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A post-Paretian concept of optimality: the "Conditional Agreement Point"

Fabrice Tricou

UMR 7235

Université de Paris Ouest Nanterre La Défense
(bâtiment G)
200, Avenue de la République
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07
Email : nasam.zaroualete@u-paris10.fr

université Paris Ouest Nanterre La Défense

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Fabrice Tricou (University of Paris Ouest Nanterre la Défense - EconomiX).

Abstract. This paper introduces and develops the concept of "conditional agreement point", defined as the dominating issue (*hard* optimality) within a certain restricted subset of the set of feasible issues (*partial* optimality). Such a concept associates individualistic independence (via the operation of individual preferences) and humanistic autonomy (via the social choice of the determined subset). As the "conditional agreement point" assumes a dualistic conception of human beings as self-interest followers and as rule makers, it acknowledges mixed or complex types of behavior and it supports some syntheses between efficiency and justice. Precisely, the notion of CAP permits the reconsideration of classical coordination or cooperation problems such as the bilateral exchange.

Keywords: efficiency and justice; independence and autonomy; bilateral exchange.

JEL classification: D60, D63, D50, C70.

1. Introduction: Utilitarian Optimum, Pareto Optimum and Conditional Agreement Point.

(a) Pareto accomplished a *tour de force* when he introduced a concept of social efficiency consistent with a strictly individualistic basis. In the Paretian line, there is no social utility *per se* and society has reached a "good" situation when it is no longer possible to improve any individual's position without damaging another individual's one (which can't be justified on economic grounds)[1]. The rise of the Pareto optimum as the key concept of normative economics meant the fall of the Utilitarian optimum, which is ambivalently related to individualism. In the Benthamite line, social utility is nothing more than the sum of individual utilities (social immanence); but the maximization of such an aggregate utility[2] requires individual sacrifices, when the losses of the losers are overcompensated by the gains of the winners (social transcendence).

The reign of Paretian ethics continues in welfare economics, even if this field is still haunted by the ghost of Utilitarianism[3]. Actually Pareto optimality faces two kinds of external criticism, beyond the selection problem that arises from the chronic multiplicity of Pareto optima (which contrasts to the general unicity of the Utilitarian optimum).

Firstly, the Paretian system presents a conservative bias. The protection of each individual freedom is strongly ensured by the veto power granted to every individual about any considered social modification. But as a consequence, the incomplete Pareto criterion is silent and inoperative in front of any conflicting social change, which induces a general paralysis and maintains the *status quo*. The Utilitarian system presents the symmetrical

[1] For Pareto, a sacrifice of some for the others cannot be justified by economic reasons (strict "ophelimity"); but it may be defended for social or ethical considerations (broad "utility"). See Pareto [1916], § 2129, p. 1339.
[2] The definition of an aggregate utility relies on the assumption of interpersonal comparisons, which requires cardinal utility. So the advancement of ordinal utility made classical Utilitarianism impossible: the old Bentham [1789] version of Utilitarianism (as opposed to the new Harsanyi [1955] version of it) was severely hit.
[3] Hausman and McPherson [1996] do not consider that Paretian efficiency surpasses Utilitarianism: "there are problems with endorsing Pareto improvements" (p. 88) and "Utilitarianism is a tempting ethical theory" (p. 106).

advantage and drawback, being able to overcome inter-individual disagreements and decide on conflictual social changes, but at the price of the sacrifice of some individual interests.

Secondly, the Paretian perspective promotes individual freedom and social efficiency, but does not value social justice: it is fairness neutral. Unlike Walras, who was concerned by social justice[4], Pareto considered the notion of justice as vague and ill-defined, so he excluded it from the hard and "logical" field of economics and placed it in the soft and "non-logical" field of sociology. As the Pareto optimum, the Utilitarian optimum is fairness neutral; but the assumption of interpersonal comparaison of utility can easily be used to ground an egalitarian optimum as the situation where the utility levels of all individuals are identical.

Endeavoring to develop a concept of optimality, we adopt the Paretian theory as a starting point, because it is the prominent reference in the field of socio-economic ethics and because its ability to produce social welfare statements on a strictly individualistic basis is quite remarkable[5]. But we depart from this perspective, as the conservatism bias and the ignorance of justice at least raise some real objections and at most constitute true failures for the Paretian ethics. In particular, the strong separation between the good "efficiency scientific propositions" on one side and the bad "justice ideological judgments" on the other side is disputable[6]. So we could say that the following research work is "post-Paretian", developed at the same time *behind* and *beyond* the Paretian perspective.

(b) To begin with, we notice that Pareto optimality is not the only notion of optimality that can be derived from the Pareto criterion. It is usual to distinguish, among Pareto optima, *weak* ones and *strong* ones (see section 2). Beyond this internal distinction, it is possible to develop two basic external distinctions generating four concepts of optimality. On the one hand, an optimum could be said *soft*, when it is not dominated by any other issue; and *hard*, when it dominates all the other issues. On the other hand, an optimum could be said *total*, when it is unrestricted or defined on the comprehensive set of feasible issues; and *partial*, when it is restricted or defined on a certain subset of issues.

Using these two oppositions, we can obviously say that the Pareto optimum (henceforth PO) is *soft* and *total*. We may also consider the class of *hard* and *partial* optima, and we propose to name such an optimum concept "*conditional agreement point*" (henceforth CAP). As a *hard* optimum, it is an *agreement point*, because the individuals unanimously prefer it to any other issue of the subset. As a *partial* optimum, it is *conditional*, because it is determined by the restriction to one specified subset.

As a concept of optimality distancing itself from the PO, the CAP can support a "third way" between the Paretian conservatism and the Utilitarian violence, transforming a conflictual situation into an agreement situation through the selection of a subset in which there is one dominating element. The selection of such a "good" subset could be obtained by the introduction of justice, either with the informal idea of some fair mutual concessions

[4] See Jaffé [1977] or Bridel [2011] about the normative orientation of Walras' works: justice is more than a concern of his "social economy"; it is a general principle which is even relevant for his "pure economy".
[5] On the way the Paretian perspective connects welfarism and individualism, see Fleurbaey [1996], pages 48-52.
[6] Efficiency displays an axiological dimension, when united with individual freedom and when traded off with equality or equity. And justice displays an objective dimension, when one explicit definition of it by a theorist is supposed to represent the implicit acknowledgement of this determination of justice by the ruling people.

(elimination of symmetrically inequal situations) or with the formal idea of a fair common submission (to the same impersonal law). This way the two mentioned flaws of the PO (conservatism and "afairness") would be addressed in one consistent move.

