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On the optimal control of pollution in a human capital growth model*

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Abstract

On the one hand, the adoption of polluting technologies can enhance the factor productivity; on the other hand, pollution lowers the stock of human capital by weakening physical and mental performances, and shortening the life expectancy at the end. To capture the impact of pollution on economic growth, we compute the optimal policy in an endogenous growth model à la Lucas (1988) and we study the effects of pollution in the short and the long run.

Keywords: pollution, human capital, endogenous growth.

JEL classification: D90, J24, 044.

1 Introduction

The seminal notion of human capital dates back to Smith and Pigou. The current meaning has been specified and popularized by Becker in his influential work *Human Capital* published in 1964. Today, the term of human capital refers to the level of education and the state of health of a given individual. Expenditures in education and intellectual training on the one hand, and, on the other hand, medical cares and physical training improve the productivity of workers and represent investments in human capital because during the life span the higher productivity results in higher wages.

Human capital accumulation is pointed out as a mechanism of perpetual growth by Lucas (1988). During the Nineties, the endogenous growth literature flourishes. Meanwhile, this optimistic view is challenged by other authors concerned by the effects of pollution on economic growth. Two decades after

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the seminal papers by Keeler, Spence and Zeckhauser (1971) and Forster (1973) on sustainable development, pollution is introduced in a model of (exogenous) growth à la Ramsey by Van der Ploeg and Withagen (1991).

Most of the papers before Van der Ploeg and Withagen (1991) addressed the issue of sustainable growth in terms of depletion of non-renewable sources (influential references are Dasgupta and Heal (1974), Stiglitz (1974) and Solow (1974)). Pollution is eventually taken into account in models of endogenous growth when the new growth theories are successful, included theories of human capital accumulation. Pollution affects human capital and hence growth through essentially three effects. First, it lowers the life expectancy and then the number of periods over which the discounting is computed (Pautrel (2009), Mariani, Pérez-Barahona and Raffin (2010)). Second and third, it reduces the physical and mental performances within a period, respectively (Gradus and Smulders (1993), Van Ewijk and Van Wijnbergen (1995)).

A common assumption in the literature on pollution and human capital accumulation is that pollution (either as a stock or a flow) is an unavoidable by-product of any consumption or production activity. In our paper, we assume that production pollutes and pollution slows human capital accumulation, but, in the spirit of Brock (1977) and Stockey (1998), the adoption of polluting technologies enhances factor productivity. Thus, pollution is considered as a production factor and becomes a control variable in the planner's program just as the worked hours. Our model preserves the simplicity of Lucas (1988) and encompasses this model as particular case.

Focusing on the case where pollution matters, we find the behavior of the economy in the short run though a stability analysis and in the long run through the comparative statics. In particular, we highlight a positive relation between worked hours and pollution level.

2 Polluting technology

We introduce a pollution mechanism *à la* Stockey (1998) in a model *à la* Lucas (1988), an optimal growth model with human capital and no physical capital. On the one hand, a polluting technology enhances labor productivity, on the other hand pollution slacks human capital accumulation. Thus, a trade-off between these opposite effects takes place.

We denote the individual labor supply by l_t and normalize the size of population to one. Then, l_t is also the aggregate labor supply. Labor services l_t enters the production function jointly with another input: a technology index a_t. Increasing this index means an improvement of labor productivity but also the adoption of a more polluting technology.

Assumption 1 *Technology*[1] *is represented by a production function* $y_t =$

[1] In Stockey (1998), the technology index a_t is bounded from above. This bound ensures the existence of a competitive equilibrium. Without upper bound, firms, bearing no pollution costs, would choose an infinite index. We focus on the social optimum and we assume that the exogenous upper bound is larger than the optimal value of a_t.

$f(l_t, a_t)$, *with* $\partial f/\partial l_t > 0$ *and* $\partial f/\partial a_t > 0$.

Pollution depends on the type of technology (more or less polluting), but also on the amount of production: $p_t = p(a_t, y_t)$, with $\partial p/\partial a_t, \partial p/\partial y_t > 0$. Notice that here the pollution is not a stock, but a flow.

We find

$$p_t = p(a_t, f(a_t, l_t)) \equiv q(a_t, l_t) \tag{1}$$

that is $a_t = a(p_t, l_t)$ with

$$\left(\frac{\partial a}{\partial p_t}, \frac{\partial a}{\partial l_t}\right) = \left(\frac{1}{\frac{\partial p}{\partial a_t} + \frac{\partial p}{\partial y_t}\frac{\partial f}{\partial a_t}}, -\frac{\frac{\partial p}{\partial y_t}\frac{\partial f}{\partial l_t}}{\frac{\partial p}{\partial a_t} + \frac{\partial p}{\partial y_t}\frac{\partial f}{\partial a_t}}\right) \tag{2}$$

and, finally,

$$y_t = f(l_t, a(p_t, l_t)) \equiv y(l_t, p_t) \tag{3}$$

In this sense, given the labor supply l_t, adopting a technology index a_t is equivalent to fixing a pollution level p_t. In other terms, p_t can be assimilated to an input. From the Implicit Function Theorem, we obtain

$$\frac{\partial y}{\partial l_t} = \frac{\partial f}{\partial l_t} + \frac{\partial f}{\partial a_t}\frac{\partial a}{\partial l_t} \text{ and } \frac{\partial y}{\partial p_t} = \frac{\partial f}{\partial a_t}\frac{\partial a}{\partial p_t}$$

and, replacing (2),

$$\frac{\partial y}{\partial l_t} = \frac{\frac{\partial p}{\partial a_t}\frac{\partial f}{\partial l_t}}{\frac{\partial p}{\partial a_t} + \frac{\partial p}{\partial y_t}\frac{\partial f}{\partial a_t}} > 0 \text{ and } \frac{\partial y}{\partial p_t} = \frac{\frac{\partial f}{\partial a_t}}{\frac{\partial p}{\partial a_t} + \frac{\partial p}{\partial y_t}\frac{\partial f}{\partial a_t}} > 0$$

3 Human capital

Leisure time is exogenous. Non-leisure time is normalized to one and spent to work or to accumulate human capital (education and health). The individual labor services are the product of human capital and the working time: $l_t \equiv h_t u_t$. The remaining non-leisure time, $1 - u_t$, is devoted to human capital accumulation. Pollution has a negative impact on human capital accumulation.

