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Working Paper
2012-02

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An Environmental-Economic Measure of Sustainable Development

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December 14, 2011

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Abstract

A central issue in the study of sustainable development is the interplay of growth and sacrifice in a dynamic economy. This paper investigates the relationship among current consumption, growth, and sustained consumption in two canonical, stylized economies and in a more general context. It is found that the maximin value measures what is sustainable and provides the limit to growth. Maximin value is interpreted as an environmental-economic carrying capacity and current consumption or utility as an environmental-economic footprint. The time derivative of maximin value is interpreted as net investment in sustainability improvement. It is called durable savings to distinguish it from genuine savings, usually computed with discounted utilitarian prices.

Key words: sustained development, growth, maximin, sustainability indicator

1 Introduction

Sustained development is a phrase that describes growth out of poverty toward a developed state that can be sustained for what Solow (1993) calls the *very long run*. In an efficient economy, growth or development entails the diversion of resources from consumption by the current generation to investment that will increase productivity in the future. For development to be sustainable, the path followed by the economy must be within environmental and technological constraints.

Although the implications for a poor society are not usually stressed, a policy proposal of greater current investment and less consumption has been advanced in several economic models that assume efficiency in the attainment of a specified goal, usually maximizing discounted–utilitarian welfare. The current standard of living in a less developed country may be so low, however, that one cannot contemplate reducing it. Sacrificing the interests of the present may be inconsistent with the Brundtland Report's (World Commission on Environment and Development, 1987) famous dictum on sustainability, which balances and protects the interest of the present as well as the future.[1] Optimal growth theory, however, neither specifies the extent of sacrifice envisaged, nor values growth *per se* in the definition of welfare, although growth is considered good.

The question of how to express the notion of sustained development formally has been partially addressed in other contexts in economics. Growth theorists have specified as parameters certain variables that could have been modeled as choices. Among them are a constant savings ratio, a constant capital-output ratio, balanced growth, or a constant "bliss" level of utility. Holding a variable constant in this way has simplified complicated dynamic problems and has allowed for many revealing

[1]One way to avoid this invidious trade-off is to assume that, while the present generation is poor, there is some possibility of improvement from the base of the present. Llavador et al. (2010), for example, find that sustainable consumption for the USA was higher than actual consumption in 2000. A possible reason is inefficiency. As Llavador et al. indicate, the long-term solution is to address the inefficiency, not necessarily to invest more in the present. In the present paper, we are not focusing on inefficiency, but on investment.

analyses.

Other analysts have found tentative evidence that there is a preference among consumers for wage or consumption profiles that increase through time (Lowenstein and Sicherman, 1991; Frank and Hutchens, 1993). Examples used are of consumption growing at a constant rate over an individual's lifetime. If such preferences can be applied to a whole society, growth of consumption at a constant rate can be considered to be a generalization of a sustained path in that its "distribution over time has some definite standard shape" (Hicks, 1946: 184).

The present paper addresses the implications of a conscious choice by a society between current sacrifice and growth. The issue is how to grow out of poverty, to improve what can be sustained. Following the tentatives in growth theory and in positive economics, we assume that the economies are not pursuing a specific objective but rather a parametric policy that seems plausible, for example, constant growth or constant employment. Sustainable development means that an acceptable standard of living is reached in the long run and then sustained. We examine the conditions for the given growth pattern to be sustainable.

Our study is motivated by two model economies that have been prominent in the study of sustainability. In a simple fishery, a fish stock is harvested and consumed directly. Open access leads to overexploitation. At any point, however, the society is assumed to be in a position to choose a level of employment in the industry, and hence of forbearance in exploiting the stock. At the beginning of the program the stock is at a low level (is "overfished") and the society wishes to rebuild its stock by limiting current consumption. In the steady state, the harvest is equal to natural growth and is thus sustained.

In the *Dasgupta-Heal-Solow* (DHS) model of an economy dependent on manufactured capital and an essential, non-renewable resource (Dasgupta and Heal, 1974; Solow, 1974), sustaining consumption at a constant level requires that investment in manufactured capital offset the depletion of the resource (Hartwick, 1977). A deviation downward from that possible constant-consumption path can allow for growth at a parametric rate through investment (d'Autume and Schubert, 2008; Asheim et al., 2007). The economy can choose from many different paths of sustained development. Overshooting the sustainable path is also possible in the model.

Since much of the discussion of sustainability in economics has been done in terms of simple models, especially these two models, it is natural for the paper to pass from the particular to the general. The findings from the simple models are the basis of a generalization to more complicated economies.

2 The Setting

Each of the two canonical models addresses a fundamental issue in environmental economics. Each implies that growth is subject to environmental constraints. Open access in the fishery leads to a tragedy of the commons. The DHS economy illustrates the fact that sustaining an economy may not involve a steady state. Each of open access and growth can lead to unsustainability and to a poverty trap.

Our analysis is based on a modification to the maximin program that allows for growth. A maximin path maximizes the standard of living of the poorest generation, looking forward from the present (Cairns and Long 2006). What is *sustained* (supported from below) along a feasible path of the economy is the minimum level of consumption of any generation over the very long run. The maximum attainable such minimum level, or the maximin level, is what is *sustainable*. Let social utility at time t be represented by $U(t)$. The sustainable or maximin level of utility at time t in a dynamic economy is given by

$$\max \bar{U} \quad \text{s.t. } U(s) \geq \bar{U} \; \forall s \geq t. \tag{1}$$

If the economy pursues the maximin objective in a *regular* maximin problem, the standard of living remains constant over the indefinite future (Burmeister and Hammond, 1977; Cairns and Long, 2006).[2] This means that if a planner decides to apply

[2]A comparison with "strong" sustainability is in order. Strong sustainability is the capacity to ensure a minimum standard of, or a minimum level of an index of, environmental quality. Maintaining a higher level of the index is considered more desirable. A maximin program could

the criterion immediately in a poor economy, future generations may be mired in a "poverty trap" involving continuing levels of the standard of living equal to the low level of the present: poverty may be sustained. The criticism implies that the present generation is considered to be at a level of poverty that is so dire that the future must be rescued from it. Our modification to the maximin program is a response to this criticism. Since, for a regular maximin program, the constant utility path is a Pareto-efficient solution, we show that for growth to occur the standard of living of the present must be reduced to an *even lower* level than that of the poverty trap. The path envisaged is one in which the society chooses a growth pattern beginning from its current, low level toward a higher, sustained level. We describe the trade-offs among present consumption, growth and long-run sustained consumption.

For the sake of definiteness, utility or the standard of living of the society is frequently interpreted herein as its level of consumption, broadly defined. We argue that the maximin level of consumption is a representation of sustainability, as it gives the highest consumption level that can be sustained from the current economic state. Even though a maximin policy may not be being pursued, at any economic state a maximin level of consumption can be determined by solving the maximin problem for the stocks at that state. The evolution of this maximin value along any trajectory plays a fundamental role in the sense that it is an indicator of what is sustainable. A current level of consumption is unsustainable if it is greater than this indicator. Furthermore, current decisions reduce what is sustainable if the maximin

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be followed for maximizing the sustained level of the environmental index (Cairns and Long, 2006). The usual criticism, that some trade-offs may not be physically possible, can be handled by constraints in the model. The fundamental difference in our perspective is that we consider sustaining a measure of human well being rather than what might be called environmental well being.

Dasgupta and Mäler (1990) assert that the current level of the environmental index is not sacrosanct. In defining what is sustained to be the minimum of utility over the indefinite future, we consider the current level of utility (or welfare) not to be sacrosanct. If one is interested in sustaining something, be it an environmental index or human well being, the maximin level and its evolution have theoretic importance.

value decreases. On the contrary, if consumption is lower than the maximin level on an interval, both the attainable maximin consumption of the economy and current consumption can increase through time. We give a sufficient condition on investment for such a sustainable growth to be possible. On such a growth path, consumption can grow as long as it stays below the dynamic maximin indicator. Once consumption catches up with the indicator's level, consumption can be sustained only at the maximin level. In this way, *growth* of consumption can be maintained until the eventual, *sustained level* of consumption is reached. There is a choice between the level of present consumption and movement toward a higher level of consumption that can be sustained, given technological and natural conditions.