As a concept of optimality staying in line with the PO, the CAP may be called up to select one PO, for instance the fair(est) one, among a multiplicity. To accomplish such a mission, the proper subset has to be determined in an adequate way that would guarantee the existence and the Pareto optimality of the CAP. The PO multiplicity issue would then be overcome, as the existence of a CAP essentially ensures its uniqueness (see section 2).

As an interpretative tool of concepts of solution, the CAP records the principle of individual liberty (or of private interest) but also acknowledges the principle of common autonomy, through the way the relevant subset of issues is determined by the community of the persons. Such a dualistic approach could be relevant to understand the mixing of competition and cooperation and also the blending of efficiency and fairness.

(c) After this introduction, the paper elaborates the formal definition of the CAP and of related concepts (section 2). It then considers the problem of bilateral exchange in the Edgeworthian box (section 3), showing that some of its solutions involve notions of CAP. The conclusion (section 4) focuses on the philosophical meaning of the concept, mixing the two modern capabilities of individualistic independence and of humanistic autonomy.

2. Formal definitions and general properties of the CAP and related concepts.

2.1. Basic framework, formal definitions and graphic representations.

Let's call X the comprehensive set of possible social issues, and x any of these possible issues: X = {x}. Let's call Y a specified subset of X, and y any element of Y: Y = {y}.

For simplicity considerations, definitions and properties will be presented in the framework of a two agent society (or economy). Let's call i and j these two individuals, and denote by k either i or j. Each individual is supposed to have a complete and transitive large preference relation (R_k for k) defined on the set X (but also valid on any subset Y of X): the proposition "k prefers x_1 to x_2 or k is indifferent between x_1 and x_2" is noted "$x_1R_kx_2$". From this large preference relation R_k, a strict preference relation P_k ($x_1P_kx_2$ being equivalent to $x_1R_kx_2$ and non[$x_2R_kx_1$]) and an indifference relation I_k ($x_1I_kx_2$ being equivalent to $x_1R_kx_2$ and $x_2R_kx_1$) can be deduced.

The Pareto principle develops an objective ranking on the basis of unanimous subjective rankings. There are two different ways to specify this view and to formally build up a social preference relation expressing the common judgment (in case there is one) on the feasible issues[7]. According to the *weak* Pareto principle, x_1 is socially preferred to x_2 if i and j strictly prefer x_1 to x_2: x_1Px_2 if "$x_1P_ix_2$ and $x_1P_jx_2$". According to the *strong* Pareto principle, x_1 is socially preferred to x_2 (x_1Px_2) not only when all prefer x_1 to x_2 ("$x_1P_ix_2$ and $x_1P_jx_2$"), but also when one strictly prefers x_1 to x_2 and the other does not strictly prefer x_2 to x_1 ("$x_1P_ix_2$ and $x_1I_jx_2$" or "$x_1I_ix_2$ and $x_1P_jx_2$").

[7] On the weak and strong forms of the Pareto principle, see Fleurbaey [1996], pages 33-34.

The social preference relation is usually incomplete[8] because of the possibility of individual disagreements about any given couple of issues (individual diversity being a direct consequence of individual independence): if "$x_1P_ix_2$ and $x_2P_jx_1$", then there is no social ranking between these issues. The social preference relation is also transitive, as the individual preference relations are transitive ("$x_1P_ix_2P_ix_3$ and $x_1P_jx_2P_jx_3$" imply $x_1Px_2Px_3$).

In such a simple framework, four concepts of optimality can be defined. One is the Pareto Optimum, but all are from the Paretian family, as all are grounded on the Paretian domination relation P.

Firstly, a feasible issue x_0 is a Pareto Optimum if there is no other element x of X such that "xPx_0". A Pareto optimum (denoted PO[X]) is *soft* (not dominated) and *total* (unrestricted).

Secondly, in case one issue is dominating on the whole set, one could name it (unconditional) Agreement Point. A feasible issue x_1 is an Agreement Point if "x_1Px", where x is any element of X different from x_1. An Agreement Point (denoted AP[X]) is *hard* (dominating) and *total* (unrestricted).

Thirdly, in case an issue is not dominated on a subset Y of X, one could name it Conditional Pareto Optimum. A feasible issue y_0 is a Conditional Pareto Optimum relatively to Y if there is no other element y of Y such that "yPy_0". A Conditional Pareto Optimum (denoted CPO[Y]) is *soft* (not dominated) and *partial* (restricted).

Fourthly, in case an issue is dominating on a subset Y of X, one could name it Conditional Agreement Point. A feasible issue y_1 is a Conditional Agreement Point relatively to Y if "y_1Py", where y is any element of Y different from y_1. A Conditional Agreement Point (denoted CAP[Y]) is *hard* (dominating) and *partial* (restricted).

The paper focuses on the CAP because it is balanced (as hard *but* partial) while the AP is overdetermined and tight (it is rare and raises peculiar coordination issues)[9] and while the CPO is underdetermined and loose (it is abundant and raises many coordination issues)[10].

To get a graphic representation of the four concepts, let's introduce the set W of considered issues, which are either all possible issues (if W = X) or just some of them (if W = Y). We want to represent the cAP (the AP if W = X and the CAP if W = Y) and the cPO (the PO if W = X and the CPO if W = Y). Any element w of W is associated to a couple of satisfactions (u_i^w; u_i^w) that can be represented in the (u_i; u_i) plane. The set of reachable utilities (for W) will be represented by the curve IJ[11] and its South-West, the North-East of the frontier

[8] The social preference relation is complete in at least two degenerate but noteworthy cases: if society is made of identical individuals (uniformity) and if society is submitted to the will of a leader (dictatorship).

[9] A unanimously preferred situation is obviously selected in a social choice context, but does not necessarily emerge in a strategic interaction context. It is a Nash equilibrium but not necessarily the only one, which raises a selection problem expressed by coordination games like the "stag hunt" or the "meeting at the mall".

[10] First the selection of the subset and second the choice of a CPO in the selected subset.

[11] It is assumed that each couple of utilities of this frontier corresponds to one and *one only* reachable issue.

corresponding to couples of utilities that can't be reached. We consider three typical shapes for the IJ curve: (IMNJ) in fig. 1, (IPJ) in fig. 2 and (IQJ) in fig. 3.