Assumption 2 *The law of human capital accumulation is given by* $\dot{h}_t/h_t = g(1 - u_t, p_t)$, *where* g *denotes the growth rate, with* $\partial g/\partial(1 - u_t) > 0$ *and* $\partial g/\partial p_t < 0$.

4 Preferences

The assumption of constant elasticity of intertemporal substitution in consumption is common in the growth literature and allows us to avoid mathematical obstacles.

Assumption 3 *Preferences are rationalized by a smooth strictly increasing and strictly concave felicity function* $v_t = v(c_t)$ *with a constant elasticity of intertemporal substitution* $\sigma = -v'(c_t)/[c_t v''(c_t)]$ *with* $\sigma \leq 1$.

The restriction $\sigma \leq 1$ is justified on the empirical ground.[2] The logarithmic case ($\sigma = 1$) is included.

5 Social optimum

The planner maximizes $\int_0^\infty e^{-\rho t} v\left(y\left(h_t u_t, q\left(a_t, h_t u_t\right)\right)\right) dt$, an intertemporal welfare functional, subject to the law of motion $\dot{h}_t = h_t g\left(1 - u_t, q\left(a_t, h_t u_t\right)\right)$, where q is given by (1). Given $h_t u_t$, instead of choosing a_t, the planner can directly compute $p_t = q\left(a_t, h_t u_t\right)$. His program reduces to

$$\max \int_0^\infty e^{-\rho t} v\left(y\left(h_t u_t, p_t\right)\right) dt \text{ subject to } \dot{h}_t = h_t g\left(1 - u_t, p_t\right) \text{ with } h_0 \text{ given}$$

where h_t is the state, while u_t and p_t become controls (indeed, p_t replaces a_t). The current-value Hamiltonian writes: $H_t \equiv v\left(y\left(h_t u_t, p_t\right)\right) + \lambda_t h_t g\left(1 - u_t, p_t\right)$ where λ_t is a costate variable.

We derive the first-order conditions:

$$\partial H_t / \partial \lambda_t = \dot{h}_t,\ \partial H_t / \partial h_t = \rho \lambda_t - \dot{\lambda}_t,\ \partial H_t / \partial u_t = 0,\ \partial H_t / \partial p_t = 0 \tag{4}$$

and the transversality condition $\lim_{t \to \infty} e^{-\rho t} \lambda_t h_t = 0$.

From (4), we obtain the arbitrage

$$\frac{\partial g / \partial p_t}{\partial g / \partial\left(1 - u_t\right)} h_t = -\frac{\partial y / \partial p_t}{\partial y / \partial l_t} \tag{5}$$

Arrow-Mangasarian (sufficient) conditions ensure the concavity of the Hamiltonian. In particular, we require $\varphi\left(h_t, u_t, p_t\right) \equiv v\left(y\left(h_t u_t, p_t\right)\right)$ to be concave with respect to $\left(h_t, u_t, p_t\right)$ and $\tilde{g}\left(u_t, p_t\right) \equiv g\left(1 - u_t, p_t\right)$ to be concave with respect to $\left(u_t, p_t\right)$.

Let us introduce the first and second-order elasticities of the functions $g_t = g\left(1 - u_t, p_t\right)$, $y_t = y\left(l_t, p_t\right)$:

$$\begin{bmatrix} g_1 & g_2 \\ y_1 & y_2 \end{bmatrix} \equiv \begin{bmatrix} \frac{1-u_t}{g_t} \frac{\partial g}{\partial(1-u_t)} & \frac{p_t}{g_t} \frac{\partial g}{\partial p_t} \\ \frac{l_t}{y_t} \frac{\partial y}{\partial l_t} & \frac{p_t}{y_t} \frac{\partial y}{\partial p_t} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \equiv \begin{bmatrix} \frac{1-u_t}{\partial g/\partial(1-u_t)} \frac{\partial^2 g}{\partial(1-u_t)^2} & \frac{p_t}{\partial g/\partial(1-u_t)} \frac{\partial^2 g}{\partial p_t \partial(1-u_t)} \\ \frac{1-u_t}{\partial g/\partial p_t} \frac{\partial^2 g}{\partial(1-u_t)\partial p_t} & \frac{p_t}{\partial g/\partial p_t} \frac{\partial^2 g}{\partial p_t^2} \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \equiv \begin{bmatrix} \frac{l_t}{\partial y/\partial l_t} \frac{\partial^2 y}{\partial l_t^2} & \frac{p_t}{\partial y/\partial l_t} \frac{\partial^2 y}{\partial p_t \partial l_t} \\ \frac{l_t}{\partial y/\partial p_t} \frac{\partial^2 y}{\partial l_t \partial p_t} & \frac{p_t}{\partial y/\partial p_t} \frac{\partial^2 y}{\partial p_t^2} \end{bmatrix} \tag{8}$$

[2] The existing literature does not provide a definitive estimate for σ. Although many standard RBC models consider values around unity, recent empirical works suggest values around 0.5 (see Campbell (1999) among the others).

and the following reduced variables:

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} \equiv \begin{bmatrix} y_{21} - y_{11} \\ y_{21} - y_{11} + g_{11} - g_{21} \\ y_{22} - y_{12} + g_{12} - g_{22} \end{bmatrix} \text{ and } \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \equiv \begin{bmatrix} \frac{y_1}{\sigma} - y_{11} \\ \frac{y_2}{\sigma} - y_{12} \end{bmatrix} \tag{9}$$

Proposition 1 *System (4) reduces to a two-dimensional system:*

$$\dot{u}_t = f_u(u_t, p_t) \equiv \frac{(1 - A_0)(1 - u_t)}{A_0 - A_1 u_t} g(1 - u_t, p_t) u_t - \frac{A_2 (1 - u_t)}{A_0 - A_1 u_t} \frac{u_t}{p_t} f_p(u_t, p_t) \tag{10}$$

$$\dot{p}_t = f_p(u_t, p_t) \equiv \frac{[1 - B_1 + (A_0 - 1) Z(u_t)] g(1 - u_t, p_t) + \frac{\partial g}{\partial (1 - u_t)} u_t - \rho}{g_{12} + B_2 - A_2 Z(u_t)} p_t \tag{11}$$

with

$$Z(u_t) \equiv \frac{B_1 - (g_{11} + B_1) u_t}{A_0 - A_1 u_t} \tag{12}$$

Proof. See the Appendix. ■

Computing the ratio

$$\frac{\dot{u}_t}{\dot{p}_t} = \frac{f_u(u_t, p_t)}{f_p(u_t, p_t)} \equiv F(u_t, p_t)$$

and solving the differential equation

$$\frac{du}{dp} \equiv F(u, p) \tag{13}$$

we find a functional solution $u_t = u(p_t)$. Replacing it in equation (11), we obtain a one-dimensional pollution dynamics $\dot{p}_t = f_p(u(p_t), p_t) \equiv \psi(p_t)$. Substituting in $\dot{h}_t = h_t g(1 - u(p_t), p_t)$, we obtain the human capital growth path from the initial condition h_0.

6 Steady state

At the steady state, the pollution level and the working time are stationary, while the other variables (h_t, y_t, c_t) grow at constant rates. $\dot{p}_t = 0$ and (11) give

$$\frac{\dot{h}_t}{h_t} = g = \frac{\rho}{1 - B_1 + (A_0 - 1) Z(u) + g_1 \frac{u}{1-u}} \tag{14}$$

The transversality condition evaluated along the Regular Growth Path (RGP) is satisfied.[3]

[3] Along the RGP, we have $\dot{\lambda}_t / \lambda_t = \rho - g - u \partial g / \partial (1 - u_t)$ and the transversality condition writes $\lim_{t \to \infty} e^{-\rho t} \lambda_t h_t = \lambda_0 h_0 \lim_{t \to \infty} \left(e^{g - \rho + \dot{\lambda}_t / \lambda_t} \right)^t = 0$, that is $g - \rho + \dot{\lambda}_t / \lambda_t = -u \partial g / \partial (1 - u_t) < 0$.

g is the growth rate for human capital, while the growth rate for production and consumption is different.

Proposition 2 *At the steady state, growth is regular: the economy does not experience the same (constant) growth rate for c_t and h_t:*

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{y}_t}{y_t} = y_1 \left(1 + u_1\right) \frac{\dot{h}_t}{h_t} \tag{15}$$

with

$$u_1 = \frac{\left(1 - A_0\right)\left(1 - u\right)}{A_0 - A_1 u} \tag{16}$$

Proof. See the Appendix. ■

More precisely, if $u_1 > -1$, we have $0 = \dot{u}_t/u_t = \dot{p}_t/p_t < \dot{y}_t/y_t = \dot{c}_t/c_t \neq \dot{h}_t/h_t = g$.

7 Example

Productivity is enhanced by the adoption of a polluting technology: $y_t = a_t A l_t^{\alpha}$ with $0 < \alpha < 1$, but production pollutes: $p_t = a_t^{\gamma} y_t$, with $\gamma > 0$. (3) becomes

$$y_t = y\left(l_t, p_t\right) = A^{\frac{\gamma}{1+\gamma}} l_t^{\alpha \frac{\gamma}{1+\gamma}} p_t^{\frac{1}{1+\gamma}} \tag{17}$$

Lucas (1988) represents the case without pollution ($y_t = A l_t^{\alpha}$) and could be recovered as a limit case with $\gamma = +\infty$.

Focus now on a multiplicative human capital accumulation: $g\left(1 - u_t, p_t\right) \equiv B\left(1 - u_t\right)^{\beta}\left(p_{\max} - p_t\right)^{\pi}$ with $p_t \leq p_{\max}$. This form simplifies to

$$\frac{\dot{h}_t}{h_t} = C\left(1 - u_t\right)^{\beta}\left(1 - x_t\right)^{\pi} \tag{18}$$

with $C \equiv B p_{\max}^{\pi}$, where $x_t \equiv p_t/p_{\max}$ is the *relative pollution.*

Specification (18) implies that, *ceteris paribus*, pollution has always a negative impact on the human capital accumulation rate and this rate never becomes negative. In the limit, when p goes to $p_{\max}$, the human capital accumulation stops. This specification is different from that introduced by Gradus and Smulders (1993) where pollution enters additively the accumulation rate and can make it negative.

The second-order conditions for the planner's maximization can be checked under some restriction in the parameter space.

Assumption 4 $\beta, \pi \in (0, 1)$ *and* $\beta + \pi < 1$.

Assumptions 3 and 4 ensure the Arrow-Mangasarian second-order (sufficient) conditions for Hamiltonian maximization to be verified.