The maximin indicator is a very-long-run indicator of what is sustainable, of the sort that Solow (1993) seeks. At least two other indicators have been proposed to evaluate sustainability.

On the one hand, *genuine savings* extends the concept of savings in the national accounts to include changes in the quantities of capital goods, especially environmental goods, that do not have market prices.[3] It is equal to the current change in social welfare, which is usually defined to be the integral of discounted social utility. Non-negative genuine savings is sometimes considered to be an indicator of sustainability because current welfare does not decrease. If genuine savings are non-negative it is, however, not possible to say whether welfare will be sustained in the long-run (Asheim, 1994). Even if negative genuine savings means that the current utility is not sustainable, the opposite is not true (Pezzey, 2004). The welfare integral can increase at the current moment but eventually decrease, even if the environment is incorporated into optimal decisions (Dasgupta and Heal, 1979). Moreover, on the trajectory being followed by the economy, the change in social welfare may be negative over a short period of time, but then turn upward. Genuine savings with a discounted utility objective function is not the long-run measure sought in considering sustainability.

[3]The comprehensive vector of capital stocks accounted for is then the same as the vector of capital stocks used to define the maximin value. The value of each stock is, however, different.

On the other hand, the *ecological footprint* has been proposed as an indicator of the environmental limit to sustainable output. It seeks to compare the level of current utilization of environmental resources (i.e., the ecological footprint) with the available flow of environmental services (i.e., the ecological carrying capacity), evaluated in terms of land of a given quality. If the level of utilization is greater than the flow of available services, the society depletes the stock and is considered to be unsustainable at its current level of utilization.

The planning exercise envisaged in the present paper has a flavor of these two approaches. The idea of the footprint is made more comprehensive through the analysis of evolving environmental *and* technological constraints. The current level of consumption corresponds with the environmental-economic footprint. The maximin value may be considered to be a dynamic, *environmental-economic limit or indicator*. As predicted by analyses of the ecological footprint, society faces diminishing long-run prospects, or diminishing sustainability, if consumption exceeds the indicator. Current decisions modify the limits to growth.

Our contribution to the economic analysis of sustainable development is to use the current maximin value as the sustainability indicator, whether or not the planner pursues a maximin path, i.e., whatever the objective of the society and whether or not the economic trajectory is efficient in pursuing that objective. *Sustainable development* is defined, not as non-decreasing of current discounted utility but as non-decreasing of the current maximin value. Sustainability declines when current utility overshoots the maximin value. In a definition of sustainable development, increasing what can be sustained (or what may be called "improving sustainability") is as much a concern as immediate growth.

3 The Economics of Unsustainability

In this section, we illustrate how it is possible to use the maximin value to characterize the unsustainability of a development path in the two canonical economies.

3.1 The Simple Fishery

We first consider a fishery under open access. Denoting the natural rate of growth of the fish stock $S(t)$ at time t by $S(t)[1 - S(t)]$, fishing effort by $E(t)$ and the consumption (harvest) of the resource by $C(t) = S(t)E(t)$, we study the following simple model of the evolution of the stock:[4]

$$\dot{S}(t) = S(t)[1 - S(t)] - S(t)E(t). \tag{2}$$

We assume that the effort E belongs to the interval $[0, 1]$. The open-access regime has $E(t) = E_0 = 1$.

In this model, the highest sustainable level of consumption is called the "maximum sustainable yield" (MSY). Its value is

$$C_{MSY} = \max_S [S(1 - S)] = \tfrac{1}{4}.$$

The associated stock is $S_{MSY} = \frac{1}{2}$ and the equilibrium level of effort is $E_{MSY} = \frac{1}{2}$. In this model, the MSY stock is a benchmark for both ecological and economic overexploitation.[5] If the initial state S_0 is lower than that associated with the MSY, the maximin criterion (1) leads to a constant harvest in equilibrium, $C(t) = S_0(1 - S_0)$. If the initial state is above the MSY level, the maximin value is the MSY harvest. We thus define the maximin value, given the state S at time t, of this economic system as

$$m(S) = \begin{cases} S_{MSY}(1 - S_{MSY}) & \text{if } S > S_{MSY}, \\ S(1 - S) & \text{if } S \leq S_{MSY}. \end{cases} \tag{3}$$

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[4]This model is often written using the parameters r, S_{sup} and q to represent the natural growth rate of the resource, its carrying capacity, and the catchability of the resource, so that

$$\dot{S}(t) = rS(t)\left(1 - \frac{S(t)}{S_{sup}}\right) - qS(t)E(t).$$

In our model, without loss of generality we define units of time, of effort and the resource such that $r = 1$, $S_{sup} = 1$, and $q = 1$. The expressions are less cumbersome, but one must be careful to keep track of the units in which the variables are measured.

[5]We do not consider the cost of effort, for sake of simplicity. It implies that the Maximum Economic Yield (golden rule) coincides with the Maximum Sustainable Yield.

If $S \leq S_{MSY}$, the level of effort, E^{mm}, on a maximin path is such that the harvest is equal to the natural growth, so that $E^{mm} S = S(1-S)$, or $E^{mm} = 1 - S$.

We shall now consider a consumption path that exhausts the resource and is thus unsustainable. For the sake of simplicity, we suppose that $S(0) = 1$ and that the resource is initially in open access, i.e., the effort level is $E_0 = 1$. That level of effort is maintained so long as there is a net benefit to fishing.[6] The dynamics of the exploited resource becomes

$$\dot{S}(t) = S(t)[1 - S(t)] - S(t) = -[S(t)]^2$$

The stock evolves as

$$S(t) = \frac{1}{1+t}. \tag{4}$$

Consumption, $C(t) = E_0 S(t) = 1/(1+t)$, decreases toward zero as the stock decreases toward zero.

To characterize the sustainability of this unsustained path we study the evolution of the maximin value. During a first period ($t \in [0, 1]$), until the stock falls to the level $S(1) = S_{MSY} = \frac{1}{2}$, the stock decreases but is still above S_{MSY}. The maximin value is thus constant at the MSY level. After $t = 1$, the maximin value decreases as the stock decreases below $S_{MSY} = \frac{1}{2}$. An analytical expression of the maximin value as a function of time is easily derived from equation (4) for any $t \geq 1$:

$$m(S(t)) = S(t)[1 - S(t)] = \frac{t}{(1+t)^2}. \tag{5}$$

Fig. 1 (Open Access trajectory of the fishery) presents the evolution of fish stock, consumption rate, and maximin value over time under open access. The

[6] A different story, which would have resulted in the same outcome, is the following. Consider that the resource stock is initially at its carrying capacity, and the initial catch level C_0 is greater than the Maximum Sustainable Yield. As the resource stock is large, it is initially easy to catch C_0, but as the stock is depleted, more effort is needed. Defining the fishing effort as a feedback rule depending on C_0 and $S(t)$, one generates a constant harvesting trajectory, with decreasing stock and increasing effort. After some time, the effort cannot increase above its upper limit. The system then reaches the open-access equilibrium, with decreasing catches.

figure illustrates an important result: once the MSY stock is overshot, the maximin value decreases. We interpret this decrease as an indicator of overshooting of the environmental capacity to provide fish over the very long run.

