h)$, i.e. the first best level of care, with the corresponding SOC, $\frac{\partial^2 \pi_i}{\partial \theta_i^2(x)}(x) < 0$, holding for both market structures. This implies that the stage-3 output level simplifies to

$$\begin{array}{l} \text{either } q_i^d(x) = \frac{1}{3}\left(a - h\theta - c(\theta) - 2C_i + C_j\right), \\ \text{or } q_i^m(x) = \frac{1}{2}\left(a - h\theta - c(\theta) - C_i\right). \end{array} \tag{5'}$$

At stage 1, firms choose their location $x_i \in [0,1]$ in order to maximize their total profit, anticipating they will play their Cournot-Nash or monopoly quantities at stage 3 and their care activities at stage 2, defined by (1)-(5'). We assume w.l.o.g. that $0 \leq x_1 \leq x_2 \leq 1$. Therefore total profit for firm i over the whole unit line writes as

$$\Pi_i = \int_0^{x_1} (q_i(x))^2 \, dx + \int_{x_1}^{x_2} (q_i(x))^2 \, dx + \int_{x_2}^{1} (q_i(x))^2 \, dx \tag{ΠG}$$

under (1)-(5'). At any location x where firm i operates, differentiating its profit w.r.t. x_i yields $\frac{\partial \pi_i(x)}{\partial x_i} = 2q_i(x)\frac{dq_i(x)}{dx_i}$, and thus depending on the market structure we have either $\frac{dq_i^d(x)}{dx_i} = -\frac{2}{3}\frac{dC_i}{dx_i}$, or $\frac{dq_i^m(x)}{dx_i} = -\frac{dC_i}{dx_i}$. As a consequence, it comes that

$$\frac{\partial \pi_i(x)}{\partial x_i} = \begin{cases} -\frac{4}{3} q_i^d(x) \frac{dC_i}{dx_i} & \text{if } q_j(x) > 0, \\ -q_i^m(x) \frac{dC_i}{dx_i} & \text{if } q_j(x) = 0. \end{cases}$$

We now contrast two situations : either there is full coverage of the unit linear market by both firms, or there exists a monopoly at some locations. We show below that some restrictions are required on the size of the unit cost of accident for these markets structures to support an equilibrium with product differentiation.

4.1 Full market coverage by the duopoly

Assume full coverage of the market line by both firms, meaning both firms sell positive quantities at each local market on the market line (e.g. "full duopoly" at any location). The derivative of total profit for firm 1 w.r.t. x_1 is given by:[8]

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{3}\left(-\int_0^{x_1} q_1^d(x)dx + \int_{x_1}^{x_2} q_1^d(x)dx + \int_{x_2}^{1} q_1^d(x)dx \right).$$

Similarly for firm 2, the derivative of total profit w.r.t. x_2 is

$$\frac{\partial \Pi_2}{\partial x_2} = \frac{4}{3}\left(-\int_0^{x_1} q_2^d(x)dx - \int_{x_1}^{x_2} q_2^d(x)dx + \int_{x_2}^{1} q_2^d(x)dx \right).$$

[8] In Cournot spatial models with location choice it has long been established that the FOCs simply translate what is now routinely called the quantity-median property, i.e. total quantity sold by a firm to the left of its location needs to equal that to the right if this location is to be optimal. To see this, note that taking the derivative of firm's 1 total profit w.r.t x_1, and setting $\frac{\partial \Pi_1}{\partial x_1} = 0$, it comes that : $\int_0^{x_1} q_1^d(x)dx = \int_{x_1}^{x_2} q_1^d(x)dx + \int_{x_2}^{1} q_1^d(x)dx$, which means that total output sold to the left of x_1 must be equal to total output delivered to the right of x_1 (the same argument applies to firm 2).

In the proof of the propositions we show that any candidate to be a SPNE outcome in terms of location is a couple x_1, x_2, with $0 \leq x_1 \leq x_2 \leq 1$, characterized as a solution to the system of firms' best responses functions, respectively $\frac{\partial \Pi_1}{\partial x_1} = 0 \Leftrightarrow x_2 = f_{F1}(x_1)$ for firm 1, and $\frac{\partial \Pi_2}{\partial x_2} = 0 \Leftrightarrow x_2 = g_{F2}(x_1)$ for firm 2. Indeed we find there may exist multiple equilibria[9] - in this case, we select as the *unique* SPNE the candidate passing the stability test, i.e. the one that satisfies the condition $|f'_{F1}(x_1)| > |g'_{F2}(x_1)|$. We have the next result:

Proposition 2 *(Strict Liability and Duopoly at each location)*
There exists a unique stable location equilibrium with both firms serving each local market, which is either: a) $x_1 = \frac{1}{2} = x_2$ *if* $\theta h + c(\theta) < a - \frac{3}{2}$*; or b)* $x_1 = \frac{1}{2}(a - h\theta - c(\theta)) - \frac{1}{4} < x_2 = \frac{5}{4} - \frac{1}{2}(a - h\theta - c(\theta))$ *if* $\theta h + c(\theta) \in [a - \frac{3}{2}, a - \frac{11}{10})$.

