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Entrepreneurs and wage-earners: a monetary  
approach

Jean Cartelier



UMR 7235

Université de Paris Ouest Nanterre La Défense  
(bâtiment G)  
200, Avenue de la République  
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07  
Email : [nasam.zaroualete@u-paris10.fr](mailto:nasam.zaroualete@u-paris10.fr)

université  
**Paris** | **Ouest**  
  
Nanterre La Défense

# Entrepreneurs and wage-earners: a monetary approach

Jean Cartelier  
University of Paris-Ouest  
EconomiX

## Abstract

The purpose of the paper is to offer a logical genesis of the differentiation of agents in two classes: capitalist entrepreneurs and wage-earners. The model presented here does not follow the Classical (or Marxian) tradition (where the two opposed classes are the straight consequence of the concentration of the means of production in the hands of a limited fraction of people). It does not follow mainstream economists either (no difference according to general competitive equilibrium or a difference taken as given in labour economics in general).

Models belonging to those traditions fail to reproduce a major stylised fact: wage-earners cannot be distinguished from entrepreneurs when they are in the market for commodities but they radically differ in the ‘market for labour’ or in production (wage-earners do not produce for their own account but for that of entrepreneurs who get profits, not wages).

Modern tentatives to deal with the differentiation of agents (Matsuyama for instance) explain it by a progressive differentiation of the level of wealth up to a threshold which makes some agents able to accumulate and others not.

We propose a different view based on the process of issuance of money. If a fraction of agents have not a *direct* access to that process they cannot act in the market for their own account. The alternatives they have are limited to autarky or to work for the account of those who have additional alternatives due to their direct access to money (to be independent producers or entrepreneurs hiring wage-earners).

The model makes explicit the necessary and sufficient conditions for the existence of an E-equilibrium in which co-exist heterogeneous agents (entrepreneurs and wage-earners) starting from a population homogeneous except for bank rationing. These reasonable conditions are: an efficient monetary system, a sufficient gap between productivity of production in mass compared to other types and a possibility to induce wage-earners to work significantly more than they would as free producers.

A non-Marxian notion of exploitation is suggested to conclude.

# 1 Introduction

The trend of growth since the 18th century in Europe and after in the rest of the world is not mainly due to independent producers (not to slaves either) but to waged workers hired by capitalist entrepreneurs<sup>1</sup>. Adam Smith was so conscious of that fact that he conceived the process of growth as one of extension of the proportion of waged labour in the total labour expended (he called *productive labour* the labour exchanged against capital and non productive the labour exchanged against income). Does waged labour imply a qualitative difference between economic agents? Do entrepreneurs and wage-earners belong to different classes? Economic theory provides various answers to these questions. Schematically, stories told by economists about the difference between capitalist entrepreneurs and wage-earners<sup>2</sup> follow one of these two traditions: English Classical tradition or Neoclassical approach.

Economists following the English Classical tradition generally do not consider wage-earners proper but the commodities they consume. Real wages are ingredients of the production on the same footing as raw products and machines. The latter are determined by the technique, the former by physiology, custom or class struggle. Market has no role here; it comes after. Capitalist entrepreneurs are the only ones able to take economic decisions (to consume or to accumulate, to hire or not hire wage-earners). Wage-earners do not exist as economic agents, no more than horses or machines. Entrepreneurs and wage-earners belong to absolutely different classes. Why? Because the former own or have the control over the means of production whereas the latter have not. Equilibrium in Classical models is of von Neuman-Sraffa type.

Classical view has the merit to stress the dependence of wage-earners and their radical difference with entrepreneurs. However, it is not relevant for our ‘open’ societies where castes do not exist. Ownership of the means of production should not be presupposed. Ownership in our market economies is the outcome of economic activity; it has to be validated by the market. The fact that only a fraction of people own the means of production has to be explained and should not be considered as given. Moreover, wage-earners are free to choose their consumption goods in the limit of their incomes. For these reasons, Classical tradition is not well-suited for thinking the heterogeneity between entrepreneurs and wage-earners in our modern societies.

Most of modern academic economists adopt an opposite view. Wage-earners and entrepreneurs are described as bargaining wages and employment on a market for labour. Even if they acknowledge some asymmetries, academic econo-

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<sup>1</sup>I am grateful to Carlo Benetti, Vincent Bignon, Antoine Rebeyrol, Mariana Rojas Breu and participants to the seminars *Philosophie et théorie économique* and *Echanges* for their remarks and critiques. I have a special debt to Régis Breton who has made me understand the lacunae of a first version of this paper. I remain responsible for its main thesis and for the remaining errors.

<sup>2</sup>Here the term *entrepreneurs* indifferently denotes managers and capital owners. It is true that the separation between property and management entails many important consequences in the real life. But, at the high level of abstraction of the present paper it would not be worth considering them with great details.

mists do not think that the asymmetries put into question the basic fact that wage-earners and entrepreneurs have market relations with each other. They are on the same footing and their relations are ruled by equivalence. These asymmetries are not viewed as generating an heterogeneity of the condition of these agents. In general equilibrium theory, capital and labour as ‘factors of production’ differ in the same way as ‘corn’ and ‘silver’ differ as commodities. Agents endowed with capital are not in a different position as agents endowed with labour (entrepreneurs exist there only as a coordination principle). In more specialized theories asymmetries are introduced but they do not generate a qualitative difference between entrepreneurs and wage-earners. Neoclassical approach is relevant in that it clearly shows that wage-earners are not cattle or machines and are free to spend wages as they please but does not account for the hierarchy in production: wage-earners do not decide anything in the sphere of production<sup>3</sup>. Therefore, Neoclassical tradition is not in accordance with a basic feature of our economies.