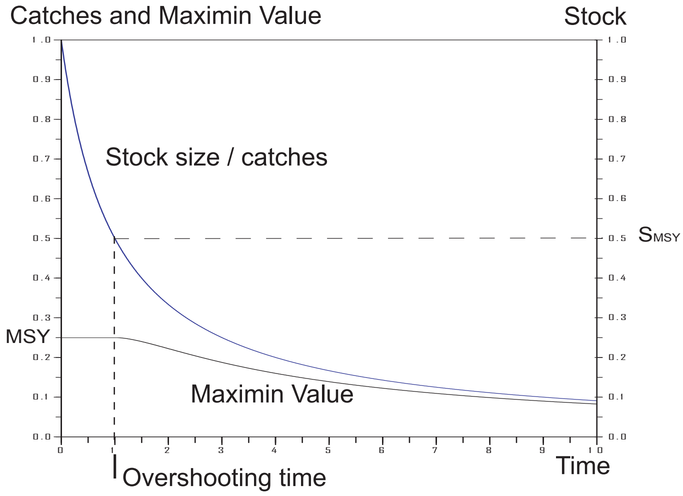

Figure 1: Open Access trajectory of the fishery

3.2 The Dasgupta-Heal-Solow Model

Consider a society that has stocks of a non-renewable resource, S_0, and of a manufactured capital good, K_0, at its disposal at time $t = 0$. It produces output (consumption c and investment $\dot{K}$) by use of the capital stock and by depleting the resource stock at rate

$$r(t) = -\dot{S}(t), \tag{6}$$

according to a Cobb-Douglas production function:

$$c + \dot{K} = F(K, r) = K^{\alpha} r^{\beta}, \text{ with } 0 < \beta < \alpha < \alpha + \beta \leq 1. \tag{7}$$

This model has been used by many authors to study the implications of exhaustibility of an essential resource, including how to sustain consumption in the face of exhaustibility. If the discounted-utility criterion is applied to this economy, consumption decreases asymptotically toward zero (Dasgupta and Heal 1974, 1979). Analysis of how consumption can be sustained requires a different approach from discounted utilitarianism. For given levels of the capital and resource stocks, Solow (1974) and Dasgupta and Heal (1979) show that the maximal consumption that the economy can sustain, the *maximin* level, is given by

$$m(S,K) = (1-\beta)\,(\alpha-\beta)^{\frac{\beta}{1-\beta}}\,S^{\frac{\beta}{1-\beta}}K^{\frac{\alpha-\beta}{1-\beta}}. \tag{8}$$

Since this aggregate of the two stocks measures the capacity of the economy to sustain the standard of living $m(S,K)$ for the long term, we interpret it as the index of sustainability. It is an increasing function of both stocks. Let the initial level of consumption be

$$c\,(0) = c_0 < m\,(S_0, K_0)\,.$$

To illustrate the economics of unsustainability in this model, we assume that the society unmindfully pursues growth at a constant rate $g > 0$, so that the consumption path is

$$c(t) = c_0 e^{gt}. \tag{9}$$

To complete the set of decision rules constituting the *resource–allocation mechanism* of the society, we choose a rate of resource use determined as follows:

$$r(K,S) = (\alpha-\beta)^{\frac{1}{1-\beta}}S^{\frac{1}{1-\beta}}K^{-\frac{1-\alpha}{1-\beta}}. \tag{10}$$

This rule is the optimal feedback rule in a maximin program and is discussed more formally in the next section.

We study the evolution of the maximin value $m[S(t),K(t)]$ along the path defined by eqs. (9) and (10). Fig. 2 (Exponential consumption and Maximin value function) presents a growth path for a given level of c_0 and a value of g.

The development path can be described by three phases.

1. The first phase is before the point labeled "overshooting time." Since at the outset the level of consumption is less than the maximin value $m(S_0,K_0)$,

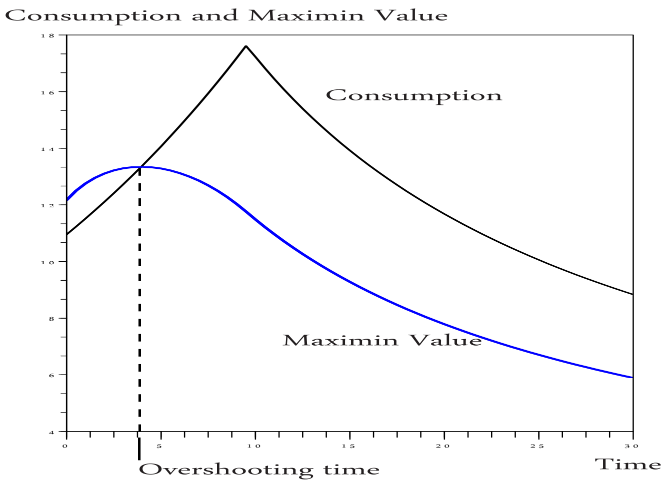

Figure 2: Exponential consumption and Maximin value function

the capital stock can be built faster than along the hypothetical maximin path starting from the same initial state, on which $c(t) = m(S_0, K_0)$, and the sustainable consumption of the economy, i.e., the maximin indicator $m\,(S(t), K(t))$, can increase over time.

2. Once the consumption $c(t)$ is greater than the maximin value $m(S(t), K(t))$, the maximin value decreases, meaning that the sustainable productive capacity of the economy decreases. Consumption growth in this second phase can still, however, be myopically pursued until a consumption peak when the consumption level reaches the total production level and investment is zero.

3. The third phase starts after this consumption peak. Consumption is bounded above by the decreasing level of production. There is no more investment. Consumption decreases toward zero, and remains above the currently sustainable level given by eq. (8).

This example illustrates an overshooting of long-term productive or environmental–economic carrying capacity represented by the maximin value. Consumption represents the economic footprint. As long as the footprint is lower than the carrying capacity, the latter can increase over time as a result of investment in productive capacity. Once the footprint is higher than the productive capacity, however, the decrease of the maximin value indicates an unsustainable development path.

The analysis of these two models suggests that the maximin value can be used as an indicator of unsustainability even when the policy is not to sustain the economy by following the maximin path. Unsustainability occurs when the maximin value decreases. In the next section, we examine how it can characterize sustained development paths in the same two illustrative models. Later we show in fairly general models that it has the same properties along any development path that satisfies certain conditions.

4 Sustained Development

In this section, we examine the conditions for a development path to be sustainable. In the two models under study, we assume that a given growth pattern is pursued. Sustainable development is defined as follows. Consumption increases according to the assumed growth pattern as long as it is lower than the maximin value, which represents generalized economic carrying capacity. When consumption catches up the maximin value, the economy stops growing and follows the maximin path starting from the economic state reached. As the maximin value is dynamic, there is a trade-off between initial consumption, the pursued growth rate, and the duration of the growth period or equivalently the level of sustained consumption that is reached in the long run.

4.1 The Simple Fishery

Let the initial state S_0 be lower than the MSY biomass, i.e., $S_0 < S_{MSY}$, as may have occurred if the economy has been facing a "tragedy of the commons" for some time because of an initial open access to the resource. The stock can be considered to be over-exploited, or vulnerable to over-exploitation, and a poverty trap. If the stock recovers from over-exploitation, the maximin value can increase.

Let a level of effort be chosen and remain constant at the level $E_0 \in]0,1[$. Such a strategy could aim at increasing the available resource and sustainable consumption while maintaining an acceptable level of employment in the fishery. Consumption is given by $C(t) = E_0 S(t)$ and the dynamics of the exploited resource becomes

$$\dot{S}(t) = S(t)\left(1 - E_0 - S(t)\right). \tag{11}$$

Along this trajectory, the stock evolves as

$$S(t) = \frac{1}{\frac{1}{1-E_0} + \left(\frac{1}{S_0} - \frac{1}{1-E_0}\right) e^{-(1-E_0)t}}. \tag{12}$$

The system tends toward a limit, $S_\infty = 1 - E_0$.

The rule of constant effort completely determines the trajectory of this fishery. By equation (3), when $S \leq S_{MSY}$, the maximin level of effort is given by $E^{mm}(S) = 1 - S$. This level of effort maintains the stock at a stationary level that may correspond to a "poverty trap." In order to recover from a period of overfishing, society must harvest less than the maximin harvest $m(S) = S(1-S)$ so that the stock can grow and the maximin value function can increase along the trajectory. This feature of the problem illustrates that there is no "free lunch" for the future. Current effort must be less than E^{mm}, and consumption less than $C^{mm} = S_0(1 - S_0)$.