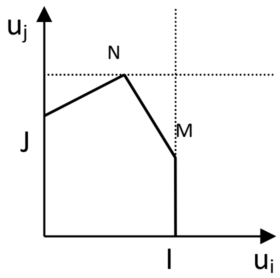

Fig. 1

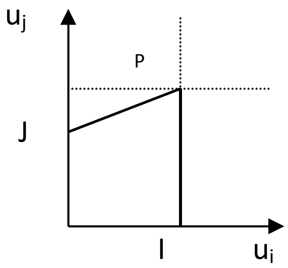

Fig. 2

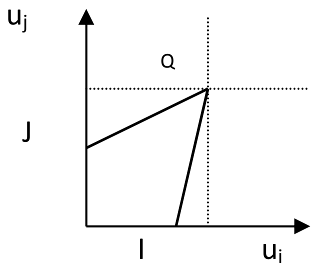

Fig. 3

Roughly speaking, a cPO is recognized by the absence of any reachable utilities at its North-East, as it is a not dominated position; and a cAP is recognized by the location of all reachable utilities at its South-West, as it is a dominating position. These basic elements have to be refined by a focus on the horizontal / vertical line (indifference for j / for i), which involve the distinction between the weak and the strong versions of the Pareto criterion.

Under the *strong* Pareto principle, a cPO is a situation where it is impossible to improve the welfare of one without degrading the welfare of the other, so the set of cPO is (NM) in 1, {P} in 2 and {Q} in 3; and a cAP is *a situation strictly preferred to any other one by one individual and largely preferred to any other one by the other individual*, so the set of cAP is {} in 1, {P} in 2 and {Q} in 3. Under the *weak* Pareto principle, a cPO is a situation where it is impossible to improve the welfare of all, so the set of cPO is (NI) in 1, (PI) in 2 and {Q} in 3; and a cAP is *a situation strictly preferred to any other one by all*, so the set of cAP is {} in 1, {} in 2 and {Q} in 3. Compared to the strong Pareto principle, the weak Pareto principle assumes a more restrictive definition of the domination, which may enlarge the set of not dominated issues (the cPO) and may shrink the set of dominating issues (the cAP).

2.2. Existence and uniqueness of a CAP in a given subset Y.

The direct way to engage the problem of the CAP consists in considering a given subset Y and then looking for a CAP[Y]. So the preliminary question is the determination of Y, and then the two raised questions are: does Y contain a CAP? And if so, is it unique?

The subset Y is determined by a restriction that divides all feasible issues of X into two subsets (forming a partition of X): eligible elements of Y and ineligible elements of Z. The subset Y can be positively defined as the subset of appointed elements, Z being the residual subset of non selected elements (Z = X - Y); or Y can be negatively defined as the subset of non excluded elements (Y = X - Z), Z being the primary subset of barred elements. Static definitions of a subset Y (wondering if it displays a CAP) and also dynamic determinations of a sequence of Y (searching for a Y displaying a CAP) are suggested in the following sections, under specified frameworks. Strictly speaking, the CAP is a concept of optimality; but under a broader view embracing its election or emergence conditions, the CAP also appears as a concept of solution or as an equilibrium, specified by the way Y is socially determined.

Proposition of *existence*: “For any given subset Y, a CAP[Y] may exist or not”. Fig. 4 displays a case of existence of a CAP: $y_1P_iy_2$ and $y_1P_jy_2$ so y_1Py_2 (in the weak and the strong senses of the Pareto principle) and y_1 = CAP[{y_1; y_2}]. Fig. 5 displays a case of inexistence of a CAP: $y_4P_iy_3$ and $y_3P_jy_4$ so y_3 and y_4 can’t be socially ranked (in the weak and the strong senses of the Pareto principle) and there is no CAP[{y_3; y_4}]. Fig. 6 displays a limit case: $y_5I_iy_6$ and $y_5P_jy_6$ so y_5 = CAP[{y_5; y_6}] following the strong Pareto principle, but there is no CAP[{y_5; y_6}] according to the weak Pareto principle.

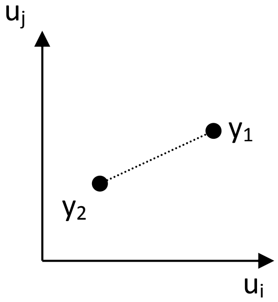

Fig. 4

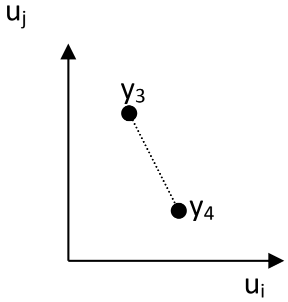

Fig. 5

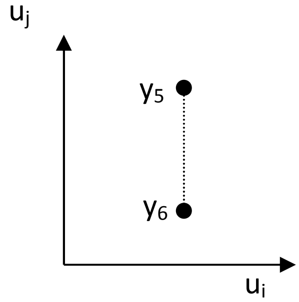

Fig. 6

Proposition of *uniqueness*: “For any given subset Y, if a CAP[Y] exists then it is unique”. This intuitive statement can easily be demonstrated *ab absurdo*. Let’s assume that y_7 and y_8 are two CAP[Y]. As y_7 is CAP, we have y_7Py_8; and as y_8 is CAP, we have y_8Py_7. According to the *weak* Pareto principle, it is impossible to have y_7Py_8 and y_8Py_7 at the same time, as it would mean that each individual strictly prefers y_7 to y_8 and y_8 to y_7 (contradiction). Following the *strong* Pareto principle, y_7Py_8 means “$y_7P_iy_8$ and $y_7I_jy_8$” or “$y_7I_iy_8$ and $y_7P_jy_8$” or “$y_7P_iy_8$ and $y_7P_jy_8$” (situations 1-2-3) and y_8Py_7 means “$y_8P_iy_7$ and $y_8I_jy_7$” or “$y_8I_iy_7$ and $y_8P_jy_7$” or “$y_8P_iy_7$ and $y_8P_jy_7$” (situations 4-5-6). As each situation 1-2-3 is inconsistent with every situation 4-5-6, it is impossible to have y_7Py_8 and y_8Py_7 at the same time (contradiction). QED.

2.3. The implementation of any issue as a CAP and the discovery of the Nash solution as the “top” CAP.

After wondering if a given subset displays a CAP or not, let us now re-engage the problem reversely, considering any given feasible issue and then looking for the subsets such that this issue is their CAP. We will especially focus on the “biggest” of these subsets.

Any subset containing a given issue y_0 (of X) plus some elements of X such that y_0Px displays y_0 as its CAP. Among such subsets, the biggest one gathers y_0 plus all the feasible issues dominated by y_0. Let us denote by Y_0 this biggest subset producing y_0 as its CAP. If y_0 is the CAP of Y_0, then y_0 is also the CAP of every subset of Y_0 containing y_0; but there is no subset of X containing all the elements of Y_0 plus at least one other element (of X - Y_0) such that y_0 would be the CAP of that subset.