Proposition 3 *Under Assumptions 3 and 4, the second-order conditions of the planner's maximization are satisfied.*

Proof. See the Appendix. ■

Restriction $\beta + \pi < 1$ is a fundamental condition of the model. Not only, it ensures the concavity of the program, but also, as we will see, it allows us to prove the existence of the steady state and to solve unambiguously the comparative statics and the stability analysis.

Computing the elasticities (6) to (8) gives:

$$\begin{bmatrix} g_1 & g_2 \\ y_1 & y_2 \end{bmatrix} \equiv \begin{bmatrix} \beta & -\pi\frac{p_t}{p_{\max}-p_t} \\ \alpha\frac{\gamma}{1+\gamma} & \frac{1}{1+\gamma} \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \equiv \begin{bmatrix} \beta-1 & -\pi\frac{p_t}{p_{\max}-p_t} \\ \beta & (1-\pi)\frac{p_t}{p_{\max}-p_t} \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \equiv \begin{bmatrix} \alpha\frac{\gamma}{1+\gamma}-1 & \frac{1}{1+\gamma} \\ \alpha\frac{\gamma}{1+\gamma} & -\frac{\gamma}{1+\gamma} \end{bmatrix} \tag{21}$$

and

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -\frac{p_{\max}}{p_{\max}-p_t} \end{bmatrix} \text{ and } \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 1+\alpha\frac{1-\sigma}{\sigma}\frac{\gamma}{1+\gamma} \\ \frac{1-\sigma}{\sigma}\frac{1}{1+\gamma} \end{bmatrix}$$

System (10)-(11) becomes

$$\dot{u}_t = \frac{\left(\beta\frac{u_t}{1-u_t} - \alpha\frac{1-\sigma}{\sigma}\frac{\gamma}{1+\gamma}\right) g\left(1-u_t, x_t p_{\max}\right) - \rho}{\Delta\left(u_t, x_t\right)} \left(1-u_t\right) u_t \tag{22}$$

$$\dot{x}_t = \frac{\left(\beta\frac{u_t}{1-u_t} - \alpha\frac{1-\sigma}{\sigma}\frac{\gamma}{1+\gamma}\right) g\left(1-u_t, x_t p_{\max}\right) - \rho}{\Delta\left(u_t, x_t\right)} \left(1-x_t\right) x_t \equiv \tilde{\psi}\left(u_t, x_t\right) \tag{23}$$

where

$$\begin{aligned}
\Delta_t &= \Delta\left(u_t, x_t\right) \\
&\equiv 1 + \frac{1+\alpha\gamma}{1+\gamma}\frac{1-\sigma}{\sigma} - \left(\beta + \alpha\frac{1-\sigma}{\sigma}\frac{\gamma}{1+\gamma}\right) u_t - \left(\pi + \frac{1-\sigma}{\sigma}\frac{1}{1+\gamma}\right) x_t \\
&> 1 + \frac{1+\alpha\gamma}{1+\gamma}\frac{1-\sigma}{\sigma} - \left(\beta + \alpha\frac{1-\sigma}{\sigma}\frac{\gamma}{1+\gamma}\right) - \left(\pi + \frac{1-\sigma}{\sigma}\frac{1}{1+\gamma}\right) \\
&= 1 - \beta - \pi > 0
\end{aligned}$$

because $u_t, x_t \in (0,1)$.

Dividing (22) by (23) side by side, we obtain

$$\frac{du}{dx} = \frac{u}{x}\frac{1-u}{1-x} \tag{24}$$

(24) is a first-order differential equation whose solution is given by

$$u_t = u\left(x_t\right) = \frac{x_t}{x_t + \left(1-x_t\right) c} > 0 \tag{25}$$

where c is an integration constant.

As usual in the endogenous growth literature, we obtain a reduced dynamics. Replacing (25) in (23), system (22)-(23) reduces to a single equation

$$\dot{x}_t = \tilde{\psi}\left(u\left(x_t\right), x_t\right) \equiv \psi\left(x_t\right) \tag{26}$$

7.1 Steady state

At the steady state, $\dot{x}_t = 0$. Equation (23) gives

$$\frac{\dot{h}_t}{h_t} = g = \frac{\rho}{\beta\frac{u}{1-u} - s} \text{ with } s \equiv \alpha\frac{\gamma}{1+\gamma}\frac{1-\sigma}{\sigma} \tag{27}$$

The regular growth rates are ranked according to (15), (16) and (19):

$$0 = \frac{\dot{u}_t}{u_t} = \frac{\dot{p}_t}{p_t} < \frac{\dot{c}_t}{c_t} = \frac{\dot{y}_t}{y_t} = y_1\left(1+u_1\right)g = \frac{\alpha\gamma}{1+\gamma}g < \frac{\dot{h}_t}{h_t} = g$$

(in the Lucas (1988) model: $\gamma = +\infty$ (no pollution) and $\dot{c}_t/c_t = \alpha g$).

Focus now on (25) and (26). $\dot{x}_t = \psi\left(x_t\right) = 0$ gives

$$\eta\left(u\right) \equiv \left(1-u\right)^{\beta}\left(\beta\frac{u}{1-u} - s\right)\left(\frac{\alpha\gamma}{\alpha\gamma + \frac{\beta}{\pi}\frac{u}{1-u}}\right)^{\pi} = \frac{\rho}{C} \tag{28}$$

$\eta\left(u\right) > 0$ requires $u \in \left(\underline{u}, 1\right)$ with $\underline{u} \equiv s/\left(\beta+s\right)$. $u \in \left(\underline{u}, 1\right)$ is equivalent to $g > 0$: in our example, the growth rate is always positive.