Under a strategy of constant fishing effort, with $E(t) = E_0 < 1 - S_0 = E^{mm}(S_0)$, fish consumption increases with the stock size. Fig. 3 depicts the following trajectories through time:

- The natural growth of the stock from the initial state $S_0 = 0.1$ (without harvesting).[7]
- The growth of the resource stock with constant fishing effort $E_0 = E^{MSY} = \frac{1}{2}$. The stock tends toward S_{MSY}. This trajectory is labeled "stock recovery."

[7]With no consumption ($C(t) = 0$, i.e., $E(t) = 0$), the dynamics of the resource stock is given

- The trajectory of the maximin value function along the trajectory for $E_0 = \frac{1}{2}$. The maximin value increases toward the MSY level.

- The consumption pattern, which increases as the stock increases and catches up to the maximin value. Consumption tends toward the MSY.

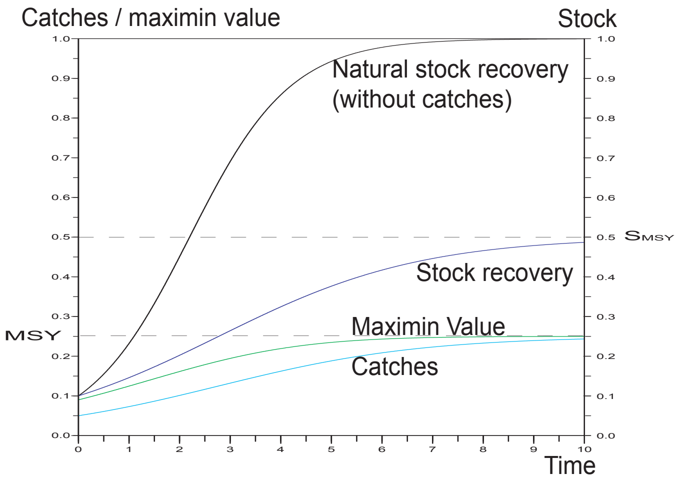

Figure 3: Evolution of the maximin value function along a constant effort trajectory leading to Maximum Sustainable Yield

We stress that the recovery of the fishery (and thus the increase in consumption) is possible only because consumption is lower than the maximin level at all times. The long-run consumption depends on the reduction of the present consumption, the constant fishing effort being between the maximin value $E^{mm}(S_0) = 1 - S_0$ and the MSY value $E^{MSY} = \frac{1}{2}$. A lower fishing effort, and hence current consumption,

by

$$S(t) = \frac{1}{1 + e^{-t}(\frac{1}{S_0} - 1)}. \tag{13}$$

The stock recovers faster, but present generation does not consume at all.

entails a higher long-run consumption.[8] Fig. 4 presents the trajectories of maximin value and catches for three different recovery strategies (for three different effort levels) with, again, an initial fish stock $S_0 = 0.1$. For this stock, the initial maximin value is $0.1(1 - 0.1) = 0.09$.

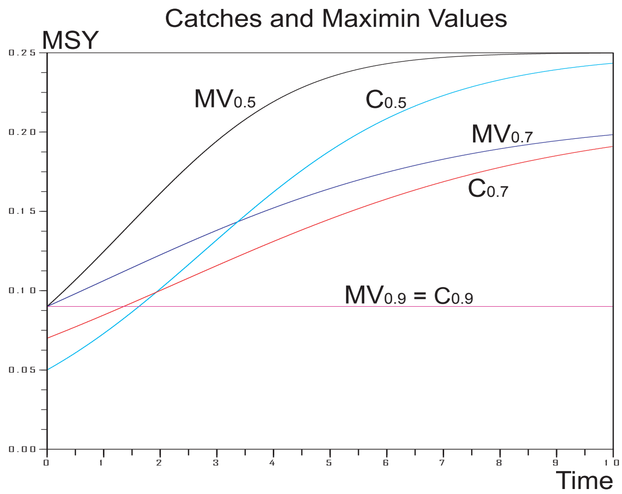

Figure 4: Sensitivity analysis (with respect to the constant effort level)

- The first strategy (trajectories denoted by $M_{0.9}$ and $C_{0.9}$) corresponds to a constant fishing effort $E_0 = E^{mm} = 0.9$. At this effort level, the stock is in equilibrium at the initial value, i.e., $S_\infty = S_0 = 0.1$. The harvest is equal to the maximin value from the initial stock at all times.

- The second strategy (trajectories denoted by $M_{0.7}$ and $C_{0.7}$) corresponds to a constant fishing effort $E_0 = 0.7 < 0.9$. The fish stock increases asymptotically toward a limit, $S_\infty = 1 - E_0 = 0.3$ (not represented on the figure). The harvest increases toward the maximin harvest for this stock, $S_\infty(1 - S_\infty) = 0.21$, which is lower than the MSY.

[8]Effort below $\frac{1}{2}$ are not considered as they would results in lower catches both for present and future generations.

- The third strategy (trajectories denoted by $M_{0.5}$ and $C_{0.5}$) is that depicted in Fig. 3, with the fishing effort set constant at the MSY equilibrium effort, 0.5. The maximin value increases asymptotically toward the MSY value and the harvest increases toward the MSY, which is 0.25.

There is a non-linear relationship between C_0 and C_∞ which is determined by the chosen (constant) effort level. Recovery effort belongs to $[E^{MSY}, E^{mm}(S_0)]$. If the effort is small and equal to E^{MSY}, present consumption is low ($C_0 = E^{MSY} S_0$) and the limiting consumption is the MSY. If the effort is equal to $E^{mm}(S_0)$, the stock remains at the initial level S_0, and the present and limiting consumption are equal. (There is no growth.) This is the maximin path, sometimes criticized as possibly entrenching a poverty trap. Intermediate cases are defined according to the relationship

$$C_\infty = \lim_{t\to\infty} E_0 S(t) = E_0 (1 - E_0) = \frac{C_0}{S_0}\left(1 - \frac{C_0}{S_0}\right), \tag{14}$$

for $C_0 \in [S_0/2, S_0]$, i.e., for $E_0 \in [1/2, 1]$. The possibility frontier between present and future consumption is described by Fig. 5.

Any pair (C_0, C_∞) that is achievable with constant effort belongs to this frontier. Social preferences between present and future consumption can be given by a function $\Psi(C_0, C_\infty)$, which can be maximized along the frontier. Several particular solutions are represented in Fig. 5, including the Green Golden Rule (Chichilnisky, Heal and Beltratti, 1995) corresponding to $\Psi(C_0, C_\infty) \equiv C_\infty$; myopic behavior from open access, corresponding to $\Psi(C_0, C_\infty) \equiv C_0$; and the maximin, corresponding to $\Psi(C_0, C_\infty) \equiv [\min(C_0, C_\infty)]$. Once initial and final consumption are chosen, the (logistic) growth rate is endogenous under the assumption that effort is constant.

4.2 The Dasgupta-Heal-Solow Model

Sustained development in the DHS model can be represented as follows. Suppose that the economy chooses an initial level of consumption c_0 that is less than the sustainability indicator provided by the maximin value $m(S_0, K_0)$. This choice allows for growth. We assume that the social planer aims at pursuing consumption

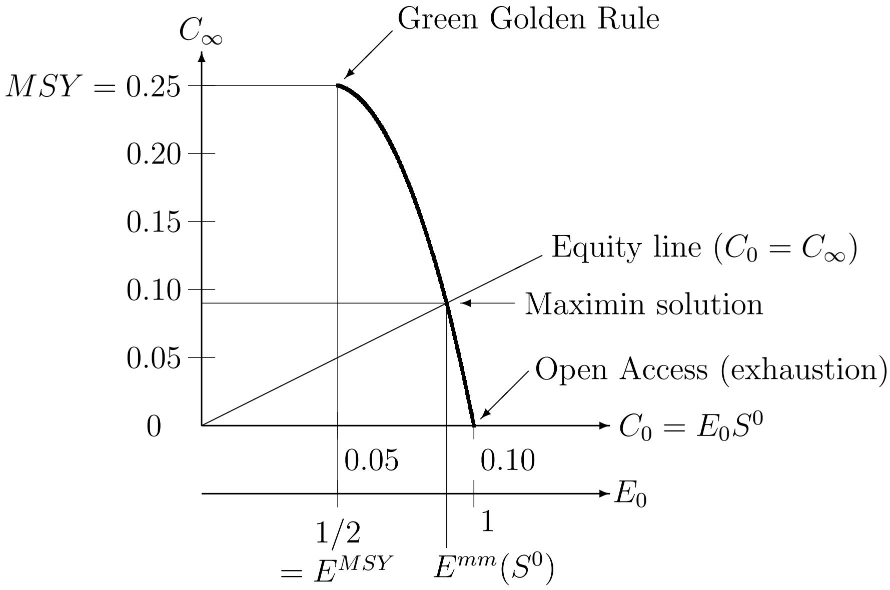

Figure 5: Trade-off between present consumption and long-run consumption in a fishery with constant effort and $S_0 = 0.1$.