To visualize the problem, let us draw X in the (u_i; u_i) plane and let us partition X into four subsets relatively to y_0 (see fig. 7): the set of x such as y_0Px (South West of issues dominated by y_0 ie SW(y_0)), the set of x such as xPy_0 (North East of issues dominating y_0 ie NE(y_0)) and the two sets of x such as there is no Pareto domination between y_0 and x (South

East SE(y_0) and North West NW(y_0)). Obviously, Y_0 = SW(y_0): the subtraction of the three subsets NE(y_0), SE(y_0) and NW(y_0) from the initial set X is the necessary and sufficient condition determining Y_0 as the biggest subset displaying y_0 as its CAP.

Two specific points deserve clarification. First, if y_0 is not the only feasible issue corresponding to the couple of utilities (y_a; y_b), then the other feasible issue(s) corresponding to this pair of satisfactions must be removed from Y_0 to get y_0 as the CAP[Y_0]. Second, the implementation of y_0 as a *weak* CAP requires the removal from Y_0 of all the elements located on the frontiers [$y_{0a}y_0$[and [$y_{0b}y_0$[; but these elements remain in Y_0 for the implementation of y_0 as a *strong* CAP.

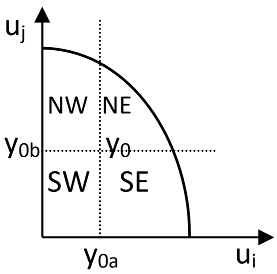

Fig. 7

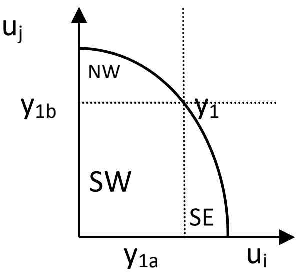

Fig. 8

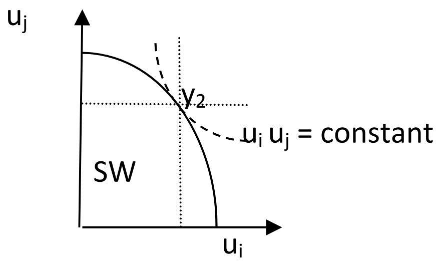

Fig. 9

The determination of the biggest subset displaying any given feasible issue as its CAP enables to associate this issue to this specific subset. Yet, the generation of any possible issue as a CAP may be regarded as a multiplicity issue, since from this angle there are as many CAP as feasible issues. Such a problem could be solved through the selection of the issue y* being the CAP associated to the "largest South West" Y*[12]. This search of the top CAP may be operated in two steps.

Firstly, if a feasible issue is Pareto suboptimal (denoted y_0), then it can't be the "largest South West" generator (denoted y*). Indeed, the Pareto domination of y_0 by some issue y (yPy_0) implies the inclusion of Y_0 in Y, so Y_0 is smaller than Y (which contains all elements of Y_0 plus at least y)[13]. As long as the considered issue is Pareto suboptimal (y_0), it presents a non-empty North East containing issues associated to a larger South West. But if the considered issue is Pareto optimal (denoted y_1), then its North East is empty (see fig. 8)[14] so there is no issue y that would be the CAP of a subset Y including Y_1. If x is not a PO[X], it can't definitely generate the largest South West: this result reduces but does not eliminate the multiplicity issue…

Secondly, among the Pareto optima (y_1), is it possible to select one of them, the *optimum optimorum* (y*), the PO(X) that "could be said" to be associated to the largest South West Y*? If X is a countable set, the *optimum optimorum* could be determined as the Pareto optimum associated to the South West presenting the greater cardinal. If not, the definition of the top Pareto optimum requires further specifications.

[12] A symmetrical and equivalent formulation of the criterion of the maximization of the size of the subset Y would be the minimization of the size of the subset Z = X - Y.

[13] If x is in Y_0 (y_0Px) then it is necessarily also in Y (yPy_0Px); but if x is in Y (yPx) then it is either in Y_0 (when yPy_0Px) or not (when yPxPy_0).

[14] As NE(y_1) = {}, only SE(y_1) and NW(y_1) have to be removed from X to get Y_1 (the biggest subset displaying y_1 as its CAP).

If X is a continuous set, the solution could be given by the maximization of the surface area of the South West of the possible issues. This notion of "area of Y" is meaningful if the ranking of these areas for the different Pareto optima does not depend on the choice of the two utility functions. For ordinal utility functions, defined up to an increasing transformation, this is not true so the "surface of Y" does not mean anything and the quest of the *optimum optimorum* is vain. But for some cardinal utility functions, the search of the top Pareto optimum could make sense: precisely, the ranking of the South West areas is invariant when the utility functions are defined up to a (positive) linear transformation: "$u_i(y_2)\ u_j(y_2) > u_i(y_1)\ u_j(y_1)$" is equivalent to "$[au_i(y_2)]\ [\alpha u_j(y_2)] > [au_i(y_1)]\ [\alpha u_j(y_1)]$" (a and α being positive multiplying factors). Under this assumption, the *optimum optimorum* can indeed be defined as the Pareto optimum maximizing the product $u_i\ u_j$.

One further step may be taken. Let's introduce the disagreement point x_0, $u_i(x_0)$ and $u_j(x_0)$ being the utility levels obtained by i and j in case they do not reach an agreement. In such a bargaining context, the relevant South West of any issue x would actually be not only South West of x but also North East of x_0, an area measured by the product $[u_i(x) - u_i(x_0)]\ [u_j(x) - u_j(x_0)]$. The maximization of this product is "zero" and "one" resistant: any change of origin or of unit would not modify the determination of the optimal situation. In other terms, assuming cardinal utility functions (defined up to a (positive) affine transformation) and a bargaining situation, the South West of any y dominating x_0 is soundly measured by the "Nash product" and the *optimum optimorum* is indeed the "Nash solution" (see fig. 9). Determining the Nash solution as the CAP y* presenting the biggest subset (under proper assumptions), one can interpret it as the solution maximizing the "area of consensus" defined as the field of issues y such as y^*PyPx_0.

2.4. The CAP as related to the AP, to the CPO and to the PO.

We conclude this general first section presenting some statements about the relation of the CAP with the three near concepts of optimality.

About conditional and unconditional agreement points, it is obvious that if there is one AP[X], then it is also the CAP of every subset Y containing this dominating element.

About conditional agreement points and conditional Pareto optima, it can be noticed that if there is a CAP[Y], then this issue is a CPO[Y]; but if there is a CPO[Y], then this issue is not necessarily a CAP[Y]. It can also be stated that if there is at least two CPO in Y, then there is no CAP[Y]; and if there is a unique CPO in Y, then it is the CAP[Y].