Proof. Using the definition of the duopoly output in $\frac{\partial \Pi_1}{\partial x_1}$ and integrating by part yields:

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{4}{9}\left(\left(a - h\theta - c(\theta) - \frac{1}{2}\right) + (x_2 - x_1)^2 - 2\left(a - h\theta - c(\theta) - 1\right)x_1 - x_2\right).$$

The SOC is satisfied since under Assumption 2, for any $0 \leq x_1 \leq x_2 \leq 1$, we obtain:

$$\frac{\partial^2 \Pi_1}{\partial x_1^2} = \frac{4}{9}\left(-2\left(a - h\theta - c(\theta) - 1\right) - 2(x_2 - x_1)\right) < 0.$$

Similarly for firm 2, the derivative of total profit w.r.t. x_2 is:

$$\frac{\partial \Pi_2}{\partial x_2} = \frac{4}{9}\left(\left(a - h\theta - c(\theta) - \frac{1}{2}\right) - (x_2 - x_1)^2 - 2\left(a - h\theta - c(\theta) - 1\right)x_2 - x_1\right).$$

Again the SOC is satisfied since $\frac{\partial^2 \Pi_2}{\partial x_2^2} = \frac{\partial^2 \Pi_1}{\partial x_1^2}$. Hence firms' best reply functions may be written as :

$$\left.\begin{array}{r} \left(a - h\theta - c(\theta) - \frac{1}{2}\right) + (x_2 - x_1)^2 - 2\left(a - h\theta - c(\theta) - 1\right)x_1 - x_2 = 0 \\ \left(a - h\theta - c(\theta) - \frac{1}{2}\right) - (x_2 - x_1)^2 - x_1 - 2\left(a - h\theta - c(\theta) - 1\right)x_2 = 0 \end{array}\right\} \quad (6)$$

[9] Strategic location models examine firms choosing their distance to each delivery location and therefore minimizing overall transport cost of shipping their product to consumers in order to maximize overall profits. This explains why firms agglomerate in a central location when no other costs are considered (see Anderson and Neven (1991) and Mayer (2000)). This behavior is complicated here by the cost of liability incurred by firms.

a) It is straightforward to see that $x_1 = \frac{1}{2} = x_2$ is a natural solution to the system (i.e. it solves $\frac{\partial \Pi_i}{\partial x_i} = 0$). The stability condition is : $2(a - h\theta - c(\theta) - 1) > \frac{1}{2(a-h\theta-c(\theta)-1)}$. It can be verified after rearranging that it is equivalent to $\left(a - h\theta - c(\theta) - \frac{3}{2}\right)\left(a - h\theta - c(\theta) - \frac{1}{2}\right) > 0$, which holds iff $h\theta + c(\theta) < a - \frac{3}{2}$.

In order for the condition of full market coverage be met for both firms (i.e. both firms sell positive outputs at any local market), it must be that $q_1^d(x = 1) = \frac{1}{3}\left(a - h\theta - c(\theta) - \frac{1}{2}\right) = q_2^d(x = 0) > 0$, which holds under Assumption 2.

b) Let us examine now whether a solution involving spatial dispersion exists, such that $x_2 - x_1 = \epsilon > 0$. Solving (6) for ϵ gives $\epsilon = \frac{3}{2} - (a - h\theta - c(\theta)) > 0$ only if $h\theta + c(\theta) > a - \frac{3}{2}$; then substituting in (6) and solving for x_2, x_1 yields $x_2 = \frac{5}{4} - \frac{1}{2}(a - h\theta - c(\theta))$, and $x_1 = \frac{1}{2}(a - h\theta - c(\theta)) - \frac{1}{4}$. The stability condition is now written as $\frac{1}{2(a-h\theta-c(\theta)-1)} > 2(a - h\theta - c(\theta) - 1) \Leftrightarrow \left(a - h\theta - c(\theta) - \frac{3}{2}\right)\left(a - h\theta - c(\theta) - \frac{1}{2}\right) < 0$, which holds iff $h\theta + c(\theta) > a - \frac{3}{2}$.

In order for the full market coverage condition be met for both firms, it is now necessary that $q_1^d(x = 1) = \frac{1}{3}\left(\frac{5}{2}(a - h\theta - c(\theta)) - \frac{11}{4}\right) = q_2^d(x = 0) > 0$, which holds iff $\frac{5}{2}(a - h\theta - c(\theta)) - \frac{11}{4} > 0 \Leftrightarrow h\theta + c(\theta) < a - \frac{11}{10}$, which is more restrictive than Assumption 2. ■

Proposition 3 shows that central agglomeration (e.g. the minimum differentiation) prevails at low levels of consumers' harm/cost of care. In contrast, firms' dispersion (i.e. spatial/product differentiation) occurs for higher levels of the external harm/cost of care, the scope of differentiation being increasing with the harm/cost of care (since according to b), x_1 increases while x_2 decreases with $a - h\theta - c(\theta)$).

However, product differentiation/location choices are suboptimal in a full duopoly equilibrium. To illustrate this, the table below provides some simple calculations for the dispersion equilibrium (case b):

$a-h\theta-c(\theta)$	x_1	x_2	ϵ
1.1	0.3	0.7	0.4
1.15	0.325	0.675	0.35
1.2	0.35	0.65	0.3
1.25	0.375	0.625	0.25
1.3	0.4	0.6	0.2
1.35	0.425	0.575	0.15
1.4	0.45	0.55	0.1
1.45	0.475	0.525	0.05

Table 1 – The scope of product differentiation – full duopoly

Table 1 indicates that the highest degree of differentiation that can be obtained (for $a-\theta h-c(\theta)$ close to $\frac{11}{10}$) falls short of the socially efficient one. The impact of $a-h\theta-c(\theta)$ on quantities is also worth noticing.

We compute below the individual outputs $q_1^d(x), q_2^d(x)$ for the dispersion equilibrium when $a-h\theta-c(\theta) \in \{1.2, 1.3, 1.4\}$ at three different local markets, i.e. $x \in \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$:

$a-h\theta-c(\theta)$	$x=\frac{1}{4}$	$x=\frac{1}{2}$	$x=\frac{3}{4}$
1.15	0.475	0.325	0.125
1.2	0.46667	0.35	0.16667
1.25	0.45833	0.375	0.20833
1.3	0.45	0.4	0.25
1.35	0.44167	0.425	0.29167
1.4	0.43333	0.45	0.33333
1.45	0.425	0.475	0.375

Table 2a – Firm 1 output at local markets $x \in \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$ – full duopoly