growth at a constant rate $g > 0$ until some time T when it reaches the maximin level $m(S(T), K(T))$.[9]

Let us introduce formally the way our problem deviates from the maximin problem, and the resulting resource–allocation mechanism. In a maximin problem, the objective is mathematically expressed as the maximization of the Hamiltonian $H(c, r, S, K) = \lambda\dot{S} + \mu\dot{K}$ subject to the constraint $c\,(t) \geq \bar{c}$, where $\bar{c}$ is the maximin consumption (Cairns and Long, 2006). It is equivalent to maximize the Lagrangean:

$$\begin{aligned} L(c, r, S, K, \nu) &= H(c, r, S, K) + \nu\,(c - \bar{c}) \\ &= \lambda\dot{S} + \mu\dot{K} + \nu\,(c - \bar{c}) \ . \end{aligned}$$

Note that the term $\nu\,(c - \bar{c})$ corresponds to the complementarity slackness condition,

[9]Another possibility is to imagine a path for which the rate of growth smoothly approaches the maximin value. For example, the path followed could be a logistic growth curve. This path would be more difficult to solve than the path proposed in the text but would give no more insight into the problem.

and is always equal to zero. Cairns and Long (2006, Proposition 1) show that the co-state variables of a maximin problem, λ and μ, are equal to the derivatives of the maximin value function with respect to the state variables, i.e., $\lambda = \frac{\partial m}{\partial K}$ and $\mu = \frac{\partial m}{\partial S}$. One thus has $\dot{m}(S,K) = \lambda \dot{S} + \mu \dot{K}$. The previous expression of the Lagrangean is thus equivalent to

$$L(c, r, S, K, \nu) = \dot{m}(S,K) + \nu (c - \bar{c}) \ .$$

The problem is tantamount to maximizing the net investment at maximin shadow values $\dot{m}(S,K)$ subject to the constraint that consumption is no less than the maximin value. In the maximin problem, this maximin value is a parameter of the optimization, and it is increased as much as possible. Hartwick's (1977) rule is that, at the maximum, $H(c,r,S,K) = \dot{m}(S,K) = \lambda \dot{S} + \mu \dot{K} = 0$. The minimal consumption $\bar{c}$ is increased up to the point at which the maximal net investment is nil.

We here deviate from this maximin optimization problem in the sense that we do not maximize the minimal consumption over time. On the contrary, we consider a given consumption pattern. We assume, however, that the social planer does not waste sustainability improvement, and maximizes net investment accounted at the maximin shadow values subject to the consumption pattern constraint.[10] In the present problem, the constraint is that $c(t) = \tilde{c}(t) = c_0 e^{gt}$ and the aim is to maximize the Lagrangean,

$$\begin{aligned} \tilde{L}(c, r, S, K, \tilde{\nu}) &= \dot{m}(S,K) + \tilde{\nu}(c - \tilde{c}), \\ &= -\frac{\partial m}{\partial S} r + \frac{\partial m}{\partial K}(K^{\alpha} r^{\beta} - c) + \tilde{\nu}\left(c - c_0 e^{gt}\right), \end{aligned}$$

That is to say, the program is to maximize $\dot{m}(S,K)$ subject to the modified constraint.[11]

[10]We interpret this objective and the associated resource allocation mechanism in the general model of next section.

[11]Note that the shadow values are the same as that of the maximin problem, and that the modified complementarity slackness condition is also equal to zero.

Resource-allocation mechanism. The society chooses the level of extraction of the resource that maximizes $\dot{m}$, the net investment at the maximin shadow values, conditional on the consumption path.

As the consumption pattern is given by equation

$$c\,(t) = c_0 e^{gt} \tag{15}$$

this resource allocation mechanism defines the natural resource extraction. By differentiating the maximin value function (eq. 8) logarithmically with respect to time t, we express the rate of growth of the maximin value as

$$\frac{\dot{m}}{m} = \left[\frac{\alpha - \beta}{1-\beta}\frac{\dot{K}}{K} + \frac{\beta}{1-\beta}\frac{\dot{S}}{S}\right] = \left[\frac{\alpha - \beta}{1-\beta}\frac{\dot{K}}{K} - \frac{\beta}{1-\beta}\frac{r}{S}\right]. \tag{16}$$

Using this derivative we compute the extraction rule $\hat{r}(K, S)$ that maximizes the rate of growth of the maximin value (whatever is the consumption), and find that it is given by

$$\hat{r}(K, S) = (\alpha - \beta)^{\frac{1}{1-\beta}} S^{\frac{1}{1-\beta}} K^{-\frac{1-\alpha}{1-\beta}}. \tag{17}$$

This feedback rule is the same as the one along the maximin path, for which growth of both consumption and maximin value are zero. For this resource–allocation mechanism, the consumption side of the economy is determined by the growth pattern, and the production side is determine so as to maximize the instantaneous gain of sustainable utility.

The limits to growth. There is an endogenous limit to the time for which growth can be supported at rate g, given the described resource allocation mechanism. In fact, the long-run level of consumption is endogenous. To avoid the unsustainable type of trajectory described in previous section, the economy must switch at some time T from the constant-growth path to a maximin path characterized by constant consumption $c_\infty \equiv m(S(T), K(T))$. The program is an open-loop path, determined at time 0. Fig. 6 illustrates such a sustained-development path.

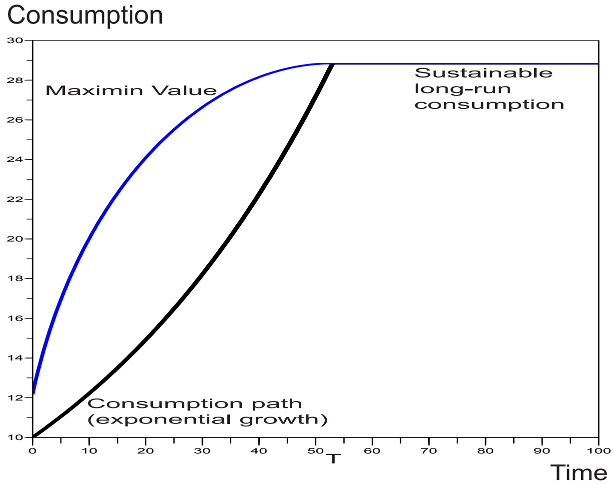

Figure 6: Exponential consumption and Maximin value function

Sustained growth at rate $g > 0$ demands that $c_0 < m(S_0, K_0)$. It will become clear how the growth rate and the duration of the growth period are linked to the initial consumption and the long-run, sustained consumption. If two of the four are given, the two others can be derived.