About conditional agreement points and Pareto optima, it is reasonable to impose *ex ante* or to verify *ex post* that the CAP[Y] should be or is indeed a PO[X]. This connection is obvious when the logistics of the CAP is summoned to select one PO[X] among a multiplicity; but in any case the concept of CAP is sounder when the dominating element of a distinguished subset is also a non dominated element of the whole set. And there are noteworthy identifications between one CAP[Y] and one PO[X], especially in the case of the bilateral exchange considered in the Edgeworth box…

3. Two fair and efficient solutions to the problem of bilateral exchange as CAP: the Walrasian equilibrium and the Egalitarian exchange point.

Two individuals i and j are initially endowed with a bundle made of bread (good 1) and of wine (good 2): i owns ($\underline{x}_{1i}$; $\underline{x}_{2i}$) and j owns ($\underline{x}_{1j}$; $\underline{x}_{2j}$). So the social quantities of goods are $\underline{x}_{1i} + \underline{x}_{1j} = \underline{x}_1$ in bread and $\underline{x}_{2i} + \underline{x}_{2j} = \underline{x}_2$ in wine. The Edgeworth box[15] is a rectangle displaying horizontally quantities of bread ($\underline{x}_1$) and vertically quantities of wine ($\underline{x}_2$). It can been looked from the synoptic point of view of an omniscient observer, from the south-east point of view of i or from the north-east point of view of j.

Every point of the box corresponds to a possible way to split the social resources, so the box is the set of feasible allocations. Any exchange can be represented by the jump from the point of initial endowments (D) to the point of final allocations (E). In this simple pure exchange economy, the present exchange is separated from the past big bang of production (creation of goods) and the future big crunch of consumption (destruction of goods).

The two individuals have preferences on all conceivable bundles, and especially on the set of feasible allocations: X = {(x_{1k}; x_{2k}) / $0 \leq x_{1k} \leq \underline{x}_1$ and $0 \leq x_{2k} \leq \underline{x}_2$}. These preferences can be represented in the box by the indifference curves of i (from the south-west view) and of j (from the north-east view). The goal of each agent is to improving her/his position, going high-right for i and low-left for j. We assume these preferences are non-satiable (every individual always prefers to have more of each good than less) and convex.

3.1. The problem of bilateral exchange and the core as an unsuccessful solution.

If the isolated bilateral exchange raises a problem, it is as far as there is no unconditional Agreement Point in the set of feasible allocations (X): among all the points of the Edgeworth box, each insatiable agent prefers getting everything (letting the other with nothing). In search of a solution to the problem of "bilateral monopoly", one could try to determine a subset Y that would be relevant (in a sense to be specified) and that would display a CAP. Such a point could be chosen as the result E of the bilateral trade.

The Edgeworthian perspective proposes a way to locate one subset of feasible allocations (Y_{CO}) that should include E. First, the chosen allocation should respect a double condition of individual rationality, which corresponds to the notion of voluntary exchange, operating a Pareto improvement. Second, this selected allocation should respect a condition of interindividual rationality, which corresponds to the notion of efficient exchange, leading to a Pareto optimum. These two conditions determine Y_{CO} as the core of the economy: the subset of allocations such as no coalition (i.e. "i" / "j" / "i and j") can improve upon.

In such a small economy, the core displays more than one single element (even if the Walrasian equilibrium is unique). Moreover, *there is no CAP(Y_{CO})*, as the two individuals disagree on the ranking of the core allocations. Substantially, an unsolved "conflictual coordination" problem is displayed (as in the "battle of the sexes"): there is a common interest to achieve a mutual coordination, but a disagreement on the final settlement. With the Edgeworthian perspective, the determination of the bilateral exchange has advanced but

[15] For a presentation, see for instance Mas-Colell, Whinston and Green [1995], pp. 515 to 525.

has not been completed. And the problem is deeper than a simple indetermination issue, as the absence of agreement on the location of E (in the core) basically means the failure of exchange and leads to a final stay in D. The failure to determine E here reveals the incompleteness of the "one on one" relation between two standard rational agents.

3.2. The Walrasian equilibrium as the CAP of its budget line.

Introducing an impartial institution to manage the prices, the Walrasian *tâtonnement* is able to go further the Edgeworthian recontracting, as it determines a unique solution to the bilateral exchange problem (under general conditions concerning preferences). The application of the equilibrium prices discovered thanks to the *tâtonnement* permits to move from the point of initial endowments (D) to a competitive general equilibrium (E_1), which is shown to be optimal by the first theorem of welfare economics (E_1 is a PO(X))[16].

A complementary interpretation of the Walrasian institutional framework can be put forward, promoting the so called competitive equilibrium as a CAP[17]: E_1 is the CAP(Y_{WE}), Y_{WE} being the subset of allocations located on the equilibrium budget line. Furthermore, this property characterizes E_1, as there is no CAP(Y_{WD}), Y_{WD} being the subset of allocations located on any disequilibrium budget line. So the Walrasian equilibrium is an agreement point under a certain price p_{WE} (or on a certain subset of allocations Y_{WE}) and the *tâtonnement* is the search of such a p_{WE} (or such a Y_{WE}) solving the inter-individual conflict.

To give a graphic explanation of these results, let's consider an Edgeworth box with I and J as the south-west and north-east corners, M and N as the north-west and south-east corners. Let's suppose a given budget line cuts [MJ] (or [MI]) at A and [NI] (or [NJ]) at B, so [AB] is the budget segment. For any given relative price p, D is a point of [AB].

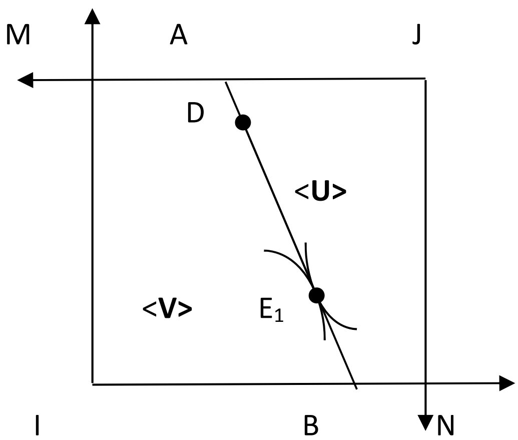

Fig. 10

Let's call E_i and E_j the optimal bundles chosen by i and by j under p. As each agent respects the budget constraint, E_i can't be east of [AB], in the U zone; and E_j can't be west of [AB], in the V zone. If each agent has non-satiable preferences, E_i can't be in the V zone; and E_j can't be in the U zone. As a conclusion, E_i and E_j are both located on [AB]. Precisely, i

[16] As E_1 dominates D, a refined statement can be made: E_1 is an element of Y_{CO}. So the Walrasian logistics may be seen as an organizational way to select one point of the core. See Hildenbrand and Kirman [1988].
[17] The appendix reconsiders the welfare economics theorems when the Walrasian equilibrium is seen as a CAP.

chooses E_i such as E_iP_iC and j chooses E_j such as E_jP_jC, C being any point located on [AB]. If $E_i = E_j = E_1$, then p is the equilibrium price and E_1 is the CAP{[AB]}: i and j agree that E_1 is better than any other point of this budget line. And if $E_i \neq E_j$, then p is not an equilibrium price and there is no CAP{[AB]}: i and j disagree on the highest ranked point of that budget line.