$a - h\theta - c(\theta)$	$x = \frac{1}{4}$	$x = \frac{1}{2}$	$x = \frac{3}{4}$
1.15	0.125	0.325	0.475
1.2	0.16667	0.35	0.46667
1.25	0.20833	0.375	0.45833
1.3	0.25	0.4	0.45
1.35	0.29167	0.425	0.44167
1.4	0.33333	0.45	0.43333
1.45	0.375	0.475	0.425

Table 2b – Firm 2 output at local markets $x \in \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$ – full duopoly

This shows that as $a - h\theta - c(\theta)$ decreases – equivalently, as $h\theta + c(\theta)$ increases – then firms'/products' dispersion increases, and thus: i) firm's 1 output decreases both at central locations, i.e. $[x_1, x_2]$, as well as at locations close to the right market border, i.e. $[x_2, 1]$; in contrast, ii) firm's 1 output increases at locations close to the left market border, i.e. $[0, x_1]$; finally, iii) symmetrical effects arise for firm 2' output (specifically, it increases at locations close to the right border, i.e. $[x_2, 1]$).

As a consequence, the relationship between liability costs and firms size needs to be more closely examined, since focusing on a single (local) market is not sufficient. As long as product differentiation occurs, increasing the liability costs triggers changes in the firms' locations/product choices, which further implies that there are segments on the unit market line where they increase (decrease) their output.

4.2 Full market coverage with a mixed market structure (duopoly-monopoly)

According to Proposition 3 product differentiation occurs in equilibrium (in pure strategies) and this equilibrium is stable for values of the external harm/cost of care that are large, but not too large. Above a certain threshold (i.e. $a - \frac{11}{10}$) it is no longer profitable for firms to deliver output at the most distant local markets. This gives rise to an alternative market structure, with a monopoly on some segments of the market line. We consider this issue here.

Assume for example that both firms compete at central locations, but extreme/border locations are monopolized by a unique firm; w.l.o.g. assume firm 1 (firm 2) is the monopoly on $[0, x_1)$ (on $(x_2, 1]$). This allows to write the derivative of total profit for firm 1 w.r.t. x_1 as

$$\frac{\partial \Pi_1}{\partial x_1} = -\int_0^{x_1} q_1^m(x)dx + \frac{4}{3}\int_{x_1}^{x_2} q_1^d(x)dx,$$

and similarly, for firm 2, the derivative of total profit w.r.t. x_2 as

$$\frac{\partial \Pi_2}{\partial x_2} = -\frac{4}{3}\int_{x_1}^{x_2} q_2^d(x)dx + \int_{x_2}^{1} q_2^m(x)dx.$$

Again, any equilibrium location pattern obtains as a solution to the system $\frac{\partial \Pi_1}{\partial x_1} = 0$, $\frac{\partial \Pi_2}{\partial x_2} = 0$. Following the same strategy, we get the next result:

Proposition 3 *(Strict Liability and the mixed Duopoly/Monopoly)*
If $h\theta - c(\theta) < a - 0.7$, the unique stable location equilibrium is such that both firms compete at each local market $x \in (x_1, x_2)$, with $x_1 = \frac{1}{2}(1-\epsilon) < x_2 = \frac{1}{2}(1+\epsilon)$ whereas firm 1 (firm 2) monopolizes all locations $x \in [0, x_1]$ (resp. $[x_2, 1]$), where

$$\epsilon = \left(\frac{50}{23}(a - h\theta - c(\theta)) - \frac{9}{23}\right) - \frac{18}{23}\sqrt{\left(\frac{25}{9}(a - h\theta - c(\theta)) - \frac{1}{2}\right)^2 - \frac{23}{9}\left((a - h\theta - c(\theta)) - \frac{1}{4}\right)}.$$

Proof. For firm 1, substituting (4)-(5') in $\frac{\partial \Pi_1}{\partial x_1}$ and integrating by part yields :

$$\frac{\partial \Pi_1}{\partial x_1} = -\frac{1}{2}\left((a - h\theta - c(\theta))\, x_1 - \frac{1}{2}x_1^2\right) + \frac{4}{9}\left((a - h\theta - c(\theta))\,(x_2 - x_1) - \frac{1}{2}(x_2 - x_1)^2\right).$$

The SOC is satisfied under Assumption 2 since for any $0 \le x_1 \le x_2 \le 1$ we have that:

$$\frac{\partial^2 \Pi_1}{\partial x_1^2} = \left(-\frac{1}{2}\right)(a - h\theta - c(\theta) - x_1) - \frac{4}{9}(a - h\theta - c(\theta) - (x_2 - x_1)) < 0$$

By the same token, for firm 2 the derivative of total profit w.r.t. x_2 is:

$$\begin{aligned}\frac{\partial \Pi_2}{\partial x_2} &= \frac{1}{2}\left(a - h\theta - c(\theta) - \frac{1}{2}\right) - \frac{1}{2}\left((a - h\theta - c(\theta) - 1)\, x_2 + \frac{1}{2}x_2^2\right) \\ &\quad -\frac{4}{9}\left((a - h\theta - c(\theta))\,(x_2 - x_1) - \frac{1}{2}(x_2 - x_1)^2\right).\end{aligned}$$

The SOC for firms 2 is also satisfied since $\frac{\partial^2 \Pi_2}{\partial x_2^2} = \frac{\partial^2 \Pi_1}{\partial x_1^2}$ given that $x_1 = 1 - x_2$ in equilibrium.