For any initial pair $(S_0, K_0) \gg 0$, there is a maximin level of consumption $m_0 = m(S_0, K_0) > 0$ given by equation (8). Also, for any initial pair of stocks it is possible at time $t = 0$ to choose any pair

$$(c_0, g) \in A(S_0, K_0) \triangleq \{]0, m_0[\times]0, \infty[\cup (m_0, 0)\}$$

The path in which $(c_0, g) = (m_0, 0)$ is the maximin (sustained) path. It has no growth. A path in which $(c_0, g) \in \{]0, m_0[\times]0, \infty[\}$ (so that $g > 0$ and $c_0 < m_0$) has growth. However, growth at a constant rate cannot go on forever.[12]

Let us define the endogenous time $T(S_0, K_0, c_0, g)$ at which consumption catches

[12]From a theoretical point of view, the described framework could be used to give a rigorous meaning to "sustainable degrowth" from an initial consumption larger than the maximin value and a negative growth rate.

up to the dynamic maximin value indicator. We have

$$c(T(\cdot)) = c_0 e^{gT(\cdot)} = m\left[S(T(\cdot)), K(T(\cdot))\right].$$

From then on, growth is no longer sustainable, and the level of consumption must remain at the maximin level; i.e., for $t \geq T(.)$, sustainability implies that $c(t) = m(S(T), K(T))$.

At time T, only the part of the resource–allocation mechanism that drives the level of consumption changes, from allowing consumption to grow at rate g to keeping consumption constant at $c_\infty = m(S(T), K(T))$. Resource use is still determined by the maximin efficiency definition.

One may view society as making a choice according to a preference ordering $\mathcal{P}(c_0, g, c_\infty)$, by which initial consumption, the rate of growth and the very long-run, sustained consumption are evaluated. Fig. 7 depicts a convex-concave correspondence from the initial pair (S_0, K_0) to the attainable frontier, $B(S_0, K_0) \triangleq \{(c_0, g, c_\infty) \text{ feasible from } (S_0, K_0)\}$. Growth is possible only if $c_0 < m(S_0, K_0)$.[13] For a given growth rate, a lower level of initial consumption allows a higher long-run level. For a given initial consumption, a lower growth rate allows a higher long-run consumption (as the actual consumption catches the maximin level more slowly). Given the initial level of consumption c_0, there is a trade-off between the eventual maximin consumption that is sustained after time T and the rate of growth that is sustained up to that level. A level of present consumption that is closer to the maximin value $m(S_0, K_0)$ entails a lower prospect for growth.

5 A General Measure of Sustainability

The general trade-off envisaged in the present paper is between social utility at time t and the ultimate utility reached in the long run. The long-run level is endogenous when one has chosen a growth pattern, as is illustrated in both models above. In the tradition of Ramsey's (1928) model of undiscounted utility, some authors assume

[13]Negative growth ($g < 0$) is required if $c_0 > m(S_0, K_0)$. (This is not represented on the figure.)

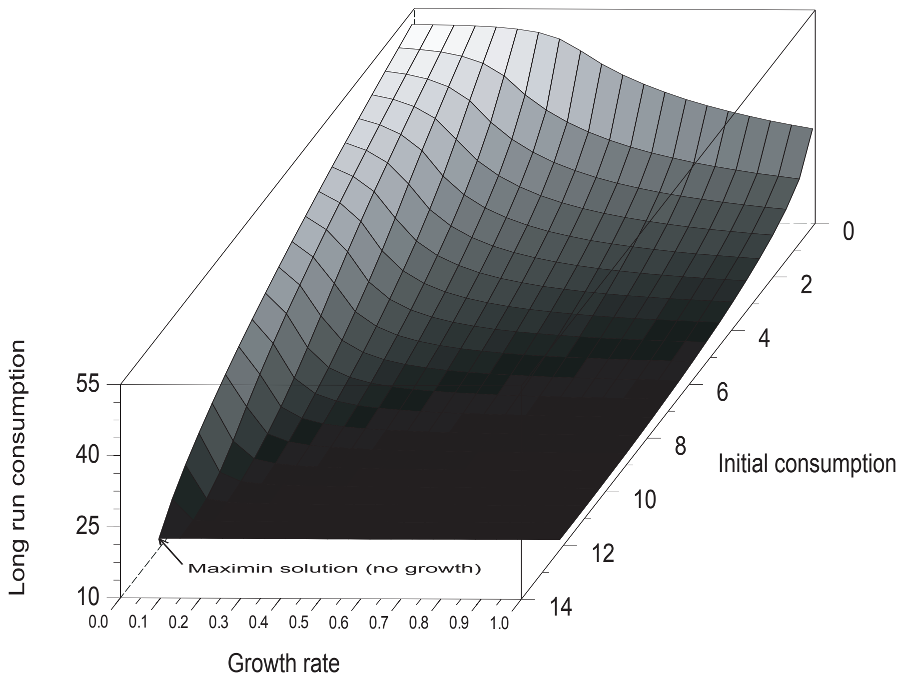

Figure 7: Necessary trade-offs between initial consumption, growth rate, and long-run (sustained) consumption in the DHS model

that growth leads the economy toward a bliss utility level (see, e.g., d'Autume and Schubert (2008) for an analysis of the DHS model in this framework). The exogenous bliss level of utility coincides with the (green) golden rule and is approached asymptotically. Our approach contrasts with this view. In our fishery model the bliss level is the MSY, which is not necessarily the long-run level chosen by the society. In the DHS model there is no exogenous bliss level and the long-run consumption is also a social choice.

In the examples above, changes in the indicator, the maximin value, have a clear meaning in terms of the sustainability of the society along the chosen trajectory.

If consumption is lower than the maximin value over some interval and the output so freed up is invested in long-term productive capacity, the sustainable level of consumption can be permanently increased. Conversely, consumption over a current interval can be increased at the expense of investment and hence of sustainable consumption in the future.

The present section examines the generalization of this conclusion to more general problems. Consider a vector of capital stocks $X \in \mathbb{R}_+^n$. The transition equation for each element X_i, $i = 1, \ldots, n$ is given by

$$\dot{X}_i = F_i (X, c) \ .$$

where $c \in C(X) \subseteq \mathbb{R}^p$ represent a vector of decision within the set $C(X)$ of admissible controls at state X. The maximin value is denoted by $m(X) \in \mathbb{R}_+$. For any feasible set of controls $c = (c_1, ..., c_n)$, the evolution of the maximin value is given by

$$\mu (X, c) = \dot{m} (X) |_c = \sum_{i=1}^{n} \dot{X}_i \frac{\partial m (X)}{\partial X_i} = \sum_{i=1}^{n} F_i (X, c) \frac{\partial m (X)}{\partial X_i}.$$

The terms $\frac{\partial m(X)}{\partial X_i}$ are the maximin shadow values of state X, and depend only on the current state and not on the economic decisions. They are thus defined whatever the economic trajectory given by functions $F_i(X, c)$.
A sustainable growth program is defined as follows.

Definition 1 Maximization of sustainability improvement. *The resource-allocation mechanism is said to maximize sustainability improvement at each instant if decisions c maximize the increase of the maximin value subject to the given growth pattern:*

$$\begin{aligned} c \quad \text{maximizes} \quad & \mu (X, c) = \sum_{i=1}^{n} F_i (X, c) \frac{\partial m (X)}{\partial X_i} \\ \text{s.t.} \qquad & U(X, c) = \bar{U}(t) \end{aligned} \tag{18}$$

The interpretation of this resource allocation mechanism is that, at each instant, the current generation increases the limit to growth as much as possible, given its current utility defined by the assumed growth pattern.

We aim to prove that, if sustainability improvement is maximized, having a lower level of utility than the maximin utility leads to an increase in the maximin value.[14] Note that the resulting path is not necessarily efficient in the usual sense.[15] This does not, however, diminish the importance of the present result. We prove that, for the growth to be sustainable, there must be a sacrifice of present utility with respect to the maximin value, and that this necessary condition is also sufficient if the resources freed up by the reduction of utility are invested optimally to increase the productive capacity of the economy.

We provide a proof for problems satisfying the following assumptions.

Assumption 1. There is a $j \in \{1, \ldots, p\}$ such that, given a state vector X and a decision vector $c = (c_1, \ldots, c_j, \ldots, c_p)$, one has $U'_{c_j} > 0$.