To sum up, the two traditions do not offer a relevant view about the wage relationship at its present stage. They do not account for the following major stylised fact: *entrepreneurs and wage-earners are in the same qualitative position in the markets for commodities* (both maximize utility under a budgetary constraint) but *they are in a qualitative opposed position in the ‘market for labour’ and in production* (wage-earners do not decide what, how neither how much to produce). The co-existence of two features of the wage relationship, one of equivalence in the market for commodities the other of subordination in production, is responsible for the symmetric failure of both traditions to give an appropriate view of it. A further question, given that stylized fact, is whether wage-earners and entrepreneurs should be treated as homogeneous agents (differing only *quantitatively* in their market allocations) or as heterogeneous agents (differing *qualitatively* in their capacity to intervene in the market). An answer to that question may be found not by presupposing either homogeneity or heterogeneity but by making explicit the process through which an *a priori* homogeneous population became split into two groups or classes: entrepreneurs and wage-earners. The nature of that process will give the key to the question above.

Matsuyama is one of the few authors to have dealt with that proces of differentiation and to have proposed a solution (see [4]). His paper is an outstanding one. At the heart of it is the evolution of the distribution of wealth. Two conditions must be fulfilled in order to become an entrepreneur: (i) entrepreneur position must yield an *income* greater than that of wage-earner (rentability condition) (ii) *wealth* must be such that a certain investment threshold may be reached thanks to some borrowing (indebtedness constraint). At a given instant, an agent’s wealth depends on his/her wealth at the preceding instant

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<sup>3</sup>For instance, Herbert Simon accounts for entrepreneur’s authority over wage-earners by a distinction between a sale and an employment contract. But he does not explain why one of the agents *B* became ‘boss’ and the other *W* the ‘worker’ (see [6]). He does not say whether the difference between *B* and *W* makes them belong or not to a same category of agents either.

(interperiodic transfer of wealth). The wage level is determined by the market for labour equilibrium and depends on the proportion of entrepreneurs (demand) and wage-earners (supply). The main result of the model is that two steady-states exist according to the level of the wage. For values less than a critical threshold, there exist two classes, entrepreneurs being the richer. For the critical value, all agents could be entrepreneurs since they fulfil rentability condition and indebtedness constraint. Not all will be actually entrepreneurs but it does not matter for Matsuyama since all have the same wealth whatever the position they have. Jean-François Jacques et Antoine Rebeyrol also propose a logical genesis of classes starting from a Solow model in which a fraction of people experience an evolution leading to a complete destruction of their capital (see [2]). Like for Matsuyama, a basic point is that *inequality of wealth is the origin of the division in classes*.

The thesis suggested in the present paper is different. That the richer become entrepreneurs and the poorer wage-earners is not the intuition at the root of the model. Even if such an intuition is sensible and in accordance with common sense, it does not help much in understanding the logic of an economy where production is due to wage-earners working for entrepreneurs. Our intuition would be inverse to Matsuyama's: it is because some individuals become entrepreneurs that they become richer and, symmetrically, because others become wage-earners that they are poorer. From a logical point of view (different from an historical one), wealth inequality is a consequence of the differentiation into classes not its cause. In the model below, there would be no sense in maintaining that equality of wealth precludes a division in classes.

Matsuyama's model is suited to the story told by Adam Smith. According to Smith, the difference in the progressive accumulation of wealth (saving is a virtue) generates the division between entrepreneurs and wage-earners. Our model is suited to the story told by Schumpeter, a very different one. According to Schumpeter it is the ability to benefit from money creation through bank credit which is the distinctive feature of entrepreneurs (they can finance innovations thanks to credit): 'l'entrepreneur est dans l'économie nationale le seul débiteur typique' ([5], p. 148). An exclusive access to new means of payment allows a would-be entrepreneur to modify the allocation of the present means of production (and the property rights), to realize an innovation and eventually profits. Entrepreneur's position is provisional and is not guaranteed by any level of wealth. The model presented below, although static, is more Schumpeterian than Smithian in that access to new means of payment is more important than the level of wealth. The purpose is not to analyse the distribution of wealth *per se* but to understand how an economy with entrepreneurs and wage-earners may come out of an economy where everybody has the same position at the start. It is more the heterogeneity of conditions than difference of wealth which matters here.

The institutional setting of our economy is quite simple. Besides individuals having the same characteristics there is a bank issuing on demand non durable means of payment. Individuals resort to money only when they decide to engage into a specialized production. Three techniques of production are available

to individuals corresponding to different social relations: autarky, specialized independent production and production with wage-earners. Here, the choice of a technique is at the same time a choice of a certain type of social relation of production.

The story told by the model goes in three stages. In the first one the bank posts the cost of the issuance of new means of payment and agents decide to go or not to go to the bank. In the second one, the bank decides to satisfy a certain proportion of individuals. Non-rationed individuals have a strategy set larger than that of rationed since they are able to engage into a specialized production. Two modes of existence only are eligible for the latter: autarky or being wage-earners. The former have two additional eligible strategies: becoming independent producers or becoming entrepreneurs (hiring rationed people giving them an indirect access to new means of payment). If rationed individuals choose to produce in autarky non-rationed agents have no choice but to be independent producers. If rationed individuals choose to be wage-earners non-rationed agents may become entrepreneurs if that position is preferred to that of independent producer. What the model is about are the conditions under which exists an equilibrium with entrepreneurs and wage-earners (an E-equilibrium).

These reasonable conditions are: an efficient monetary system, a sufficient gap between productivity of production in mass compared to other types and a possibility to induce wage-earners to work significantly more than they would as free producers. These conditions being fulfilled, a partial rationing by the bank is a necessary and sufficient condition for the existence of a constrained E-equilibrium. *A differentiation in the access to money is ever present in an economy with entrepreneurs and wage-earners; it may be a sufficient condition for the existence of an E-equilibrium depending on other parameters, namely the intensity of effort of wage-earners.*

The model is presented in section 2. Section 3 is devoted to an endogeneization of the bank and to a discussion of the role of wealth in the model. A non-Marxian notion of exploitation is suggested to conclude.

## 2 The model

A distinctive feature of the model is that money is used at equilibrium (except in autarky). Money is *essential* (in the sense of Wallace) when production is specialized. The absence of a double coincidence of wants makes barter impracticable. Transactions cannot be realized but thanks to a generally accepted means of exchange called money. Since money is necessary as an intermediary of exchange (and not as store of value) parsimony in arguments commands that money be non-durable. Consequently, agents cannot engage in a specialized production if they do not get new means of payment from the bank.