Hence the firms' best reply functions may now be written as (setting $x_2 = f_{F1}(x_1) \Leftrightarrow \frac{\partial \Pi_1}{\partial x_1} = 0$ and $x_2 = g_{F2}(x_1) \Leftrightarrow \frac{\partial \Pi_2}{\partial x_2} = 0$):[10]

$$\begin{cases} -\frac{1}{2}\left(\left(a-h\theta-c(\theta)\right)x_1 - \frac{1}{2}x_1^2\right) + \frac{4}{9}\left(\left(a-h\theta-c(\theta)\right)(x_2-x_1) - \frac{1}{2}(x_2-x_1)^2\right) = 0 \\ \frac{1}{2}\left(a-h\theta-c(\theta)-\frac{1}{2}\right) - \frac{1}{2}\left(\left(a-h\theta-c(\theta)-1\right)x_2 + \frac{1}{2}x_2^2\right), \\ -\frac{4}{9}\left(\left(a-h\theta-c(\theta)\right)(x_2-x_1) - \frac{1}{2}(x_2-x_1)^2\right) = 0 \end{cases} \tag{7}$$

It is obvious that $x_1 = \frac{1}{2} = x_2$ is no longer a possible solution. Let $\epsilon = x_2 - x_1$; equating the LHS in (7), any equilibrium candidate satisfies the following condition:

$$\left(a-h\theta-c(\theta)\right)x_1 - \frac{1}{2}x_1^2 = \left(a-h\theta-c(\theta)-\frac{1}{2}\right) - \left(a-h\theta-c(\theta)-1\right)x_2 - \frac{1}{2}x_2^2;$$

and solving leads to $x_2 = \frac{1}{2}(1+\epsilon)$ and thus $x_1 = \frac{1}{2}(1-\epsilon)$, meaning that any equilibrium is symmetric. Substituting for $x_2 = g_{F2}(x_1)$ yields

$$\frac{23}{36}\epsilon^2 - \left(\frac{25}{9}(a-h\theta-c(\theta)) - \frac{1}{2}\right)\epsilon + \left(a-h\theta-c(\theta)-\frac{1}{4}\right) = 0. \tag{8}$$

The general solution to (7) has two possible roots:

$$\epsilon = \left(\frac{50}{23}\left(a-h\theta-c(\theta)\right) - \frac{9}{23}\right) \pm \frac{18}{23}\sqrt{\left(\frac{25}{9}\left(a-h\theta-c(\theta)\right) - \frac{1}{2}\right)^2 - \frac{23}{9}\left(\left(a-h\theta-c(\theta)\right) - \frac{1}{4}\right)}.$$

The SPNE is the root for which conditions $0 < \epsilon < 1$ and $0 \leq x_1 \leq x_2 \leq 1$ (with $x_1 = \frac{1}{2}(1-\epsilon), x_2 = \frac{1}{2}(1+\epsilon)$) together with the SOCs are all satisfied; moreover, it must also be consistent with the existence of both a monopoly and a duopoly at some locations. It can be verified that : $a - h\theta - c(\theta) > x_1 \Rightarrow q_1^m(x=0) > 0$ and $q_2^d(x_1) > 0$; whereas : $a - h\theta - c(\theta) > x_2 \Rightarrow q_2^m(x=1) > 0$ and $q_1^d(x_2) > 0$, which is more demanding. As

[10] Once again, returning to the derivative of firm 1 total profit wrt x_1, and setting $\frac{\partial \Pi_1}{\partial x_1} = 0$, we obtain equivalently that $\int_0^{x_1} q_1^m(x)dx = \frac{4}{3}\int_{x_1}^{x_2} q_1^d(x)dx$, which means that its total output delivered to local markets where firm 1 is a monopoly must be equal to $\frac{4}{3}$ times its total output delivered to the local market where both firms compete. In other words, the quantity-median property is now modified to account for the mixed market structure situation.

a consequence, in the rest of the analysis, we assume that this condition is met, and in order to perform numerical computations, we impose that $a - h\theta - c(\theta) > 0.7$.[11]

Given this restriction, it can be verified that

– the candidate

$\epsilon^{+} = \left(\frac{50}{23}\left(a - h\theta - c(\theta)\right) - \frac{9}{23}\right) + \frac{18}{23}\sqrt{\left(\frac{25}{9}\left(a - h\theta - c(\theta)\right) - \frac{1}{2}\right)^2 - \frac{23}{9}\left(\left(a - h\theta - c(\theta)\right) - \frac{1}{4}\right)}.$

does not satisfy the basic requirements ($\epsilon^{+} > 1$ for any $a - h\theta - c(\theta) \in (0.7, 1)$);

– the candidate

$\epsilon^{-} = \left(\frac{50}{23}\left(a - h\theta - c(\theta)\right) - \frac{9}{23}\right) - \frac{18}{23}\sqrt{\left(\frac{25}{9}\left(a - h\theta - c(\theta)\right) - \frac{1}{2}\right)^2 - \frac{23}{9}\left(\left(a - h\theta - c(\theta)\right) - \frac{1}{4}\right)}.$

does satisfy the basic requirements. Moreover, it satisfies the stability condition as we show below. The stability condition $|f'_{F1}(x_1)| > |g'_{F2}(x_1)|$ is written as

$$\frac{\frac{1}{2}\left(\left(a - h\theta - c(\theta)\right) - x_1\right) + \frac{4}{9}\left(\left(a - h\theta - c(\theta)\right) - \epsilon\right)}{\frac{4}{9}\left(\left(a - h\theta - c(\theta)\right) - \epsilon\right)} > \frac{\frac{4}{9}\left(\left(a - h\theta - c(\theta)\right) - \epsilon\right)}{\frac{1}{2}\left(\left(a - h\theta - c(\theta)\right) - x_1\right) + \frac{4}{9}\left(\left(a - h\theta - c(\theta)\right) - \epsilon\right)},$$

given that the numerator of the LHS and the denominator of the RHS must be positive because of the SOC. Hence, the inequality always holds. Thus, the candidate ϵ^{-} associated with $x_2 = \frac{1}{2} + \frac{1}{2}\epsilon^{-}$, $x_1 = \frac{1}{2} - \frac{1}{2}\epsilon^{-}$ is the unique stable location equilibrium. ■

Proposition 4 shows that if we extend the range of admissible values for $a - h\theta - c(\theta)$ (specifically for lower values, for instance by considering much larger values of $h\theta + c(\theta)$), then firm dispersion (equivalently, product differentiation) is supported by a mixed market structure with duopoly at locations around the market center, and monopoly close to the each market border. This also implies that the scope of product differentiation is enlarged. Once again, we perform a simple numerical application to illustrate this result.