Proposition 4 *A steady state u exists.*

Proof. See the Appendix. ■

Solving equation (28), we find u. From (5), we obtain also

$$x = \frac{p}{p_{\max}} = \frac{\frac{\beta}{\pi}\frac{u}{1-u}}{\alpha\gamma + \frac{\beta}{\pi}\frac{u}{1-u}} < 1 \tag{29}$$

and, finally, through (27), we compute g.

The planner compute c_0 to ensure that the economy stays on the RGP

$$c_0 = y_0 = A^{\frac{\gamma}{1+\gamma}}\left(h_0 u\right)^{\alpha\frac{\gamma}{1+\gamma}}\left(x p_{\max}\right)^{\frac{1}{1+\gamma}}$$

where h_0 is predetermined. More explicitly, the RGP becomes

$$h_t = h_0 e^{gt},\ y_t = y_0 e^{\alpha\frac{\gamma}{1+\gamma}gt},\ c_t = c_0 e^{\alpha\frac{\gamma}{1+\gamma}gt}$$

Focus now on the comparative statics and on the impact of parameters on the steady state.

Proposition 5 *At the steady state, there is a positive relation between the pollution level and the working time: $dp/du > 0$.*

Proof. See the Appendix. ■

u has a positive direct effect on l and an indirect negative effect on l through g, while p has a direct positive effect on y and a negative effect on l through g: the arbitrage between u and p is captured by equation (5), resulting in an unambiguous relation between u and p.

Consider

$$\gamma_0 \equiv \frac{\beta}{\pi\alpha} \frac{\frac{1+s}{\beta+s} - \frac{\pi}{\beta}}{\frac{1+s}{\beta+s} + 1} < \frac{\beta}{\pi\alpha} \equiv \gamma_1 \tag{30}$$

There are two cases: (1) "pollutions matters": $\gamma_0 < \gamma < \gamma_1$, (2) "pollution does not matter": $\gamma > \gamma_1$. The second case is similar to that without pollution (with $\gamma = +\infty$ we recover the Lucas (1988) model). In the following, we will focus on the novelty of the paper, that is on the first case.

Assumption 5 *Pollutions matters:* $\gamma_0 < \gamma < \gamma_1$.

We introduce a critical value of pollution:

$$p_+ \equiv p_{\max} x \left(\sqrt{\left(\frac{a_1}{2a_2}\right)^2 + \frac{a_0}{a_2}} - \frac{a_1}{2a_2} \right)$$

where

$$a_2 \equiv \gamma_1 - \gamma,\ a_1 \equiv (\gamma - \gamma_0)\left(1 + \frac{1+s}{\beta+s}\right) \text{ and } a_0 \equiv \frac{1+s}{\beta+s}\left(\gamma + \frac{1}{\alpha}\frac{s}{1+s}\right) > 0 \tag{31}$$

Lemma 6 *Assumptions 4 and 5 imply* $p < p_+$.

Proof. See the Appendix. ■

The following critical value

$$\omega \equiv \frac{\beta u + Q - 1}{1-u} \text{ with } Q \equiv \pi x - \frac{s}{\beta\frac{u}{1-u} - s}$$

plays also a role in the comparative statics and in the stability analysis.

Lemma 7 *Under Assumptions 4 and 5,* $\omega < 0$.

Proof. See the Appendix. ■

Differentiating (28), we capture the impact of parameters on the steady state:

$$\begin{bmatrix} \frac{B}{u}\frac{\partial u}{\partial B} \\ \frac{p_{\max}}{u}\frac{\partial u}{\partial p_{\max}} \\ \frac{\rho}{u}\frac{\partial u}{\partial \rho} \\ \frac{\sigma}{u}\frac{\partial u}{\partial \sigma} \end{bmatrix} = \frac{1}{\omega}\begin{bmatrix} 1 \\ \pi \\ -1 \\ \alpha\frac{\gamma}{1+\gamma}\frac{1}{\sigma}\frac{g}{\rho} \end{bmatrix} \tag{32}$$

and

$$\begin{bmatrix} \frac{\alpha}{u}\frac{\partial u}{\partial \alpha} \\ \frac{\gamma}{u}\frac{\partial u}{\partial \gamma} \\ \frac{\pi}{u}\frac{\partial u}{\partial \pi} \\ \frac{\beta}{u}\frac{\partial u}{\partial \beta} \end{bmatrix} = \frac{1}{\omega}\begin{bmatrix} Q \\ \frac{1}{1+\gamma}Q + \frac{\gamma}{1+\gamma}\pi x \\ \pi x + \pi \ln\left[(1-x)\, p_{\max}\right] \\ 1 - Q + \beta \ln(1-u) \end{bmatrix} \tag{33}$$

The existence of a steady state requires also a lower bound for pollution: $\underline{p} \equiv p_{\max} x(\underline{u})$.

Proposition 8 *Let $p > \underline{p}$. Assumptions 4 and 5 imply*

$$\begin{aligned} \frac{\partial u}{\partial B} &< 0, \ \frac{\partial u}{\partial p_{\max}} < 0, \ \frac{\partial u}{\partial \rho} > 0, \ \frac{\partial u}{\partial \sigma} < 0 \\ \frac{\partial p}{\partial B} &< 0, \ \frac{\partial p}{\partial p_{\max}} < 0, \ \frac{\partial p}{\partial \rho} > 0, \ \frac{\partial p}{\partial \sigma} < 0 \end{aligned}$$

In addition, if $p_{\max} > e^{-x}/(1-x)$,

$$\frac{\partial u}{\partial \pi} < 0 \text{ and } \frac{\partial p}{\partial \pi} < 0$$

Proof. See the Appendix. ■

As in Lucas (1988), a model without physical capital and pollution, ρ has a positive effect on u because more impatient agents prefer to work more and consume more today, instead of to accumulate human capital for tomorrow. The higher the working time, the higher the production and pollution. We observe also that B has the same qualitative impact of $p_{\max}$ because both of these parameters enter the factor $C \equiv Bp_{\max}^{\pi}$.