This reminds us that in a monetary economy there are not only purchases and sales; there is also a specific operation which cannot be reduced to a purchase or a sale: the *issuance of money*. In all monetary systems, issuance of money is the outcome of a confrontation between demands of individuals and reactions

of a *monetary authority*. In a metallic system, individuals have to bring their precious metals to the *Mint* which accepts (or not) to transform them in legal coins. The Mint decides if the coinage is free and or costly and if melting coins is legal or not. The transformation of gold into coins is not a purchase or a sale. In the same way, in a credit sytem individuals ask to the bank to recognize that they have a certain amount of capital (a discounted sum of future reimbursement payments) by lending them that precise amount of money. Banks accept or refuse according to the characteristics of the individuals but also as a consequence of a monetary policy decided by a monetary authority.

Capital plays here the role gold plays in a metallic system. In both cases, money does not preexist to that operation which, consequently, cannot be interpreted as a purchase or a sale. Money disappears either by melting coins or by paying back the money to the bank.

Issuance of money is crucial in the model since the relative capacity of individuals to get new means of payment has important effects. That capacity may be independent of wealth and, in any case, analytically distinct from it. It will be shown that *deprivation of access to money for a minimal fraction of individuals may be a sufficient condition for the existence of a E-equilibrium; it is in any case a typical feature of such an equilibrium.*

## 2.1 Assumptions and data

Let be an economy populated by a continuum of unit mass of identical individuals characterized by their specialization in production and consumption. Individuals are equally distributed between  $N$  types  $i = 1, \dots, N$  such that  $\ln_i(q_i) > 0$  and  $\ln_i(q_{j \neq i}) = 0$  where  $\ln_i(\cdot)$  is the fonction of utility (chosen for the sake of simplicity). The  $N$  goods are identical except for the ‘colour’. Individuals can produce either their consumption good (their ‘colour’) or, more efficiently, specialize in the production of good  $i + 1$ . In the latter case, to overcome the absence of double coincidence of wants, they resort to money. They demand some money to the bank at a cost  $r$ . Money is non-durable (it must be paid back after the market) and is used for transactions only. The bank may decide to satisfy the demands of a fraction  $1 - s$  only of individuals. A fraction  $s$  is rationed.

### 2.1.1 Techniques of production

Three techniques of production exist but their utilization may be conditional upon an access to money.

All individuals can produce for themselves their consumption good with a technique  $q_i = A(x_i)^\beta$  where  $x_i$  is the effort measured in terms of utility,  $q_i$  the quantity of good,  $\beta$  a parameter  $0 < \beta < 1$  and  $A$  an index of efficiency. Net utility is  $\ln(q_i) - x_i = \ln A + \beta \ln x_i - x_i$ ; it varies with the effort. Maximization of utility in autarky leads to an optimal effort  $x_{aut}^* = \beta$ .

Autarky maximum utility function is thus:

$$U_{aut}^* = \ln A + \beta \ln \beta - \beta \tag{1}$$

All individuals, non-rationed by the bank, may produce the good in which they are more efficient in production (not their consumption good). Money they get from the bank allows them to buy their consumption good and the sale of their production allows them to pay back the bank. A specialized individual of type  $i$  will produce good  $i + 1$  with technique  $q_{i+1} = B(x_i)^\beta$  with  $B > A$ . Specialized production is more efficient than autarky and than any other ( $q_{j \neq i, i+1} = B(x_i)^\beta = 0$ ). Bank issues one unit of money for each individual choosing specialization plus  $r$  unit for itself (in view of purchasing a quantity of good  $i + 1$  as real income). Let consider symmetrical positions only. Only a fraction of production of good  $i$  is consumed by the producer of good  $i + 1$ , the bank getting a fraction  $\frac{r}{1+r}$ . The independent producer gets a fraction  $\frac{1}{1+r}$  of good  $i + 1$ . Sale of that fraction allows him/her to buy a fraction  $\frac{1}{1+r}$  of good  $i$ . The net utility of a specialized producer, in a symmetrical position, is:

$$\ln\left(\frac{Bx^\beta}{1+r}\right) - x = \ln B - \ln(1+r) + \beta \ln x - x$$

Maximization of net utility leads to an optimal effort  $x_{ind}^* = \beta$ .

Maximum utility function for an independent specialized producer is:

$$U_{ind}^* = \ln B - \ln(1+r) + \beta \ln \beta - \beta \quad (2)$$

When a fraction  $s$  of individuals of type  $i$  are rationed by the bank, non rationed individuals of type  $i$  may resort to a technique involving rationed individuals of type  $i$ . In such a case, the former are *entrepreneurs*, the latter *wage-earners*. Assume that entrepreneurs and wage-earners are of the same type in order to keep the idea of specialization. If  $\sigma = \frac{s}{1-s}$  is the dimension of an entrepreneur in terms of employment, the technique is  $q = K(\sigma x)^\beta$ , with  $x \geq \beta$  (the effort of wage-earners is assumed to be greater than that of individuals working for their own account) and  $K \geq B$  où  $K$  is an efficiency parameter. If entrepreneurs demand one unit of money per wage-earner (wage-earners are a continuum of mass  $\sigma$ ), the bank gets a fraction  $\frac{\sigma r}{\sigma(1+r)} = \frac{r}{1+r}$  of the production of each entrepreneur. What remains is shared between wage-earners and entrepreneur according to a proportion  $\frac{w}{(1+r)}$  and  $\frac{(1-w)}{(1+r)}$  respectively. Each wage-earner gets a fraction:  $\frac{w}{\sigma(1+r)}$ . Wage-earners do not decide anything about production. Their effort  $x$  is determined outside the model. The only relevant maximization is that of entrepreneurs. Consequently,  $w$  is determined by equality of marginal productivity of wage-earners  $K\beta x^\beta \sigma^{\beta-1}$  and real wage:

$$K\beta x^\beta \sigma^{\beta-1} = \frac{w}{\sigma(1+r)} Kx^\beta \sigma^\beta \rightarrow w = \beta(1+r) \text{ with } r < \bar{r} = \frac{1-\beta}{\beta} \quad (3)$$

Using the value of  $w$  given by (3), it is possible to calculate the real income of an entrepreneur and of a wage-earner, that is  $(\frac{1}{1+r} - \beta)K(\sigma x)^\beta$  and  $\beta K \sigma^{-(1-\beta)} x^\beta$  respectively.