[11] As a consequence, it is possible to rule out the obvious solutions associated with $\theta h + c(\theta) \in \left\{\frac{1}{4}, \frac{9}{50}\right\}$, that correspond to the cases where one of the bracketed term in (8) is nul. See also the Appendix for more details.

$a - h\theta - c(\theta)$	x_1	x_2	ϵ
0.7	0.31345	0.68656	0.37311
0.8	0.31491	0.68510	0.37019
0.9	0.31583	0.68417	0.36834
1	0.31647	0.68353	0.36706
1.1	0.316 94	0.68306	0.36612
1.2	0.3173	0.6827	0.3654
1.3	0.31759	0.68242	0.36483
1.4	0.31782	0.68219	0.36437
1.5	0.31801	0.68200	0.36399
1.7	0.31830	0.6817	0.36340
1.9	0.31852	0.68149	0.36297
2	0.31861	0.68140	0.36279
2.5	0.31893	0.68107	0.36214
3	0.31913	0.68087	0.36174

Table 3 – The scope of product differentiation – mixed market structure (duopoly/monopoly)

Table 3 shows that the mixed market structure: i) is consistent with product differentiation in equilibrium for a larger range of harm/cost of care, ii) leads to a larger degree of product differentiation than the full duopoly (in the range of harm/cost of care where this latter is viable), finally iii) further extends the scope of product differentiation since an equilibrium (in pure strategies) exists for a larger level of harm/cost of care. Nonetheless, the extent of equilibrium product differentiation is still lower than the socially optimal one.

In Tables 4a,4b we consider the impact of $a - h\theta - c(\theta)$ on output levels within the mixed market structure duopoly/monopoly. In Table 4a we compute $q_1^m(x)$ and $q_1^d(x)$ for $a - h\theta - c(\theta) \in \{0.8, 1, 1.2, 1.4, 2, 3\}$ in the neighborhood of local markets where firm 1 is a pure monopoly (e.g. at locations x close to $[0, x_1]$).[12] As $a - h\theta - c(\theta)$ decreases,

[12]These locations correspond to firm's 1 equilibrium locations for $a - h\theta - c(\theta) = 0.8, 1, 1.2, 1.4, 2, 3$

both firms' locations as well as the market structure change at these locations; but the increase in product differentiation is associated with a decrease in the set of local markets that firm 1 serves as a monopoly (and hence an expansion of the area where both firms compete). As a result, in the neighborhood of these locations, firm's 1 output decreases either because firm 1 is a monopoly, or because firm 1 faces now a competitor (both $q_1^m(x), q_1^d(x)$ decrease with $a - h\theta - c(\theta)$).

$a - h\theta - c(\theta)$	$x = 0.31491$	$x = 0.31647$	$x = 0.3173$	$x = 0.31782$	$x = 0.31861$	$x = 0.31913$
3	1.4979	1.4987	1.4991	1. 4993	1.4997	1.5
2	0.99815	0.99893	0.99935	0.99961	1	0.78708
1.4	0.69855	0.69933	0.69974	0.7	0.58733	0.58681
1.2	0.59881	0.59959	0.6	0.52114	0.52035	0.51983
1	0.49922	0.5	0.45486	0.45434	0.45497	0.45445
0.8	0.4	0.3885	0.38767	0.38715	0.38636	0.38584

Table 4a – Firm 1 output in the neighborhood of $[0, x_1]$ – mixed duopoly/monopoly

In turn, Table 4b considers the neighborhood of local markets where firm 2 is a pure monopoly, e.g. locations close to the segment $[x_2, 1]$. It turns out that as $a - h\theta - c(\theta)$ decreases, the effect for firm 1 at a given location depends on whether firm's 2 monopoly power is secured at that location (in which case the output for firm 1 is null), or the market structure changes into a duopoly (hence firm's 1 output makes a discrete jump from 0 to $q_1^d(x) > 0$).

$a - h\theta - c(\theta)$	$x = 0.68510$	$x = 0.68353$	$x = 0.6827$	$x = 0.68219$	$x = 0.68140$	$x = 0.68087$
3	0	0	0	0	0	0
2	0	0	0	0	0	0.42534
1.4	0	0	0	0	0.22454	0.22507
1.2	0	0	0	0.15691	0.1577	0.15823
1	0	0	0.089457	0.089967	0.090757	0.091287
0.8	0	0.02144 3	0.022273	0.022783	0.023573	0.024103

respectively. See Table 3.

Table 4b – Firm 1 output in the neighborhood of $[x_2, 1]$ – mixed duopoly/monopoly

Thus, Tables 2a,2b, 4a,4b illustrate together that the impact of liability costs on firms' output/size may be quite complex, since it depends on the magnitude of the liability cost, the actual impact of the latter on the scope of product differentiation, and finally, on the market structure itself. With minimum product differentiation, the basic result of the unilateral accident model holds (the higher the liability costs, the smaller the firms' size). As the scope of product differentiation increases, then the impact on firms' size depends on the effects of liability costs both on the extent of product differentiation and on the structure of the market/competition.

4.3 Social welfare

In this final section we discuss the impact of the liability costs (or equivalently, expected harm/cost of care) on consumers surplus and producers surplus, as well as on social welfare. We perform calculations for: i) "large" values of $a - h\theta - c(\theta) \in \{2, 2.5, 3\}$ as a matter of comparison between the full duopoly with central agglomeration equilibrium and the mixed duopoly/monopoly; ii) "intermediate" values of $a - h\theta - c(\theta) \in \{1.2, 1.3, 1.4\}$ as a matter of comparison between the full duopoly with dispersed equilibrium locations and the mixed duopoly/monopoly. We write[13] Total Industry Profit as

$$\Pi_T = \Pi_1 + \Pi_2 = \int_0^1 \sum_{i=1}^{2} (q_i(x))^2 \, dx, \tag{PS}$$

Consumers' Surplus as

$$CS = \int_0^1 \frac{(Q(x))^2}{2} dx, \tag{CS}$$

and Social Welfare as

$$W = SC + \Pi. \tag{SW}$$

The following two tables display the result of the numerical application:

[13] See the Appendix for more details on the expressions of profits, consumers' surplus and social welfare.