When π increases, the environmental quality $(p_{\max} - p)$ has a larger impact on capital accumulation. The planner reduces the pollution level and increases the time spent for education and health $(1-u)$. When β increases, the time spent for education and health $(1-u)$ has a larger impact on capital accumulation. The planner reduces the working time u and the pollution level (because, on the one side, production partially lowers and, on the other side, the environmental quality has a larger effect on capital accumulation).

Assumption 6

$$\sigma > \frac{1}{1 + \frac{\beta}{\alpha}\frac{1+\gamma}{\gamma}\frac{u}{1-u}\frac{\pi x}{1+\pi x}} \in (0,1)$$

Assumption 6 is equivalent to $Q > 0$ and is satisfied in the case of logarithmic preferences $(\sigma = 1)$.

Proposition 9 *Let $p > \underline{p}$. Assumptions 4, 5 and 6 imply*

$$\frac{\partial u}{\partial \alpha} < 0, \ \frac{\partial u}{\partial \gamma} < 0, \ \frac{\partial p}{\partial \alpha} < 0, \ \frac{\partial p}{\partial \gamma} < 0$$

When α is higher, the relative productivity of pollution in the reduced production function (17) lowers and, so, the planner adopts a less polluting technology. The higher environmental quality increases the impact of education and wealth on capital accumulation. The planner decides to reduce the working time to raise the investments in education and wealth. The same arguments work for γ because a higher γ also lowers the relative productivity of pollution in the reduced production function (17).

Finally, focus on human capital accumulation. Consider, for simplicity, the logarithmic case.

Proposition 10 *Let $\sigma = 1$. Propositions 8 and 9 hold also for the stationary growth rate g but now with reversed signs, that is $sign\partial g/\partial z = -sign\partial u/\partial z$ for $z = B, p_{\max}, \alpha, \gamma, \pi$. In addition, $\partial g/\partial \beta < 0$, while $\partial g/\partial \rho < 0$ iff*

$$\frac{\rho}{u}\frac{\partial u}{\partial \rho} > 1 - u$$

Proof. See the Appendix. ■

Focus on

$$g = \frac{\rho}{\beta}\frac{1-u}{u} \tag{34}$$

u has a negative impact on human capital accumulation (the higher the working time, the lower the investments in education and health). This explains why $sign\partial g/\partial z = -sign\partial u/\partial z$. The other parameters β and ρ have also a direct effect (positive and negative, respectively: see expression (34)).

The negative direct effect of β on g always dominates the possibly positive indirect effect through $\partial u/\partial \beta < 0$ (see Proposition 8). Under condition (34), the positive direct effect of ρ on g is dominated by a negative indirect effect through $\partial u/\partial \rho > 0$ (see Proposition 8).

7.2 Stability analysis

In the Lucas (1988) model the growth path is unique. This result also holds in our model.

Proposition 11 *Under Assumption 4 and 5, the eigenvalue of reduced dynamics (26) around the steady state (29) is positive.*

Proof. See the Appendix. ■

8 Conclusion

In this paper we have considered the effects of pollution on human capital accumulation through an endogenous growth model à la Lucas (1988) augmented by a pollution mechanism *à la* Stockey (1998).

We have found positive relation between pollution level and the working time because pollution slows down the human capital accumulation and makes less efficient the investments in education and health.

9 Appendix

Proof of Proposition 1 First-order conditions (4) write

$$\frac{\dot{h}_t}{h_t} = g_t \text{ and } \frac{\dot{\lambda}_t}{\lambda_t} = \rho - g_t - u_t \frac{\partial g}{\partial (1-u_t)} \tag{35}$$

$$\lambda_t = v'(c_t)\frac{\partial y/\partial l_t}{\partial g/\partial (1-u_t)} \tag{36}$$

jointly with condition (1). Conditions (35) and (36) look like the first-order conditions of the program without pollution. Since $y_t = y(h_t u_t, p_t)$ and $g_t = g(1-u_t, p_t)$, the implicit equation (1) allows us to locally define $u_t = u(h_t, p_t)$ with elasticities:

$$(u_1, u_2) \equiv \left(\frac{h_t}{u_t}\frac{\partial u}{\partial h_t}, \frac{p_t}{u_t}\frac{\partial u}{\partial p_t}\right)$$

From (36), we obtain $\dot{\lambda}_t/\lambda_t$:

$$\begin{aligned}\frac{\dot{\lambda}_t}{\lambda_t} &= \frac{v''(c_t)}{v'(c_t)}\dot{c}_t + \frac{\frac{\partial^2 y}{\partial p_t \partial l_t}\dot{p}_t + \frac{\partial^2 y}{\partial l_t^2}\left[\dot{h}_t u_t + \left(\frac{\partial u}{\partial p_t}\dot{p}_t + \frac{\partial u}{\partial h_t}\dot{h}_t\right)h_t\right]}{\partial y/\partial l_t} \\ &\quad + \frac{\frac{\partial^2 g}{\partial(1-u_t)^2}\left(\frac{\partial u}{\partial p_t}\dot{p}_t + \frac{\partial u}{\partial h_t}\dot{h}_t\right) - \frac{\partial^2 g}{\partial p_t \partial(1-u_t)}\dot{p}_t}{\partial g/\partial(1-u_t)}\end{aligned} \tag{37}$$