Maximum utility functions, if transactions are effectively realized, are thus:

- for an entrepreneur:

$$U_{ent}^* = \ln\left(\left(\frac{1}{1+r} - \beta\right)K\sigma^\beta x^\beta\right) = \ln\left(\frac{1}{1+r} - \beta\right) + \ln K + \beta \ln \sigma + \beta \ln x \quad (4)$$

A higher  $\sigma$  and a lower  $r$  make an entrepreneur better off<sup>4</sup>.

- for a wage-earner, taking the effort  $x$  into account:

$$U_w^* = \ln(\alpha K \sigma^{-(1-\beta)} x^\beta) - x = \ln \beta + \ln K - (1-\beta) \ln \sigma + \beta \ln x - x \quad (5)$$

A lower  $\sigma$  makes a wage-earner better off<sup>5</sup>.

### 2.1.2 Transactions

If all individuals choose autarky no transactions, no problem. When specialized production is chosen, either by independent producers or by entrepreneurs hiring wage-earners, money is used as an exclusive intermediary. Purchases and sales are realized simultaneously (introducing costs of transaction other than  $r$  will complicate the story without any advantage). Each producer of type  $i$  (either entrepreneur or independent producer) spends the money he/she has got from the bank (one unit or  $\sigma$  units indirectly *via* wage-earners) toward a producer of type  $i-1$  and pays interest ( $r$  or  $\sigma r$ ) to the bank; this is possible if he/she sells a fraction of his/her production to the bank for  $r$  (or  $\sigma r$ ) units of money and what is remaining to individuals of type  $i+1$ . In the case of entrepreneurs and wage-earners, production is partly sold to wage-earners of type  $i+1$  and partly (say a fraction  $\pi$ ) to entrepreneurs of type  $i+1$ .

The schema below describes the transactions in that case<sup>6</sup>. Dotted lines show how money enters and leaves circulation and full lines show purchases and sales.

As it clearly appears, transactions cannot all be realized but if there is a mutual compatibility between individual actions. Symmetry between different types (but not between individuals of each type) is the most obvious condition.

## 2.2 Individual strategies and equilibria

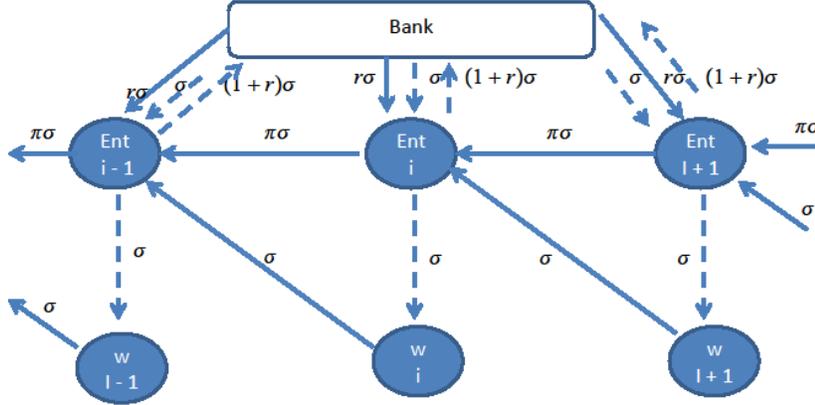
At the end of the game an individual may be in one of these four positions: autarky, independent producer, entrepreneur or wage-earner. Three stages:

<sup>4</sup>Partial derivatives are:  $\frac{\partial}{\partial \sigma}(\ln(\frac{1}{1+r} - \beta) + \ln K + \alpha \ln \sigma + \beta \ln x) = \frac{\beta}{\sigma} > 0$  and  $\frac{\partial}{\partial r}(\ln(\frac{1}{1+r} - \beta) + \ln K + \beta \ln \sigma + \beta \ln x) = -\frac{1}{(1+r)(1-(\beta+r))} \leq 0$  si  $\beta \leq \frac{1}{1+r}$  which is the case for  $0 < w < 1$

<sup>5</sup>Partial derivative is negative:  $\frac{\partial}{\partial \sigma}(\ln \beta + \ln K - (1-\beta) \ln \sigma + \beta \ln x - x) = \frac{1}{\sigma}(\beta - 1) < 0$

<sup>6</sup>Through expenditures  $\sigma$  to entrepreneur  $i-1$  wage-earners of type  $i$  get a quantity of good  $i-1$   $\frac{w}{1+r} K(\sigma x)^\beta = \beta K(\sigma x)^\beta$ , which implies a price  $p = \frac{\sigma^{1-\beta}}{\beta K x^\beta}$ . Through expenditures to entrepreneur  $i-1$ , entrepreneur  $i$  gets a quantity of good  $i-1$   $\frac{1-w}{1+r} K(\sigma x)^\beta = \frac{1-\beta(1+r)}{\beta(1+r)} K(\sigma x)^\beta$ . That quantity times price gives:  $\sigma \frac{1-\beta(1+r)}{\beta(1+r)} = \sigma \pi \rightarrow \pi = \frac{1-\beta(1+r)}{\beta(1+r)}$ .

# Money flows



1. the bank posts  $r$ , cost if issuance of one unit of money
2. money demanders learn if they are rationed
3. non-rationed individuals chose, given others' choices, their preferred position; rationed individuals chose either autarky or to be wage-earners.

At stage 1, two strategies only are possible: to go or not to go to the bank.

## 2.2.1 To go or not to go to the bank

This first choice does not make sense unless independent producer or entrepreneur positions are preferred to autarky and wage-earner positions. To keep the story simple and without loss of generality, assume that to go to the bank does not engage individuals. In the case they would not or they could not use money they may pay it back to the bank without incurring cost  $r$ . As a consequence, 'to go to the bank' is a weakly dominant strategy and is always chosen by individuals.