The vision of the Walrasian equilibrium as the CAP of its budget line throws a new light on this basic non-strategic equilibrium concept. It is commonly said that, for a "well behaved" economy, a given structure of perfect *competition* among *independent* agents can lead to an *efficient* situation. Three elements can be added to this usual statement.

First, perfect *competition* relies on a *cooperation* among agents, all agreeing on and each accepting the two Walrasian common rules of the game[18]: "let's take prices are given and let's not trade out of equilibrium". The Walrasian model thus appears as a *coopetitive* construction, with a procedural cooperation followed by a non-strategic competition.

Second, if the *tâtonnement* paves the way for an equilibrium characterized by its *efficiency*, its constituting rules appear to be rules of *justice*. First, individuals are equal in front of prices that are the same for all and manipulated by none, as they are determined by an objective institution. Second, individuals are equal in front of the achievement of their plans, as it is only when all individuals can fulfill their intentions (under a given set of prices) that actual exchange is allowed to take place.

Third, the acceptance of the rules of the game by the non-strategic players is the junction between the two moments of "law making" and "law taking". The latter moment of competition involves people as *independent* agents playing the game (as price takers): they are the *subjects* of a law they obey. The former moment of cooperation involves people as *autonomous* persons choosing the rules of the game (as market power renouncers): they are the *sovereigns* of the law they decree. In this line, the organization of the *tâtonnement* or the establishing of perfect competition is grounded on an prior economic agreement, a former "social contract": as Hobbesian individuals mutually abandon their natural power and place it in the hands of the Leviathan which ensures the political protection of their life, Walrasian individuals mutually abandon their market power and place it in the hands of the Auctioneer which ensures the achievement of their economic exchange.

As noticed by Ingrao and Israel[19], there are two opposite ways to interpret the *tâtonnement*. In the "objective-descriptive" line, it is viewed as a metaphor of the competitive process, the market spontaneously working *as if* there would be an auctioneer. In the "utopian-normative" line, the *tâtonnement* is considered as a real process based on a formal construction to be voluntarily implemented if the community of citizens wants economic exchanges to be free, fair and efficient. As Berthoud [1988] and Rebeyrol [1999], we follow this second line of interpretation that develops the Walrasian settling of the problem of (bilateral) exchange through *procedural justice*: fair trade is established though a set of just rules (beyond the prior question of distributive justice concerning initial endowments).

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[18] Beyond basic respect for the life and for the property of others (neither murder nor theft) and before the fulfillment of the contractual commitments corresponding to the optimal quantity plan for a given price system.
[19] Ingrao and Israel [1990].

3.3. The Egalitarian exchange point as the CAP of the "equality curve".

As an alternative to the Walrasian formal institutionalization of the bilateral exchange (which implements a mediation between the members of the economic society), one could consider an informal "one on one" common sense discussion that would lead the two individuals to the selection of certain conditions they would want their common exchange (from D to E_2) to respect, echoing an axiomatic bargaining[20].

The condition of voluntary exchange ($U_i(E_2) > U_i(D)$ and $U_j(E_2) > U_j(D)$) and the condition of efficient exchange (E_2 is a PO(X)) appear as basic requirements, but we already know that they together determine multiple solutions: all the elements of the core. So the idea would be to add another condition, a reasonable requirement of fair exchange, trying to reduce to one the number of allocations meeting the conditions.

As a condition of fair exchange, let's select a specific equality requirement: the condition of commutative justice expressed as "$\Delta U_i = \Delta U_j$"[21]. As classical Utilitarianism, this criterion is based on the assumption of cardinal utility and, what's more, on the assumption of inter-individual comparisons of utilities. It states a commutative rule which is adequate to the context of bilateral exchange and it is (partly) consistent with the other requirements of voluntary and efficient exchange.

Identified by the equality of the two (positive) utility gains, this requirement of fair (and voluntary) exchange can be represented by an *equality curve* [DD'], D' being the other intersection point between the indifference curves of i and of j going through D. The equality curve is decreasing[22] and the common utility raising ΔU_k (k = i, j) is indeed positive along the portion[DD'], which is included in the lens (DGD'F)[23].

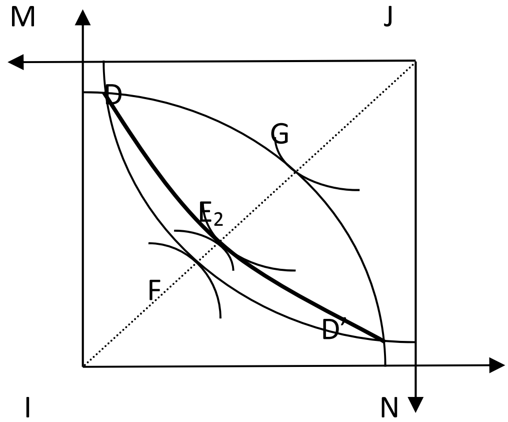

Fig. 11

[20] See for instance Osborne and Rubinstein [1990], page 29: "In the axiomatic approach, the outcome of bargaining is defined by a list of properties that it is required to satisfy".
[21] For a presentation, see for instance Mas-Colell, Whinston and Green [1995], pp. 841-842.
[22] An increasing curve would mean a betterment for one ($\Delta U_k > 0$) but a worsening for the other ($\Delta U_{-k} < 0$).
[23] One could extend the equality curve north-west of D and south-east of D', getting irrelevant portions such as $\Delta U_i = \Delta U_j < 0$: being outside the lens, these locations (for E_2) would contradict the basic principle of voluntary exchange (from D).

The principle of voluntary exchange is respected in the Pareto improving lens (DGD'F), which includes the core FG (the relevant part of the contract curve or *efficiency curve* [IJ]) and the equality curve DD' (the points of same utility raisings from D). The continuity of the efficiency and of the equality curves and the necessity for the equality curve to be above (DFD') and below (DGD')[24] together guarantee an intersection E_2 of the two curves inside the lens, determining a voluntary, efficient and fair exchange from D to E_2.