Production is entirely consumed: $c_t = y(h_t u_t, p_t)$. Taking the logarithms and deriving with respect to time we get also:

$$\frac{\dot{c}_t}{c_t} = \frac{\frac{\partial y}{\partial p_t}\dot{p}_t + \frac{\partial y}{\partial l_t}\left[\dot{h}_t u_t + h_t\left(\frac{\partial u}{\partial p_t}\dot{p}_t + \frac{\partial u}{\partial h_t}\dot{h}_t\right)\right]}{y_t} \tag{38}$$

Replacing $c_t v''(c_t)/v'(c_t) = -1/\sigma$, $\dot{h}_t/h_t = g_t$ and (38) in (37), we find:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \left[\left(\frac{u_t}{1-u_t}g_{11} - B_1\right)u_2 - B_2 - g_{12}\right]\frac{\dot{p}_t}{p_t} + \left[\left(\frac{u_t}{1-u_t}g_{11} - B_1\right)u_1 - B_1\right]g_t \tag{39}$$

Substituting in turn

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - g_t\left(1 + \frac{u_t}{1-u_t}g_1\right) \tag{40}$$

in (39) and solving for $\dot{p}/p$, we finally obtain a two-dimensional dynamic system:

$$\frac{\dot{h}_t}{h_t} = g_t \text{ and } \frac{\dot{p}_t}{p_t} = \frac{\left[1 - B_1 + \frac{u_t}{1-u_t}g_1 + \left(\frac{u_t}{1-u_t}g_{11} - B_1\right)u_1\right]g_t - \rho}{B_2 + g_{12} - \left(\frac{u_t}{1-u_t}g_{11} - B_1\right)u_2}$$

In order to compute the first-order elasticities of function u, we differentiate

$$h_t\frac{\partial y}{\partial l_t}\frac{\partial g}{\partial p_t} = -\frac{\partial y}{\partial p_t}\frac{\partial g}{\partial(1-u_t)} \tag{41}$$

with respect to (u_t, h_t, p_t), where $g_t = g(1-u_t, p_t)$, $y_t = y(l_t, p_t) = y(h_t u_t, p_t)$:

$$\begin{aligned}&\left[h_t\frac{\partial y}{\partial l_t}\frac{\partial g}{\partial p_t}\left(y_{11} - \frac{u_t}{1-u_t}g_{21}\right) + \frac{\partial y}{\partial p_t}\frac{\partial g}{\partial(1-u_t)}\left(y_{21} - \frac{u_t}{1-u_t}g_{11}\right)\right]\frac{du_t}{u_t} \\ &+ \left[h_t\frac{\partial y}{\partial l_t}\frac{\partial g}{\partial p_t}(y_{12} + g_{22}) + \frac{\partial y}{\partial p_t}\frac{\partial g}{\partial(1-u_t)}(y_{22} + g_{12})\right]\frac{dp_t}{p_t} \\ &+ \left[h_t\frac{\partial y}{\partial l_t}\frac{\partial g}{\partial p_t}(1 + y_{11}) + \frac{\partial y}{\partial p_t}\frac{\partial g}{\partial(1-u_t)}y_{21}\right]\frac{dh_t}{h_t} \\ =\ & 0\end{aligned}$$

and using (41):

$$0=\frac{A_1u_t-A_0}{1-u_t}\frac{du_t}{u_t}-A_2\frac{dp_t}{p_t}+(1-A_0)\frac{dh_t}{h_t}$$

The elasticities of u become:

$$u_1=\frac{(1-A_0)(1-u_t)}{A_0-A_1u_t}\text{ and }u_2=-\frac{A_2(1-u_t)}{A_0-A_1u_t}$$

The dynamic system writes:

$$\frac{\dot{h}_t}{h_t}=g_t\text{ and }\frac{\dot{p}_t}{p_t}=\frac{\left[1-B_1+(A_0-1)\frac{A_0-A_1}{A_0-A_1u_t}\right]g_t+\frac{\partial g}{\partial(1-u_t)}u_t-\rho}{B_2+g_{12}-A_2\frac{A_0-A_1}{A_0-A_1u_t}}$$

We observe that $u_t=u(h_t,p_t)$ and

$$\frac{\dot{u}_t}{u_t}=\frac{h_t}{u_t}\frac{\partial u}{\partial h_t}\frac{\dot{h}_t}{h_t}+\frac{p_t}{u_t}\frac{\partial u}{\partial p_t}\frac{\dot{p}_t}{p_t}=u_1g_t+u_2\frac{\dot{p}_t}{p_t}$$

Then, we obtain the following dynamic system:

$$\begin{aligned}\frac{\dot{u}_t}{u_t}&=\frac{(1-A_0)(1-u_t)}{A_0-A_1u_t}g(1-u_t,p_t)-\frac{A_2(1-u_t)}{A_0-A_1u_t}\frac{\dot{p}_t}{p_t}\\ \frac{\dot{p}_t}{p_t}&=\frac{[1-B_1+(A_0-1)Z(u_t)]g(1-u_t,p_t)+\frac{\partial g}{\partial(1-u_t)}u_t-\rho}{g_{12}+B_2-A_2Z(u_t)}\end{aligned}$$

that is (10)-(11). ■

Proof of Proposition 2 At the steady state $\dot{u}_t=\dot{p}_t=0$ and (38) give (15). ■