## 2.2.2 To be or not to be rationed by the bank

Stage 2 opens by bank decision about rationing. Individuals learn whether they are or not rationed. In other terms they get (with a probability  $\frac{1}{1+\sigma} = 1 - s$ ) or they do not get the money they have demanded (with a probability  $\frac{\sigma}{1+\sigma} = s$ ).

Bank decision (considered provisionally as exogenous) splits individuals into two categories. Non-rationed individuals have a strategy set larger than rationed ones. The former can choose between four strategies: autarky, to be wage-earner (labour suppliers), to be independent producer or to be entrepreneur (labour demanders). The latter can choose only between autarky and to be wage-earner.

Autarky yield a net utility independent of the choice of other individuals. Autarky here is an outside option. Other positions yield a net utility which highly depends on the choice of other individuals. We are looking for equilibria positions only, that is situations in which any individual cannot improve each utility if no one changes his/her strategy. In such situations nobody has an interest to deviate.

### 2.2.3 Equilibria

As the transaction schema above makes it clear, equilibria requires that individuals of different types behave symmetrically. The production of a specialized producer (either independent or entrepreneur) of type  $i$  cannot be sold but if *a same fraction* of individuals of all types has chosen to be a specialized producer.

Consider first the choice : ‘independent producer’. There is a continuum of situations such that a fraction  $\alpha$  ( $0 \leq \alpha = \alpha_i \forall i \leq 1$ ) of individuals of all types are specialized, a fraction  $1 - \alpha$  being in autarky. These situations are all equilibrium situations. Let call them  $\alpha$ -*equilibria*. When  $\alpha = 0$ , it is a *A-equilibrium* (A for autarky): nobody chooses to be specialized (a variant is: the bank rations everybody:  $\sigma \rightarrow \infty$ ). All individuals are in autarky and nobody has an interest to deviate, *i. e.* to specialize, since he could not sell his/her production. When  $\alpha = 1$ , it is a *I-equilibrium* (I for independent producer): nobody is rationed by the bank ( $\sigma = 0$ ) and nobody prefers to be entrepreneur ( $U_{ind} > U_{ent}, U_{ind} > U_w, U_{ind} > U_{aut}$ ); all individuals are independent producers and the economy is in its true sense a ‘market economy’. For positive and finite values of  $\sigma$ , a fraction  $\frac{1}{1+\sigma} = \alpha$  of individuals of each type is specialized and a fraction  $\frac{\sigma}{1+\sigma}$  prefer to produce in autarky. In all these situations nobody has an interest to deviate: to leave autarky or specialization generates a loss in utility: in the first case the good is not sold and an opportunity to produce his/her own consumption good is lost, in the second the advantage of specialization is lost.

But conditions  $U_{ind} > U_{ent}, U_{ind} > U_w, U_{ind} > U_{aut}$  may not be all fulfilled. Equilibria other than  $\alpha$ -equilibria may exist. If  $U_{ind} < U_{ent}$  non-rationed individuals prefer to be entrepreneurs rather than independent producers. If, at the same time, rationed individuals prefer to be wage-earners rather than autarky producers ( $U_w > U_{aut}$ ) and, finally, if entrepreneurs are better off than wage-earners ( $U_{ent} \geq U_w$ ) there is a room for *E-equilibria*, (E for entrepreneurs) that is for situations where entrepreneurs employ wage-earners to produce for their own account.

By contrast with A- and I-equilibria, an economy in E-equilibrium is made of two different classes of individuals. Entrepreneurs and wage-earners belong to two heterogeneous groups of people; there is a hierarchy between these groups,

the strategy set of the latter being more restricted and included in the strategy set of the former. Wage-earners do not take the lead in the market since they are deprived from what is the condition for any initiative in production, money. They have no choice but to work for others in order to get some money. Not working for their own accounts they cannot even choose freely their effort.

Let see the conditions for the existence of a E-equilibrium.

### 2.3 The existence of a E-equilibrium

Three conditions must be simultaneously fulfilled for an E-equilibrium to exist:

- $U_w > U_{aut}$  makes sure that rationed individuals participate in a entrepreneurial economy; using (1) and (5), it may be written as:

$$\ln \sigma \leq \ln \bar{\sigma} \equiv \ln\left(\left(\frac{\beta K}{A}\right)^{\frac{1}{1-\beta}}\right) + \frac{\beta}{1-\beta} \ln \frac{x}{\beta} + \frac{\beta-x}{1-\beta} \quad (6)$$

- $U_{ent} \geq U_{ind}$ , makes sure that non-rationed individuals do not become independent producers; using (4) and (2), it may be written as:

$$\ln \sigma \geq \ln \underline{\sigma} \equiv \frac{1}{\beta} \ln\left(\frac{B}{K(1-\beta(1+r))}\right) + \ln \frac{\beta}{x} - 1 \quad (7)$$

- $U_{ent} \geq U_w$  makes sure that non-rationed individuals not wishing to be independent producers do not prefer to become wage-earners; using (4) and (5), it may be written as:

$$\ln \sigma \geq \ln \sigma^* \equiv \ln\left(\frac{\beta(1+r)}{1-\beta(1+r)}\right) - x \quad (8)$$

Conditions (6) and (7) cannot be simultaneously fulfilled unless  $\bar{\sigma} > \underline{\sigma}$ , which requires in turn that:

$$\ln \beta + \frac{1}{\beta} \ln \frac{K}{B} + \ln \frac{B}{A} + \frac{1}{\beta} \ln(1-\beta(1+r)) + 1 - x + \beta \ln \frac{x}{\beta} - \frac{\beta}{x} \equiv S > 0 \quad (9)$$

and that  $\underline{\sigma} \leq \sigma \leq \bar{\sigma}$ .