	$a - h\theta - c(\theta)$	$Full\ Duopoly$	$Duopoly +$ $Monopoly$
Π_T	2	2×0.34259	2×0.40785
	2.5	2×0.56481	2×0.65779
	3	2×0.84259	2×0.96775
CS	2	0.68519	0.65558
	2.5	1.1296	1.0158
	3	1.6852	1.456
W	2	1.3704	1.47128
	2.5	2.2593	2.33138
	3	3.3704	3.3915

Table 5 – "Large values" for $a - h\theta - c(\theta)$ (central agglomeration in a full duopoly)

A first noticeable result is that the magnitude of the external harm/cost of care has no impact either on the ranking of social welfare, or on the ranking of firms' profits across the different market configurations : for any level of $a - h\theta - c(\theta)$ (whether large, intermediate, or low), the social welfare is the highest under the mixed market structure, but it is the lowest in full duopoly when $a - h\theta - c(\theta)$ takes intermediate values. In contrast, firms obtain the lowest profits in the full duopoly case. More market power together with more product differentiation is always desirable to firms. A key ingredient in this case is the fact that a different price is charged each local market - remember that firms control shipping of their product to local markets, so they can apply spatial price discrimination.

	$a - h\theta - c(\theta)$	*Full Duopoly*	*Duopoly +* *Monopoly*
Π_T	1.2	2×0.1109	2×0.13277
	1.3	2×0.13074	2×0.15875
	1.4	2×0.15083	2×0.18713
CS	1.2	0.19419	0.2456
	1.3	0.24415	0.28565
	1.4	0.297	0.32890
W	1.2	0.41599	0.51114
	1.3	0.50563	0.60315
	1.4	0.59866	0.70316

Table 6 – "Intermediate values" for $a - h\theta - c(\theta)$

In contrast, consumers' surplus is the highest with full duopoly, but for intermediate value of $a - h\theta - c(\theta)$, the mixed market structure allows a higher consumers's surplus than the full duopoly. Hence, there is no trade-off for consumers in this range: whenever $a - h\theta - c(\theta)$ is not too low, more product differentiation is desirable for consumers despite the fact that more product differentiation means facing more market power from firms (at least locally). Actually, in this case higher prices are compensated by higher product differentiation and the full market coverage.

Thus, a major conclusion is that whether consumers' surplus or social welfare is used as the criterion to conduct public policies, the conclusions may turn out to be different regarding product differentiation and market structure for industries subject to product liability whenever $a - h\theta - c(\theta)$ takes values that are not too low.

5 Conclusion

This paper examines the consequences of product liability for firms choices in terms of both output and product differentiation. We also consider the impact of liability on market structures. Our analysis points at the possibility of multiple equilibria, and argues that analyzing the impact of liability costs on firms' size may not be straightforward.

The formal framework we used here is very simple, and several alternative assumptions may be considered. For example, in our model consumers across local markets differ only w.r.t. the amount of expected harm they incur (proportional to quantities). The analysis may be extended to a situation where consumers are heterogenous in terms of their response to output and ensuing harm (i.e. heterogenous h). Specifically, whenever (the probability distribution function of) h depends on x, both product (horizontal) differentiation and safety (vertical) differentiation (see equation (1)) would occur in equilibrium, but this will not change qualitatively our conclusions. By the same token, our results are robust when allowing for a different shape of cost-of-care function, with both a variable and a fixed component (see Cosnita-Langlais and Langlais 2022, for an analysis of environmental liability).

References

[1] Anderson S.P. and Neven D.J. 1991. Cournot competition yields spatial agglomeration, *International Economic Review*, 32, 793-808.

[2] Baumann F. and Friehe T. 2015. Optimal damages multipliers in oligopolistic markets, *Journal of Institutional and Theoretical Economics*, 171 (4), 622-640.

[3] Baumann F., Friehe T., and Rasch A. 2018. Product liability in markets for vertically differentiated products, *American Law & Economics Review,* 20, 46-81.

[4] Benassi C, Chirco A., and Scrimitore M. 2007. Spatial discrimination with quantity competition and high transportation costs : A note, *Economics Bulletin*,12(1), 1-7.

[5] Bertoletti P. and Etro F. 2016. Preferences, entry and market structure, *RAND Journal of Economics*, 47, 792-821.

[6] Boyd J. 1994. Risk, liability, and monopoly, *International Journal of the Economics of Business*, 1, 387-403.

[7] Buzby J. and Franzen P. 1999. Product safety and product liability, *Food Policy*, 24, 637-651.

[8] Chen Y. and Hua X. 2017. Competition, product safety, and product liability, *Journal of Law, Economics, and Organization*, 33(2): 237-267.

[9] Cosnita-Langlais A. and Langlais E. 2022. Product choice and safety under environmental liability laws : to differentiate or not to differentiate ?, mimeo.

[10] Chamorro Rivas J.M. 2000. Spatial dispersion in Cournot competition, *Spanish Economic Review*, 2, 145-152.

[11] Daughety A. and Reinganum J. 1995. Product safety : liability, R&D, and signalling, *American economic Review*, 85(5), 1187-1206.

[12] Daughety A.F. and Reinganum J.F. 2006. Markets, torts and social inefficiency, J.H., *RAND Journal of Economics,* 37, 300-323.

[13] Daughety A.F. and Reinganum J.F. 2013. Economic Analysis of Products Liability: Theory. In: Arlen, J.H., *Research Handbook on the Economics of Torts*, Edward Elgar.