It is also possible to determine E_2 as the CAP[Y_{EQ}], where Y_{EQ} is the equality curve. First and algebraically, if C denotes any point of the equality curve, the equation of this curve is $U_i(C) - U_i(D) = U_j(C) - U_j(D)$, which is equivalent to $U_i(C) = U_j(C) + [U_i(D) - U_j(D)]$. So the allocation of Y_{EQ} preferred by i and the allocation of Y_{EQ} preferred by j are the same one (Max $U_i(C)$ and Max $U_j(C)$ inside Y_{EQ} lead to the same choice): there is one CAP[Y_{EQ}]. Second and geometrically, the CAP[Y_{EQ}] is on the efficiency curve: if this point C* was not a PO(X), then there would be a non-empty lens of unanimously preferred allocations generated from C*, so there would be in this lens some points C** such that $\Delta U_k(C^{**})$ would be greater than $\Delta U_k(C^*)$, thus ΔU_k would not have been maximized. Conclusion: The equality curve displays one CAP, which is located on the efficiency curve: E_2.

The axiomatic perspective defining E_2, the Egalitarian exchange point, displays competitive features, obviously with the requirements of Pareto improvement and Pareto efficiency. But it also displays cooperative features, clearly with the requirement of justice or equality and also with the general principle of a bilateral discussion leading to the mutual choice of common principles governing the determination of the exchange. But the blending between competition and cooperation is even deeper. In one way, the self interested principles of voluntary and efficient exchange involve the acknowledgement and the perfection of the mutual advantage. In the other way, the introduction of justice and the engagement in a reasonable discussion aim at making a deal advantageous to both: each self interest can only be promoted if a deal is finally closed by a common agreement which requires the respect of the other's self interest. Such a reasonable bargaining is definitely coopetitive (in its own way but echoing the Walrasian coopetition).

The resort to justice is essential to get an agreement in the axiomatic perspective supporting the determination of the Egalitarian exchange point. The justice in question is commutative: it is at work beyond the initial distribution of resources. It is also *substantive* and not *procedural*: if the Walrasian price is fair because it is determined under fair objective rules, the equitable-efficient price is fair because it is determined by one axiom (among others) whose content expresses a certain notion of intersubjective fairness.

The difference between procedural justice and substantive justice is rooted in a distinction between two concrete forms of autonomy. In the Walrasian perspective, the autonomy is basically *political*, as two (or much more) citizens decide on the formal laws and the general institution that will frame the economic exchange, with a disjunction between the moments of bottom up law making and of top down law taking. In the axiomatic perspective, the autonomy is rather *social*, as two actors argue about the features their mutual exchange should display, with a conjunction between the moments of axioms choosing and of axioms applying.

[24] Along (DFD'), $\Delta U_i = 0$ and $\Delta U_j > 0$. And along (DGD'), $\Delta U_i > 0$ and $\Delta U_j = 0$.

There are two different ways to interpret the solutions developed by the theory of axiomatic bargaining (and especially the Nash solution). In the dominant Rubinstein line[25], justice is not acknowledged and agents basically stay standard (expected) utility maximizers: as in non-cooperative game theory, individuals are just independent. In the Moulin line[26], justice is acknowledged and the conception of the individual is dualistic, irreducible to utility maximization: individuals are also autonomous. We here follow this second line of interpretation, considering the substance of equality contained in the presently selected rule of exchange.

4. Conclusion: Acknowledging the notion of autonomy in economics.

While modern notions of society involve both principles of independence and of autonomy[27] (a), the free market tradition of political economy tends to overestimate independence and to underestimate autonomy (b). The concept of CAP may contribute to bringing autonomy back into economic theory (c), adhering to and highlighting a simple idea: any inter-individual operation (the coordinated achievement of independent people) requires some common ground (the social basis shared by autonomous people).

(a) Against binary views of the Modern swing, Renaut [1989] proposes to distinguish humanistic autonomy and individualistic independence as the two capacities of Modernity [28]. Marked by Descartes and his *Cogito*, the first modern move places the human being as the root of his/her laws, which are no longer received from some transcendent entity such as God, Nature or Tradition. Marked by Leibniz and his Monadology, the second modern move fosters the individual being as independent from the others and from the collective body.

If the given order of the ancient society was ensured by some transcendence and framed as some social hierarchy, the open order of the modern society has to come from free and equal human beings. Autonomy and independence may be opposed as alternative principles generating two different visions of the modern society[29]: voluntary order versus emergent order.

On one side, social contract theories developed the notions of autonomy and voluntary order. Human beings and communities are both sovereigns and subjects of the laws they choose and respect. As an intuitive "fact of conscience" to be asserted and

[25] As stated by Osborne and Rubinstein [1990], page 1: "A bargaining theory is (…) not concerned with questions like "what is a just agreement?" (…)". See also Harsanyi [1989], pages 58 and 59.
[26] See Moulin [1981], pages 175 and 176. See also Ponssard [1977], pages 111 to 113. In cooperative scenarios, every player suspends his/her strategic freedom and the power of decision is given up to the community, who determines some fair sharing.
[27] About the notion of autonomy in economics and in political philosophy, see Heap *et alii* [1992], chapter 6.
[28] Renaut disagrees with the monistic views of Modernity due to Dumont, who opposes modern individualistic societies to ancient holistic societies; and due to Heidegger, who opposes modern autonomy to ancient heteronomy. But his vision echoes the dualistic view proposed by Berlin [1958], who opposes two modern forms of freedom: positive liberty (of autonomous people) exemplified by Rousseau or Hegel and negative liberty (of independent individuals) illustrated by Constant or Stuart Mill.
[29] Autonomy and independence may also be associated as cooperative principles, shaping a complex vision of modern society that would display "institutional complementarities" between the State and the Market.

sustained, society rests on the will of citizens and civism expresses the conjunction of the micro-macro levels. Following a virtuous ethics, self-interested behavior is condemned as society relies on public involvement, in line with the general will *à la* Rousseau.

On the other side, political economy developed the notions of independence and spontaneous order. Independent individuals are self-interested and emergent social phenomena come from their actions but not from their intentions[30]. As a paradoxical "fact of nature" to be unveiled and left alone, society rests on itself and social objectivity expresses the disjunction of the micro level (particular wills) and the macro level (general order). Following a consequentialist ethics, self-interested behavior is legitimized by the good[31] social results it produces, in line with the invisible hand *à la* Smith.

(b) Assuming independent rational individual choice and celebrating spontaneous market coordination, political economy has for the most part mistreated the notion of autonomy.

Firstly, the principle of autonomy is excluded when it comes to decision making, as rational choice theory only recognizes individual independence. It is excluded that prevailing exogenous preferences be transcended by ethical principles the person could want to impose to her/him-self, going towards some humanistic opening[32]. Typically, the universalization of instrumental rationality denies axiological rationality and so excludes personal autonomy: the *homo oeconomicus* does not acknowledge any deontological duty.