Recall that  $r$  is bounded above ( $r < \bar{r} = \frac{1-\beta}{\beta}$ ) and that  $K$  is not. For  $r < \bar{r}$ ,  $A, B, x$  and  $\beta$  given, it always exists values for  $K$  such that (9) is valid<sup>7</sup>. If bank rationing is sufficiently severe, namely if  $\underline{\sigma} \leq \sigma \leq \bar{\sigma}$ , a E-equilibrium exists. That a high  $K$  is a condition for the existence of a E-equilibrium is not

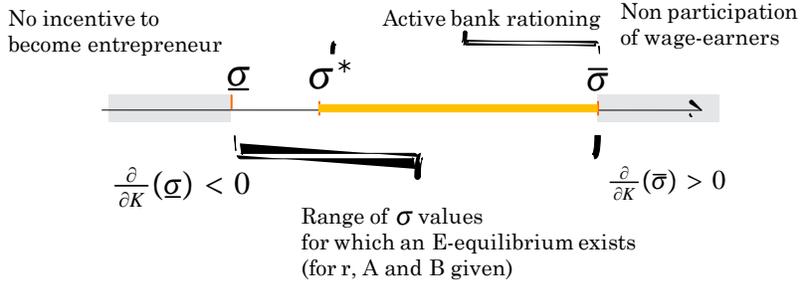
<sup>7</sup>Partial derivative  $\frac{\partial}{\partial K}(S)$  est positive ( $\frac{\partial}{\partial K}(S) = \frac{1}{K\beta} > 0$ ) and  $\frac{\partial}{\partial r}(S) < 0$ , is bounded since  $r \leq \bar{r}$ .

a surprise but that a sufficient deprivation of means of payment for a fraction of individuals may be also a condition is less trivial and, in any case, rarely mentioned.

Schema below shows how, for given values of  $r$ , an higher value of  $K$  increases the set of values of  $\sigma$  for which an entrepreneurial economy is an equilibrium.

But this is not the end of the story. Condition (8) must be considered as well. It says that entrepreneur's utility has to be at least equal to that of wage-earners. It depends only on the effort of wage-earners and on the real wage (recall that  $w = \beta(1 + r)$ ). Condition (8) has a very important meaning. It leads to define, if  $\underline{\sigma} \leq \sigma \leq \bar{\sigma}$  holds, two kinds of E-equilibrium:

1. A non-constrained E-equilibrium if  $\underline{\sigma} \leq \sigma < \sigma^* \leq \bar{\sigma}$ . Such a E-equilibrium exists with a voluntary rationing. Bank rationing is not binding since some individuals ( $\sigma^* - \sigma$ ) voluntarily decide not to use the means of payment they get from the bank<sup>8</sup>. Non-rationed individuals are indifferent between to be an entrepreneur or to be a wage-earner. Nobody has an interest to deviate: becoming a wage-earner, an entrepreneur would loose in utility ( $\sigma$  increases) and, on the other hand, a (non-rationed) wage-earner could not become an entrepreneur since he would not find wage-earners to employ. In that case, bank rationing is not responsible for the existence of an economy with heterogeneous groups of individuals, entrepreneurs and wage-earners.
2. A constrained E-equilibrium if  $\underline{\sigma} \leq \sigma^* < \sigma \leq \bar{\sigma}$ . Bank rationing is more severe than that individuals would voluntarily decide ( $\sigma > \sigma^*$ ). In that case, entrepreneurs have no interest to deviate, that is to become wage-earners ( $U_{ent} > U_w$ ) and wage-earners have no possibility to change their position since they are constrained by the bank (see schema).



A crucial point for the main thesis of the present paper is to establish the condition under which, when (9) holds, only constrained E-equilibria exist and non-constrained E-equilibria do not. The condition is obviously  $\sigma^* < \underline{\sigma} < \sigma$ . If that condition is satisfied, entrepreneurs and wage-earners have the same utility

<sup>8</sup>If 'to go to the bank' would have not been a weak dominant strategy, they would have chosen 'not to go to the bank'.

but the situation is not an equilibrium: non-rationed individuals have interest to deviate and to become independent producers and, consequently, rationed individuals have no choice but to be in autarky. There does not exist non constrained E-equilibria. If  $\sigma^* < \underline{\sigma} < \sigma$  holds, only constrained E-equilibria exist.

The effort  $x$  wage-earners must supply (not chosen by them) is decisive for condition  $\sigma^* < \underline{\sigma}$  to hold. Substituting in condition  $\sigma^* < \underline{\sigma}$  the values of  $\sigma^*$  and  $\underline{\sigma}$  derived from (8) and (7) respectively and defining  $x = \lambda\beta$  with  $\lambda \geq 1$ , allow to get :

$$T \equiv \beta\lambda - \ln \lambda - 1 - \frac{1}{\beta} \ln \left( \frac{K}{B} \right) - \ln \beta(1+r) - \frac{1-\beta}{\beta} \ln(1-\beta(1+r)) > 0 \quad (10)$$

Here, a high relative efficiency of entrepreneurial production  $K$  has a negative influence. A high  $K$  makes less likely condition (10) hold. For  $\frac{K}{B}$ ,  $\beta$  and  $r$  given, it is the relative intensity of effort of the wage-earners  $\lambda$  which matters. Partial derivative of  $T$  is  $\frac{\partial}{\partial \lambda}(T) = \beta - \frac{1}{\lambda}$  with  $\beta < 1$  and  $\lambda \geq 1$ . For low values of  $\lambda$  ( $\lambda < \frac{1}{\beta}$ ),  $\frac{\partial}{\partial \lambda}(T) < 0$ ;  $\frac{\partial}{\partial \lambda}(T) = 0$  for  $\lambda = \frac{1}{\beta}$  and is positive for  $\lambda > \frac{1}{\beta}$ , which is the case considered here. Clearly an increase of  $\lambda$  is favourable to the existence of a constrained E-equilibrium. For given values of other parameters (techniques, preferences, etc.) and if bank rationing respects condition (9)<sup>9</sup>, a sufficiently high intensity of effort of wage-earners (compared to that of an independent producer) ensures the existence of a constrained E-equilibrium.