[14] Eaton C.B. and Schmitt N. 1994. Flexible manufacturing and market structure, *American Economic Review,* 84, 875-888.

[15] Epple D. and Raviv A. 1978. Liability Rules, Market Structure, and Imperfect Information, *The American Economic Review*, 68(1), 80-95.

[16] Etro F. 2014. The theory of endogenous market structures, *Journal of Economic Surveys*, 28, 804-830.

[17] Galasso A. and Luo H. 2019. When does product liability risk chill innovation ? Evidence from medical implants. Harvard Business School, Working Paper 19-002.

[18] Greenhut J., Greenhut M.L., and Li S. 1980. Spatial Pricing Patterns in the United States. *The Quarterly Journal of Economics*, 94, 329-350.

[19] Gupta B., Pal D., and Sarkar J. 1997. Spatial Cournot competition and agglomeration in a model of location choice. *Regional Science and Urban Economics*, 27 (3), 261–282.

[20] Helland E. and Showalter M. 2009. The impact of liability on the physician labor market, *The Journal of Law & Economics*, 52, 635-663.

[21] Horstmann I. and Markusen J. 1992. Endogenous market structures in international trade (natura facit saltum), *Journal of International Economics*, 32, 109-129.

[22] Kreps D. and Scheinkman J. 1983. Quantity Precommitment and Bertrand CompetitionYield Cournot Outcomes, *Bell Journal of Economics,* 14, 326-337.

[23] Loureiro M. 2008. Liability and food safety provision : evidence from the US, *International Review of Law and Economics*, 28, 204-211.

[24] Marino A.M. 1988. Product Liability and Scale Effects in a Long-Run Competitive Equilibrium, *International Review of Law and Economics*, 8, 97-107.

[25] Marino A.M. 1991. Market Share Liability and Economic Efficiency, *Southern Economic Journal*, 57 (3), 667-675.

[26] Marjit S. and Mukherjee A. 2015. Endogenous market structures, trade cost reduction, and welfare, *Journal of Institutional and Theoretical Economics*, 171, 493-511.

[27] Mathieu C. 1995. International Enterprises and endogenous market structures, *Annales d'Economie et de Statistiques*, 47, 171-195.

[28] Matsushima N. 2001. Cournot competition and spatial agglomeration revisited, *Economics Letters*, 73, 175-177

[29] Matsumura T. and Shimizu D. 2005. Spatial Cournot competition and economic welfare: a note. *Regional Science and Urban Economics*, 35 (6), 658-670.

[30] Mayer T. 2000. Spatial Cournot competition and heterogeneous production costs across locations, *Regional Science and Urban Economics*, 20, 325-352.

[31] McBride M. 1983. Spatial Competition and Vertical Integration: Cement and Concrete Revisited, *American Economic Review*, 73, 1011-1022.

[32] Parchomovski G. and Stein A. 2008. Torts and innovation, *Michigan Law Review*, 107, 285-315.

[33] Phlips, L. 1983. "The Economics of Price Discrimination". Cambridge University Press..

[34] Polinsky A.M. and Rogerson W. 1983. Products liability, consumer misperceptions, and market power. *Bell Journal of Economics*, 14, 581–589.

[35] Ringleb A. and Wiggins S. 1990. Liability and Large-Scale, Long-term Hazards, *Journal of Political Economy*, 98, 574-595.

[36] Shimizu D. 2002. Product differentiation in spatial Cournot markets, *Economics Letters*, 76, 317-322.

[37] Spence M. 1977. Consumer Misperceptions, Product Failure and Producer Liability. *Review of Economic Studies*, 44, 561-572.

[38] Spulber D. 1988. Products Liability and Monopoly in a Contestable Market, *Economica*, 55, 333-341.

[39] Sutton J. 1991. *Sunk costs and market structure*, Cambridge MA, MIT Press.

[40] US House. 2003. The medical liability insurance crisis: A review of the situation in Pensylvania, *Hearing before the Subcommittee on Oversight and Investigations*, Committee on Energy and Commerce, 108th Congress, 1st Session, February 10.

[41] US Senate. 2004. The medical liability crisis and its impact on patient care, *Hearing before the Committee on the Judiciary* 108th Congress, 2d Session, August 20.

[42] Viscusi W. and Moore M. 1993. Product Liability, Research and Development, and Innovation, *Journal of Political Economy*, 101 (1), 161-184.

[43] Viscusi W. 2012. Does product liability make us safer ?, *Regulation*, Spring, 24-31.

[44] Viscusi W. 1995. Insurance and catastrophes: The changing role of the liability system, *The Geneva Ppaers on Risk and Insurance Thery*, 20(2), 177-184.

6 Appendix

Complement to the proof of Proposition 4. Let us examine two specific solution candidates to the equation (8), i.e. for which one of the two bracketed terms is nul. Note however that they both require that Assumption 2, as a sufficient condition, be relaxed – and thus the SOC for the location choice need to be weakened to $a - h\theta - c(\theta) - \epsilon > 0$; nevertheless, we show that no equilibrium (in pure strategies) exists in these cases.

– case where $a - h\theta - c(\theta) = \frac{1}{4}$. There exist two candidates for (8) : $\left\{\frac{7}{23}, 0\right\}$. For $\epsilon = \frac{7}{23}$, then $x_2 = \frac{15}{23}$ and $x_1 = \frac{8}{23}$, but the SOCs are not satisfied. For $\epsilon = 0$, we have that $x_2 = \frac{1}{2} = x_1$, and again the SOC is not satisfied (in the strong form).

– case where $\frac{25}{9}(a - h\theta - c(\theta)) = \frac{1}{2} \Leftrightarrow a - h\theta - c(\theta) = \frac{9}{50} = 0.18$; then there is a unique positive root for (8) : $\epsilon = \frac{3}{115}\sqrt{7}\sqrt{23} = 0.33101$; associated to $x_2 = 0.665\,51$ and $x_1 = 0.33450$ – but the SOC is not satisfied, hence it cannot be the solution.