Assumption 1 means that there is at least one control variable which influences utility continuously around a given level. We assume that the control has a positive effect on utility for the sake of simplicity, i.e., $U'_{c_j} > 0$. Our result, however, holds for a negative effect, by redefining c_j as $-c_j$. Note that when Assumption 1 is satisfied, there is an interval $[\underline{c}_j, \bar{c}_j]$, such that $U\big((c_1, \ldots, \underline{c}_j, \ldots, c_p), X\big) < U\left((c_1, \ldots, \bar{c}_j, \ldots, c_p), X\right)$.

Assumption 2. For all capital stocks X_i, $i = 1, ..., n$,

$$\frac{\partial m\,(X)}{\partial X_i} \frac{\partial F_i\,(X, c)}{\partial c_j} \leq 0\ . \tag{19}$$

Assumption 2 means that the control c_j does not increase (decrease) investment in stocks having a positive (negative) contribution to the maximin value.

Assumption 3. For at least one capital stock (think of "manufactured capital" for

[14]The opposite result, i.e., having a higher level of utility than the maximin utility leads to an decrease in the maximin value, is straightforward from the usual maximin problem.

[15]In particular, it does not maximize the long-run utility given the initial consumption and growth pattern. The definition of such an efficient sustainable growth path is a task for future research.

concreteness), the condition in Assumption 2 is strictly satisfied:

$$\frac{\partial m\left(X\right)}{\partial X_i}\frac{\partial F_i\left(X,c\right)}{\partial c_j} < 0\ . \tag{20}$$

Assumption 3 means that the control c_j has an effect on the maximin value.

A simple example is if the control c_j is consumption in the DHS model, consumption increases utility and comes from forgone investment in manufactured capital. Investment in manufactured capital contributes directly to the maximin value and does not affect the change in the resource.

These assumptions are, in fact, quite general. If assumption 1 is not satisfied, utility does not depend on the decisions at the considered economic state.

Assume that Assumption 2 is not satisfied, i.e., there is some $i \in \{1,\ldots,n\}$ such that $\frac{\partial m(X)}{\partial X_i}\frac{\partial F_i(X,c)}{\partial c_j} > 0$, while Assumption 1 is. It would then be possible to increase both current utility and the maximin value by increasing c_j. In that case, the maximin problem has no solution as there would be always one control increasing utility and the maximin. Assumptions 1 and 2 entail that there is no free lunch in a maximin problem.

Assume that Assumption 3 is not satisfied while Assumptions 1 and 2 are, i.e, $\frac{\partial m(X)}{\partial X_i}\frac{\partial F_i(X,c)}{\partial c_j} = 0$ for $i = 1,...,n$. It would then be possible to increase the utility at current time without modifying the maximin value. The problem is non-regular in this state (Cairns and Tian 2010, Martinet and Doyen 2011). Taken together, Assumptions 1-3 simply mean that our result is valid on the regular parts of maximin problems, i.e., around states for which the maximin value is affected by the current decisions.

Proposition 1 *On an trajectory with maximal sustainability improvement in which Assumptions 1-3 hold,*

$$\text{sgn}\ \dot{m}\left(X(t)\right) = \text{sgn}\ \left[m\left(X\left(t\right)\right) - \bar{U}\left(t\right)\right].$$

Proof of Proposition 1 *Denote by* $c^m(X(t)) = \left(c_1^m(X(t)), \ldots, c_j^m(X(t)), ..., c_p^m(X(t))\right)$ *the controls associated with the hypothetical maximin program starting from the stocks* $X(t)$ *at time* t. *Since the maximin value does not decrease in a maximin program, for these controls the change of the maximin value in that problem,* $\mu(X(t), c^m(X(t)))$, *is non-negative and given by*

$$\mu(X, c^m) = \sum_{i=1}^{n} F_i(X, c^m) \frac{\partial m(X)}{\partial X_i} \geq 0. \tag{21}$$

Let the state at time t *be given and the chosen utility level be equal to* $\bar{U}(t) < m(X(t))$, *exogenously fixed by a chosen development pattern. Consider the set of decisions* $\mathcal{C}(X(t))$ *that make it possible to attain exactly utility* $\bar{U}(t)$:

$$\mathcal{C}(X(t)) = \left\{c \left| U(X(t), c) = \bar{U}(t)\right.\right\} \subset C(X(t)).$$

Under Assumption 1, there is a vector of decisions $\tilde{c} = (c_1^m, \ldots, \tilde{c}_j, \ldots, c_n^m) \in \mathcal{C}(X(t))$ *that achieves the utility constraints and differs from the maximin decisions* $c^m(X(t))$ *only for the* j^{th} *control.*[16] *If the decisions* $\tilde{c}$ *are applied, then*

$$\mu(X(t), \tilde{c}) = \sum_{i=1}^{n} F_i(X(t), \tilde{c}) \frac{\partial m(X(t))}{\partial X_i}.$$

Comparing this expression with $\mu(X(t), c^m(X(t)))$ *yields*

$$\begin{aligned} \mu(X, \tilde{c}) - \mu(X, c^m(X)) &= \sum_{i=1}^{n} \left[F_i(X, \tilde{c}) - F_i(X, c^m)\right] \frac{\partial m(X)}{\partial X_i} \\ &= \sum_{i=1}^{n} \left[\frac{F_i(X, \tilde{c}) - F_i(X, c^m)}{\left(\tilde{c}_j - c_j^m\right)} \left(\tilde{c}_j - c_j^m\right)\right] \frac{\partial m(X)}{\partial X_i} \end{aligned} \tag{22}$$

As the functions $F_i(X, c)$ *are continuous and differentiable in* c_j, *we know from the mean value theorem (Lagrange's finite-increment theorem) that, for* $i = 1, \ldots, n$,

[16] If it is not possible to modify c_j sufficiently to reduce utility to level $\bar{U}(t)$, i.e., if $\tilde{c}_j \notin [\underline{c}_j, \bar{c}_j]$, one needs to proceed by steps, and take the intermediate value $U\left((c_1^m, \ldots, \underline{c}_j, \ldots, c_p^m), X\right)$ as a reference (instead of $\bar{U}(t)$) in the previous definition of admissible states and in what follows, and repeat the process (with a new control) until utility is reduced to $\bar{U}(t)$.

there is a value $\mathfrak{c}_j^i \in (\tilde{c}_j, c_j^m)$ *if* $\tilde{c}_j < c_j^m$, *or* $\mathfrak{c}_j^i \in (c_j^m, \tilde{c}_j)$ *if* $\tilde{c}_j > c_j^m$ *such that* $\frac{F_i(X,\tilde{c})-F_i(X,c^m)}{\left(\tilde{c}_j - c_j^m\right)} = \frac{\partial F_i\left(X,\mathfrak{c}^i\right)}{\partial c_j}$. *Eq. (22) becomes*

$$\begin{aligned}\mu(X,\tilde{c}) - \mu(X, c^m(X)) &= \sum_{i=1}^{n}\left[\frac{\partial F_i(X,\mathfrak{c}^i)}{\partial c_j}\left(\tilde{c}_j - c_j^m\right)\right]\frac{\partial m(X)}{\partial X_i} \\ &= \left(\tilde{c}_j - c_j^m\right)\sum_{i=1}^{n}\frac{\partial F_i(X,\mathfrak{c}^i)}{\partial c_j}\frac{\partial m(X)}{\partial X_i}\ .\end{aligned}$$

By Assumptions 2 and 3, the sum term is strictly negative. Since, by Assumption 1, $U'_{c_j} > 0$, *one necessarily has that* $\tilde{c}_j < c_j^m$ *since* $\bar{U}(t) < m(X(t))$, *and thus* $\left(\tilde{c}_j - c_j^m\right) < 0$. *The product of these two negative terms is thus positive, and one concludes that* $\mu(X,\tilde{c}) - \mu(X, c^m(X)) > 0$. *Since* $\mu(X(t), c^m(X(t))) \geq 0$, *one deduces that* $\mu(X(t),\tilde{c}) > 0$.[17]

Under the resource-allocation mechanism that maximizes sustainability improvement, the path satisfies the following problem:

$$\max_{c\in\mathcal{C}(X(t))}\sum_{i=1}^{n}F_i(X(t),c)\,\frac{\partial m(X(t))}{\partial X_i}\ . \tag{23}$$

Since $\tilde{c} \in \mathcal{C}(X(t))$, *one has*

$$\max_{c\in\mathcal{C}(X(t))}\mu(X(t),c) \geq \mu(X(t),\tilde{c}) > 0.$$

QED.