To sum up, necessary and sufficient conditions for the existence of a constrained E-equilibrium are<sup>10</sup>:

$$\left\{ \begin{array}{l} r < \bar{r} = \frac{1-\beta}{\beta} \\ \underline{\sigma} \leq \sigma \leq \bar{\sigma} \\ T \equiv \beta\lambda - \ln \lambda - 1 - \frac{1}{\beta} \ln \left( \frac{K}{B} \right) - \ln \beta(1+r) - \frac{1-\beta}{\beta} \ln(1-\beta(1+r)) > 0 \end{array} \right\} \quad (11)$$

These formal conditions may be translated in plain language: (i) a minimal efficiency of the process of issuance of money implying a cost not too high, (ii) a relatively high productivity of wage-earners compared to that of independent producer is responsible for a large range of possible values of  $\sigma$ , (iii) a rigorous discipline imposed to wage-earners (a high  $\lambda$ ) makes sure that entrepreneurs have a better position than wage-earners (moreover a low efficiency of production in autarky deprives rationed individuals from the outside option of autarky).

<sup>9</sup> Partial derivative:  $\frac{\partial}{\partial r}(T)$  is positive ( $\frac{\partial}{\partial r}(T) = \frac{r}{(1-\beta(1+r))(1+r)} \geq 0$ ).

<sup>10</sup> A numerical example may illustrate what is said in the text. Take  $A = 3.4161$  ( $\ln A = 1.2285$ ),  $B = 3.7577$  ( $\ln B = 1.3238$ ) and  $K = 6.6859$  ( $\ln K = 1.9$ ); assume  $\beta = 0.8$ ; it follows that  $\bar{r} = 0.25$ ; let  $r = 0.05 < \bar{r}$ , the value of  $x$  such that it is indifferent to be entrepreneur or wage-earner is  $x^* = 2.0081$ . Let  $x = 2.1$ . It is easy to derive:  $w = 0.8(1.05) = 0.84$ ,  $\underline{\sigma} = 1.1794$  and  $\bar{\sigma} = 6.447$ . A constrained E-equilibrium exists for all values of  $\sigma$  such that  $1.1794 \leq \sigma \leq 6.447$ .

### 3 Beyond the model

Beyond the conditions for the existence of a constrained E-equilibrium, two questions must be evoked: (i) should bank's behaviour be introduced into the model? (ii) Is a comparison between entrepreneur's and wage-earner's utility legitimate?

#### 3.1 The bank and the closure of the model

So far the bank has been treated as exogenous and its decisions have been considered like those of 'nature'.

It is certainly possible to be content with that. The bank would symbolize a more or less favourable environment for an entrepreneurial economy to emerge. A severe rationing would mean a great scarcity of means of payment as so many observed in the past. A high cost  $r$  could be interpreted as signalling a primitive and poorly efficient monetary system.

But, economists do not like exogenous variables. They prefer a closed model to an open one. Moreover, they need to check the consistency of the model. The question is: would really a profit maximizing bank ration money such that a constrained E-equilibrium exists? Is there not a contradiction between bank's own interest and the values of  $r$  and  $\sigma$  associate to a constrained E-equilibrium?

The point is worth examining since the bank does not get the same amount of goods (or the same utility, which is equivalent if the bank is assumed to be indifferent to the "colour" of goods and its utility function is increasing in goods) in an entrepreneurial economy and in a pure market economy. Assume that the bank desires to get the greater quantity of goods: what are the terms of arbitrage? Let compare the quantity of goods the bank gets in a I-equilibrium,  $\frac{r}{1+r}B\beta^\beta$ , and in a E-equilibrium  $\frac{1}{1+\sigma}\frac{r}{1+r}K\sigma^\beta x^\beta$ . Note that the rate of interest (if  $< \bar{r}$ ) does not play any role.

If  $x = \beta$ , the bank will favour an entrepreneurial economy if:

$$\frac{r}{1+r}B\beta^\beta \leq \frac{1}{1+\sigma}\frac{r}{1+r}K\sigma^\beta\beta^\beta \rightarrow \frac{K}{B} \geq \frac{1+\sigma}{\sigma^\beta}$$

In the general case,  $x = \lambda\beta$  avec  $\lambda > 1$ , the condition is weaker:

$$\frac{K}{B} \geq \frac{1+\sigma}{\lambda^\beta\sigma^\beta} \tag{12}$$

If  $\frac{K}{B} < \frac{1+\sigma}{\lambda^\beta\sigma^\beta}$ , the bank wishes a market economy of independent producers; it posts a rate of interest  $r = \frac{B}{A} - 1 \leq \bar{r}$  and  $\sigma = 0$ . As for the rate of interest bank's advantage is at its best for the higher level of  $r$  compatible with a positive demand for new means of payment ( $\frac{r}{1+r}$  increases with  $r$ ). The condition  $\frac{K}{B} \geq \frac{1+\sigma}{\lambda^\beta\sigma^\beta}$  holds for a certain range of  $\sigma$ . Whether this range partially overlaps the range for which exists a constrained E-equilibrium depends on the value of the parameters of efficiency of the two techniques of production<sup>11</sup>.

<sup>11</sup>In our numerical example, condition is  $1 \leq \sigma \leq 841.64$ . This interval encompasses

A step further in the endogeneization of the bank consists in looking for its optimal behaviour (which is tantamount to assuming that the bank follows the same rationality as individuals, forgetting that it acts as a monetary authority). Assume the bank maximizes the quantity of goods it receives in an entrepreneurial economy, that is  $\frac{1}{1+\sigma} \frac{r}{1+r} K \sigma^\beta x^\beta$ . Bank's behaviour is given by a FOC

$$\frac{\partial}{\partial \sigma} \left( \frac{1}{1+\sigma} \frac{r}{1+r} K \sigma^\beta x^\beta \right) = \frac{K r x^\beta}{\sigma^{\beta-1} (1+\sigma)^2 (1+r)} (\beta(1+\sigma) - \sigma) = 0 \rightarrow \sigma_{opt} = \frac{\beta}{1-\beta}$$

For all  $\sigma > 0$  and  $r < \bar{r}$ , the optimal bank rationing is  $\sigma_{opt} = \frac{\beta}{1-\beta}$ <sup>12</sup>.