Secondly, the principle of autonomy is rarely recognized in economics when it comes to the common rules of interaction. Economic models basically focus on individual decisions and their social results under given social rules that are generally treated as exogenous constraints or elements of a preexisting structure. So agents are basically "rule takers" (except maybe in the contemporary theory of bilateral contractual arrangements).

Thirdly, the principle of autonomy may be recognized in economics when it comes to social results. With the Hayekian "spontaneous order" or with the Keynesian "no bridge" macroeconomics, and also with contemporary "agent based models", the pattern of *emergence* states social results as based on social rules and individual actions but irreducible to these beginning elements: macro consequences are autonomous from their micro foundations (the macroeconomic working has its proper laws). A society does not behave the way its individual components do, as shown by the appearance or the disappearance of properties when one aggregates the micro behaviors into the macro result.

(c) The concept of CAP may be useful to reveal the way economics treats or mistreats the notions of autonomy. But more importantly, it should be relevant to explore the ways to articulate independence and autonomy not as substitutable principles, but as complementary abilities. The CAP can operate such a "conceptual cooperation", as it carries

[30] Often quoted by Hayek, Ferguson viewed social phenomena as "the result of human action but not the execution of any human design".

[31] If social order is *produced by* individual impulses, its good orientation has to be *ensured by* some providential guidance which steps in between bottom intentions and top consequences.

[32] The closing of the self concern can be broken by some opening to intersubjectivity: at least the recognition of otherness and at most the rise to universality (as expressed by the Kantian moral philosophy).

a notion of efficiency but balances this pillar of individual independence with another pillar of social autonomy in the way the restriction is determined. This abstract definition of autonomy as a self restriction (from the set X to a certain subset Y displaying one CAP) has been applied to some political and social modes of autonomy; and it could also be applied to some moral mode of it in game theory (especially for the prisonner's dilemma).

Philosophically, the CAP perspective acknowledges that the modern subject attains self-determination only in taking a distance from her/his immediate propensity to follow her/his interest. Formally, the room for maneuver opened by the choice of the restriction enables to select many subsets Y such that a CAP[Y] exists (and is unique); so the problem of inexistence is often avoidable, and if existence is ensured so is uniqueness. Substantially, the room for maneuver about the restriction may also be considered not as an arbitrary trick of the synoptic model designer, but rather as a way to take into account the ability of human beings to build together their interaction and to solve together their coordination problems.

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Appendix: Another look at the welfare economics theorems in the Edgeworth box.

The two fundamental theorems of welfare economics relate the notions of Walrasian equilibrium (E_1) and of Pareto optimum. Simple alternative demonstrations of these equivalence theorems can be presented, in the context of the Edgeworth diagram, when the Walrasian equilibrium is identified as the CAP of its budget line.

Consider this statement of the first theorem: “if E_1 is the CAP[AB], then it is a PO(X)”.

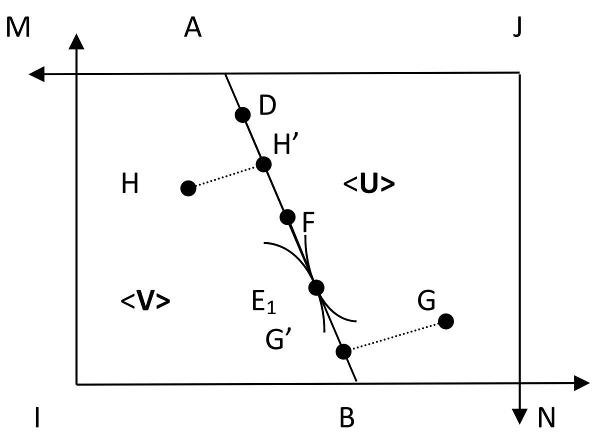

Fig. 12

A competitive general equilibrium E_1 corresponds to a feasible allocation such as E_1P_iC and E_1P_jC, C being any point of the budget line [AB]. Let's consider any point of the box: F on [AB], G in the north-east of [AB] (U zone) and H south-west of [AB] (V zone). Firstly, as both agents prefer E_1 to F, F is dominated by E_1, so it can't dominate E_1. Secondly, let's define H' as a point of [AB] such as $x_{1i}(H')\geq x_{1i}(H)$ and $x_{2i}(H')\geq x_{2i}(H)$. As i's preferences are non-satiable, we have $H'P_iH$. We also have E_1P_iH', as E_1 is the CAP[AB]. By transitivity, we have E_1P_iH; so H can't dominate E_1. Thirdly and symmetrically, defining G' as a point of [AB] such as $x_{1j}(G')\geq x_{1j}(G)$ and $x_{2j}(G')\geq x_{2j}(G)$, we can also easily show that $E_1P_jG'P_jG$, so G can't dominate E_1. It follows that there is no feasible issue dominating E_1, which is therefore Pareto optimal.

The second theorem of welfare economics considers a Pareto optimum (let's say O) and focuses on the possible implementation of it as a Walrasian equilibrium (under the assumption of convex preferences). The Pareto optimality of O allows a partition of the Edgeworth box in three zones, with the indifferences curves of i and j as borders.

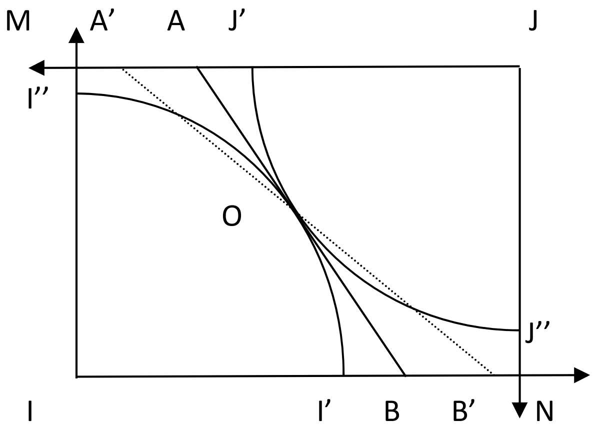

Fig. 13

There is no Pareto domination between O and any allocation located in the areas (OI'II'') and (OJ'JJ''), corresponding respectively to the subsets Y_{SW} and Y_{NE}. But O dominates all the allocations located in the aera (OI''MJ'J''NI'), which constitutes the subset Y': O is the CAP[Y'].

To implement O as a Walrasian equilibrium (identified as the CAP of its budget line), one has to select a continuous linear subset of Y' going through O and attaining the axes. Considering the tangency of (I'OI'') and (J'OJ'') in O, such a line [AB] is unique. Locating D on any other segment [A'B'] going through O may intersect Y' but would also necessarily intersect Y_{SW} and Y_{NE}, precluding to get O as the CAP[A'B'].

As a consequence, to get O as the CAP of its budget line (the Walrasian equilibrium), D has to be placed on the common tangent of the indifference curves (I'OI'') and (J'OJ'').