Numerical application. We denote $A = a - h\theta - c(\theta)$. We perform calculations for "large" $A \in \{2, 2.5, 3\}$ as a matter of comparison between the full duopoly with central agglomeration (I), and the mixed structure duopoly/monopoly (III); for "intermediate" $A \in \{1.2, 1.3, 1.4\}$, as a matter of comparison between the full duopoly with equilibrium dispersion (II), and the miedx duopoly/monopoly(III).

(I) In an equilibrium supporting full duopoly at each location, firms are located at the market center when A is large enough. The relevent range for A is $A > \frac{3}{2}$. The individual output at each local market x is $q_i(x) = \frac{1}{3}\left(A - \left|\frac{1}{2} - x\right|\right)$ for $i \in \{1, 2\}$, while the aggregate output is $Q(x) = \frac{2}{3}\left(A - \left|\frac{1}{2} - x\right|\right)$. Then total individual profits are defined by

$$\Pi_1 = \frac{1}{9}\left(\int_0^{\frac{1}{2}} \left(A - \frac{1}{2} + x\right)^2 dx + \int_{\frac{1}{2}}^1 \left(A + \frac{1}{2} - x\right)^2 dx\right) = \Pi_2,$$

and consumers' surplus is given by

$$CS = \frac{1}{2}\frac{4}{9}\left(\int_0^{\frac{1}{2}} \left(A - \frac{1}{2} + x\right)^2 dx + \int_{\frac{1}{2}}^1 \left(A + \frac{1}{2} - x\right)^2 dx\right).$$

(II) In an equilibrium supporting full duopoly at each local market, firms are dispersed when A is low enough, but not too low. The relevent range for A is $A \in \left[\frac{3}{2}, \frac{11}{10}\right]$. The individual output at each location is $q_i(x) = \frac{1}{3}(A - 2|x_i - x| + |x_j - x|)$ for $i \in \{1, 2\}$, while the aggregate output is $Q(x) = \frac{1}{3}(2A - |x_1 - x| - |x_2 - x|)$, with $x_1 = \frac{1}{2}A - \frac{1}{4}$ and $x_2 = \frac{5}{4} - \frac{1}{2}A$. Firms' equilibrium locations for the different values of $A \in \{1.2, 1.3, 1.4\}$ used for the calculations are as follows:

$a - h\theta - c(\theta)$	x_1	x_2
1.2	0.35	0.65
1.3	0.4	0.6
1.4	0.45	0.55

Then total individual profits are defined by

$$\Pi_1 = \frac{1}{9}\left(\begin{array}{c} \int_0^{x_1} (A - 2 \times x_1 + x_2 + x)^2\, dx + \int_{x_1}^{x_2} (A + 2 \times x_1 + x_2 - 3x)^2\, dx \\ + \int_{x_2}^{1} (A + 2 \times x_1 - x_2 - x)^2\, dx \end{array}\right),$$

$$\Pi_2 = \frac{1}{9}\left(\begin{array}{c} \int_0^{x_1} (A - 2 \times x_2 + x_1 + x)^2\, dx + \int_{x_1}^{x_2} (A - 2 \times x_2 - x_1 + 3 \times x)^2\, dx \\ + \int_{x_2}^{1} (A + 2 \times x_2 - x_1 - x)^2\, dx \end{array}\right),$$

and consumers' surplus is given by

$$CS = \frac{1}{2}\frac{1}{9}\left(\begin{array}{c} \int_0^{x_1} (2 \times A - x_2 - x_1 + 2x)^2\, dx + \int_{x_1}^{x_2} (2 \times A - x_2 + x_1)^2\, dx \\ + \int_{x_2}^{1} (2 \times A + x_2 + x_1 - 2 \times x)^2\, dx \end{array}\right).$$

(III) In an equilibrium supporting the duopoly at central locations and a monopoly at each market border, firms are dispersed whatever the size of A. The individual output at each local amrket x is: $q_i(x) = \frac{1}{3}(A - 2|x_i - x| + |x_j - x|)$ for $i \in \{1, 2\}$, while the aggregate output is $Q(x) = \frac{1}{3}(2A - |x_i - x| - |x_j - x|)$ if firms compete at that location; or, the aggregate output at each location is $q_i(x) = Q(x) = \frac{1}{2}(A - |x_i - x|)$ for $i \in \{1, 2\}$ if firm i is a monopoly at that location. Firms' equilibrium locations for the different values of $A \in \{0.8, 0.9, 1, 1.2, 1.3, 1.4, 2, 2.5, 3\}$ used for the calculations are as follows:

$a - h\theta - c(\theta)$	x_1	x_2
0.8	0.31491	0.68510
0.9	0.31583	0.68417
1	0.31647	0.68353
1.2	0.3173	0.6827
1.3	0.31759	0.68242
1.4	0.31782	0.68219
2	0.31861	0.68140
2.5	0.31893	0.68107
3	0.31913	0.68087

Then total individual profits are:
$\Pi_1 = \frac{1}{4}\int_0^{x_1} (A - x_1 + x)^2 \, dx + \frac{1}{9}\int_{x_1}^{x_2} (A + 2 \times x_1 + x_2 - 3x)^2 \, dx,$
$\Pi_2 = \frac{1}{9}\int_{x_1}^{x_2} (A - 2 \times x_2 - x_1 + 3x)^2 \, dx + \frac{1}{4}\int_{x_2}^{1} (A + x_2 - x)^2 \, dx,$
and consumers' surplus is given by
$CS = \frac{1}{2}\left(\frac{1}{4}\int_0^{x_1} (A - x_1 + x)^2 \, dx + \frac{1}{9}\int_{x_1}^{x_2} (2 \times A + x_1 - x_2)^2 \, dx + \frac{1}{4}\int_0^{x_1} (A + x_2 - x)^2 \, dx\right).$