Proof of Proposition 3 Under the monotonic transformation $k_t\equiv\ln h_t$, the planner's program writes equivalently: $\max\int_0^\infty e^{-\rho t}v\left(y\left(e^{k_t}u_t,p_t\right)\right)dt$ subject to $\dot{k}_t=g(1-u_t,p_t)$, where k_t is the new state. The Hamiltonian writes: $H_t\equiv v\left(y\left(u_te^{k_t},p_t\right)\right)+\mu_tg(1-u_t,p_t)$ where μ_t is the new costate variable. In order to apply the Arrow-Mangasarian Sufficiency Theorem, we require $\varphi(k_t,u_t,p_t)\equiv v\left(y\left(u_te^{k_t},p_t\right)\right)$ to be concave with respect to (k_t,u_t,p_t) and $\tilde{g}(u_t,p_t)=B(1-u_t)^\beta(p_{\max}-p_t)^\pi$ to be concave with respect to (u_t,p_t). Under Assumption 3, $v(c_t)=c_t^{1-1/\sigma}/(1-1/\sigma)<0$ and the principal diagonal minors of the Hessian matrix $D^2\varphi$ have alternating signs: $m_1=v_t\tau^2y_1^2<0$, $m_2=v_t^2\tau^3y_1^3/u_t^2>0$, $m_3=v_t^3\tau^4y_1^3y_2/(u_tp_t)^2<0$ with $y_1=\alpha\gamma/(1+\gamma)$, $y_2=1/(1+\gamma)$ and $\tau\equiv(1-\sigma)/\sigma$. Thus, φ is strictly concave. Under Assumption 4, the principal diagonal minors of the Hessian matrix $D^2\tilde{g}$ also have alternating signs: $n_1=-\beta(1-\beta)g/(1-u_t)^2<0$ and $n_2=\beta(1-\beta-\pi)\pi g^2/[(1-u_t)(p_{\max}-p_t)]^2>0$. Thus $\tilde{g}$ is strictly concave too. The strict concavity of Hamiltonian implies the uniqueness of solution. ■

Proof of Proposition 4 A steady state is solution of $\eta(u) = \rho/C$. We observe that

$$\eta(\underline{u}) = 0 \text{ and } \lim_{u \to 1} \eta(u) = \beta \left(\alpha\gamma \frac{\pi}{\beta} \right)^{\pi} \lim_{u \to 1} \frac{u^{1-\pi}}{(1-u)^{1-\beta-\pi}} = +\infty$$

because $\beta + \pi < 1$. Thus a steady state exists because u is a continuous function over $(\underline{u}, 1)$ and $\rho/C > 0$. ■

Proof of Proposition 5 From (25) and (29), we have $u = \beta u / [\beta u + c\pi\alpha\gamma(1-u)]$. Then $u \in (0,1)$ iff $c > 0$. In this case, we obtain

$$x(u) = \frac{cu}{cu + 1 - u} \text{ and } x'(u) = \frac{c}{(cu + 1 - u)^2} > 0$$

Finally, notice that $dp/du = p_{\max} x'(u)$. ■

Proof of Lemma 6 By definition, $p = x p_{\max}$. Then, $p < p_+$ is equivalent to

$$\sqrt{\left(\frac{a_1}{2a_2}\right)^2 + \frac{a_0}{a_2}} > 1 + \frac{a_1}{2a_2} \tag{42}$$

Under Assumption 5, $a_0, a_1, a_2 > 0$ and (42) becomes equivalent to $a_0 > a_1 + a_2$. Replacing (30) and (31), $a_0 > a_1 + a_2$ writes $\beta + \pi < 1$. Thus, under Assumption 5, $p < p_+$ is equivalent to $\beta + \pi < 1$ (Assumption 4). ■

Proof of Lemma 7 We observe that $\omega < 0$ iff $Q < 1 - \beta u$. Replacing g and p, we find that $\omega < 0$ iff $a_2 u^2 + a_1 u - a_0 < 0$. Under Assumption 5, we have $a_2, a_1, a_0 > 0$. Let

$$u_{\pm} \equiv -\frac{a_1}{2a_2} \pm \sqrt{\left(\frac{a_1}{2a_2}\right)^2 + \frac{a_0}{a_2}}$$

Then, $u_- < 0 < u_+$ and, so, $\omega < 0$ iff $u < u_+$. Since $p_+ \equiv p_{\max} x(u_+) > 0$ and $x'(u) > 0$, then $\omega < 0$ iff $p < p_+$. ■

Proof of Proposition 8 Consider the elasticities (32) and (33) and apply Lemma 7. Notice that $dp/du = p_{\max} x'(u) > 0$. ■

Proof of Proposition 9 Consider the elasticities (33) and apply Lemma 7. Notice that $dp/du = p_{\max} x'(u) > 0$. ■

Proof of Proposition 10 In the logarithmic case, (34) holds and, for any parameter z, excepted β, ρ, we have

$$\frac{\partial g}{\partial z} = -\frac{\rho}{\beta} \frac{1}{u^2} \frac{\partial u}{\partial z} \tag{43}$$

Apply Propositions 8 and 9, and take into account the sign reversal in (43).

Focus now on $\partial g/\partial\beta$. We have

$$\frac{\partial g}{\partial \beta} = \frac{\rho}{\beta}\frac{1-u}{u}\frac{u+\ln(1-u)}{1-(\beta u+\pi x)}$$

In the last fraction, the numerator is negative while, under Assumption 4, the denominator is positive.

Finally, observe that

$$\frac{\partial g}{\partial \rho} = -\frac{\rho}{\beta}\frac{1}{u^2}\frac{\partial u}{\partial \rho} + \frac{1}{\beta}\frac{1-u}{u}$$

■

Proof of Proposition 11 The eigenvalue of reduced dynamics (26) around the steady state (29) is given by

$$\psi'(x) = \frac{1}{\Delta}\left(\beta u \frac{g}{1-u} - \beta u \rho - \pi \rho x\right) = -\omega \frac{\rho(1-u)}{\Delta}$$

Assumption 4 implies $\omega < 0$ (Lemma 7) that is $\psi'(x) > 0$. ■

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