### 3.2 Is a comparison between entrepreneurs and wage-earner's utility legitimate?

We have taken for granted that condition (8) plays a central role in the existence of an entrepreneurial economy. That condition,  $U_{ent} \geq U_w$ , states that at equilibrium entrepreneur's utility cannot be less than wage-earner's utility. That point deserves a little bit of a discussion.

In the model, heterogeneity between individuals concerns the strategy set: non-rationed individuals may choose strategies non-eligible to rationed individuals. The latter have no choice but submit themselves to non-rationed individuals or quit. How is it possible to express that heterogeneity using standard analytical tools?

We are put into a dilemma: to accept a comparison in terms of utility, as it is done in the model, is tantamount to acknowledging an homogeneity between entrepreneur and wage-earner, which is precisely contrary to the model's intuition; to refuse a comparison in terms of utility is equivalent to presuppose that heterogeneity and to renounce to endogeneize it, which is precisely what the model aims at.

Two false solutions must be discarded. The first would consist in giving a positive utility to the dimension of the strategy set. As a result entrepreneur's utility would be arbitrarily increased in order to legitimate a comparison in terms of utility. The second false solution would consist in considering that entrepreneur's position has an absolute advantage on wage-earner's position (lexicographic utility). This would be equivalent to presuppose an heterogeneity instead of making it emerge from the model.

There is no solution to our problem. But it is however useful to stress that the comparison between utilities is not symmetrical. It does not make sense for rationed individuals becoming wage-earners since they cannot become entrepreneurs. If an entrepreneur can switch for wage-earner's position, the inverse switch is not available to the wage-earner. Deprived from a direct access to the bank, a wage-earner must accept an inequality between both situations utility. *Inequality of utilities (or of wealth) is a consequence of a differentiation between entrepreneurs and wage-earners but not the cause.* What matters is less

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the range of  $\sigma$  associate to a constrained E-equilibrium. Here, bank's interest is perfectly compatible with the existence of an entrepreneurial economy.

<sup>12</sup>In our numerical example, a constrained E-equilibrium exists (and is unique) since  $\sigma_{opt} = \frac{\beta}{1-\beta} = 4$  belongs to the 'good' range of values of  $\sigma$ .

utility than status. This appears also in the fact that wage-earners do not determine their effort and that the wage level is given by the profit maximization of entrepreneur's profit.

## 4 Conclusion

Some brief remarks to conclude.

1. An economy of independent producers (I-equilibrium) and an entrepreneurial economy (E-equilibrium) are both monetary economies in which relations between individuals are expressed by payments. That fact gives a misleading homogeneity between the two economies or between entrepreneurs and wage-earners. Relations between independent producers or between entrepreneurs are *market relations* ruled by the *principle of equivalence*. Relations between entrepreneurs and wage-earners are ruled by another logic, that of *monetary submission*. Equivalence does not play any role since wage-earners do not sell anything (they are wage-earners precisely because they cannot produce for the market). Entrepreneurs not wage-earners are responsible for the production (recall that wage-earners do not choose their effort). Consequently the sanction for a failure is not the same: an entrepreneur may go bankrupt, a wage-earner loses his/her employment.
2. Condition (8) compares entrepreneur and wage-earner's utility but one may think that the comparison is biased since there is no effort from the entrepreneur. We have compared  $\ln w \frac{K(\sigma x)^\beta}{1+r} - x$  for the wage-earner and  $\ln(1-w) \frac{K(\sigma x)^\beta}{1+r}$  for the entrepreneur. An objection may be that an entrepreneur also supplies an effort. Taking this into account would have the merit not to attribute from the start a rent to the entrepreneur. Consider then the entrepreneur's effort and assume that its level is  $\beta$  (the same effort as an independent producer). A comparison between entrepreneurs and wage-earners on this new basis makes appear what may be called an *exploitation* of the latter by the former, in a sense different from Marx.

Let compare  $\ln w \frac{K(\sigma x)^\beta}{1+r} - \lambda\beta$  for wage-earners, on the one hand, and  $\ln(1-w) \frac{K(\sigma x)^\beta}{1+r} - \beta$ , for entrepreneurs, on the other. For  $w = \frac{1}{2}$ , entrepreneurs' position differ from wage-earners' position by  $x - \beta = \lambda - 1$ . An equal sharing of production does not generate an equality in position as it should be if wage-earners and entrepreneurs had an homogeneous status. The gap between the two positions in case of equal sharing of production reveals an exploitation due to the difference between the amount (and the nature) of the effort supplied. Here we do not compare paid and unpaid labour as Marx did but a free activity with an effort realized as a consequence of a monetary submission. *That exploitation does not depends on the*

*wage but on the status.* If a relative high efficiency of the entrepreneurial production is a condition for the existence of a constrained E-equilibrium, another important condition is a high degree of exploitation.

3. At the outset, individuals are all alike and are free to choose their position. The difference in their position vis-à-vis of the bank makes them differ in their strategy set. Once they become either entrepreneurs or wage-earners they are really different. Entrepreneurs have all the prerogatives traditionally acknowledged in economic theory: they can maximize their advantage (utility or profit) taking into account the two legs of a budgetary constraint (resources and expenses). Wage-earners do not work for themselves. They maximize their advantage (utility) but under a different constraint: they control only one leg of the constraint not the effort  $x$  and not the resource ( $w$  is the outcome of a profit maximization by entrepreneurs). It is precisely the argument Keynes invokes in *General Theory* to reject what he called ‘the second classical postulate’(see [3], chapter 2). As wage-earners budgetary constraint is not of the same type as that of entrepreneurs, Walras Law (*i. e.* the sum of budgetary constraints over all individuals) applies in Keynes economics only in a restricted way. ‘Restricted Walras Law’ does not encompass the ‘market for labour’ (see [1]). It is this very fact, and this very fact only, which gives sense to Keynes’ main proposition, namely the existence of a general competitive equilibrium with involuntary unemployment.

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