The path of the economy is said to be a sustainable development at time t if $\dot{m}\,|_t \geq 0$ and $\dot{u}(t) \geq 0$. Under Assumptions 1 and 2, and given the maximization of sustainability improvement, a necessary and sufficient condition for sustainable development is that $u(t) \leq m\,|_t$. This condition, applied to the maximin value, is what Pezzey (1997) uses to define sustainability. Sustainable utility decreases ($\dot{m}\,|_t < 0$) if $u(t) > m\,|_t$.

[17] If $\bar{U}(t)$ was not feasible by modifying only one control, one must iterate the reasoning to show the successive additional improvements of maximin value with respect to the previous vector of decisions, until $\bar{U}(t)$ is reached, and the associated control belongs to $\mathcal{C}(X(t))$. See footnote 16.

In the regular part of the fishery, the control, E, can be used to increase or to decrease the level of consumption, $C(S,E) = SE$. Moreover, $\partial C(S,E)/\partial E = S > 0$, and $\partial F(S,E)/\partial E = \partial [S(1-S) - SE]/\partial E = -S < 0$. The proposition applies.

In the DHS model there are two stocks, resource S and manufactured capital K, and two controls, consumption c and extraction r. Utility is given by $U(S,K,c,r) = \bar{c}$. Moreover, one has $(\partial F_1/\partial c)(\partial U/\partial c) < 0$. Reducing consumption allows for an increase in the capital stock and an increase in the maximin value. Again, the proposition applies.

Since the maximin value m is a function of the stocks X, its change through time *on any path*,

$$\mu(X,c) = \sum_i \frac{\partial m(X)}{\partial X_i} \dot{X}_i,$$

is a weighted measure of change in the stocks. Sustainable growth entails positive net investment when evaluated at the sustainability prices $\partial m/\partial X$. The current level of well being is not being sustained if $\mu(X,c) < 0$, i.e., if net investment at sustainability prices is negative. The proposition confirms that a policy of sustainable growth in an efficient program costs the current generation as compared to pursuing a maximin policy; sustainable growth occurs only if $u < m$. There is no free lunch for the future.

The criterion involving net investment closely resembles the instantaneous criterion that has erroneously been applied to genuine savings or genuine investment as determined from a green extension of the national accounts (e.g., World Bank, 2006; Dasgupta, 2009). Green accounting is an improvement to the traditional national accounts in that it generalizes them to included non-marketed goods. The issue regarding sustainability, however, turns not solely on the assets to be included but also on the shadow or accounting prices at which investment is evaluated. We disagree with the World Bank (2006: 41) when they write, "Economic theory tells us that there is a strong link between changes in wealth and the sustainability of development—if a country (or a household, for that matter) is running down its assets, it is not on a sustainable path. For the link to hold, however, the notion of wealth must be truly comprehensive." It is not enough for the notion of wealth to be comprehensive (to include all assets, not just marketed assets). In sustainability analysis it is equally vital *to get the accounting prices right.* An increase of the integral of discounted utility implies that genuine savings, computed at competitive prices, are positive at a given instant. However, constancy or increase of welfare signaled by nonnegative genuine savings may not be lasting or durable. Rather, the genuine savings indicator can be positive along a competitive path even though consumption exceeds the maximin level (Asheim, 1994). Welfare measured as discounted utility may ultimately turn downward in spite of the positive, current, genuine savings (Pezzey, 2004). Even though it is comprehensive, genuine savings as it is usually computed, i.e., using discounted utilitarian prices, is not equal to investment evaluated at the shadow values of the maximin program, $\sum_i \frac{\partial m(X)}{\partial X_i} \dot{X}_i$. We distinguish genuine investment, be it applied to maximized social welfare or the level of welfare generated by the resource–allocation mechanism of a real economy, from investment calculated from the maximin value by calling the latter sustaining or *durable investment.*

Durable investment is the indicator of the current change in sustainability. It is comprehensive investment evaluated at maximin shadow prices, along any particular path of the economy. It is the statistic that is appropriate in expressing sustainability improvement. For sustained development at t the economy must have both $\dot{u}(t) \geq 0$ and $\dot{m}(t) \geq 0$. This last condition means that the maximal sustainable utility, i.e., the set of sustainable utility opportunities for future generations, increases at the current time. Current growth does not jeopardize the capacity of future generations to sustain utility.

According to the generalized concept of genuine savings indicator formalized by Asheim (2007), non-negative net investment (accounted at the shadow values of a given welfare function) is associated with non-decreasing welfare at the current time. There is, however, no normative reason to have a non-negative net investment when welfare is defined as discounted utility. Discounted utility does not *require* non-negative investment. Maximin does. Non-negative investment at maximin prices is a characteristic of maximin paths, and thus of the maximin value function. Pursuing non-negative investment at maximin prices, even in a sub-optimal economy, is consistent with sustainability and with the optimality concept of maximin. Pursuing non-negative investment at discounted utility prices is not a criterion for sustainability and is inconsistent with the optimality concept of discounted utility.

6 Conclusion

The discussion stresses a property of a growth path that is not stressed by proponents of sustainable growth out of poverty. If the maximin path is not pursued, but instead some growth path is followed, then earlier generations must be deprived in order to divert toward investment the resources needed to sustain growth. Whether this deprivation is consistent with the vague criterion enunciated in the Brundtland report in terms of "needs" is not obvious. Growth is possible only at a cost. Open access, which in abstract terms is the main environmental problem facing humanity, is an inefficiency that cannot be overcome without current sacrifice. Growth is possible only within limits given by the technology and the environment. Otherwise, it can cause overshooting.

We come to affirm the conclusion drawn by Solow (1993: 172): From an empirical point of view it makes sense to approach sustainability from the dual, that is to say, to use the approach of the footprint rather than the one based on the national accounts. The reason has to do with the prices obtained from extending national accounting toward green accounting. The prices of green accounting, which are the shadow values for discounted utility, are not the "right" accounting prices. The right accounting prices are the maximin shadow values, which are based on what is sustainable. The ecological footprint uses physical measures that can be measured correctly. Through its set of explicit trade-offs that make land the numeraire, ecological footprint analysis has implied a form of substitutability among natural and other stocks. The ecological footprint has no explicit objective, although an implicit objective is to sustain the society. This lack of an explicit objective is what leads to the derivation of accounting prices from the (natural) constraints facing the society. A dynamic and fully comprehensive footprint, using physical measures, would be dual to a measure based on prices. Pricing in units of land can be interpreted as a pricing system that is equivalent to pricing with a specific numeraire. Maximin analysis puts the insights of the ecological footprint on a sounder, more comprehensive footing, based not on land capacity but on "generalized capacity to produce economic well-being" (Solow, 1993).

The definition of durable savings "works" for any resource-allocation mechanism. But durable savings must be evaluated at "the right prices", the durable (or sustaining) prices. If there is a suspicion that the market is not producing a sustainable result, then market prices are likely wrong. How to get the prices is a difficult question, even in very simple models. The difficulty is no reason to use genuine savings with discounted utilitarian prices to measure long-term sustainability. This practice can be misleading and send an incorrect message as genuine savings can be positive even if current utility exceeds the maximal sustainable utility (Asheim, 1994; Pezzey, 2004), and the maximin value indicator is decreasing.

The indicator of sustainability on any program, optimal or not, is the maximin value. Durable investment, the change in the maximin value, is the indicator of whether or not the level of well-being that can be sustained is increasing or decreasing.

References

[1] Asheim, G. (1994), "Net National Product as an Indicator of Sustainability," *Scandinavian Journal of Economics* 96: 257-265.

[2] Asheim, G. (2007), "Can NNP be used for welfare comparisons?," *Environment and Development Economics*, 12(1):11-31.

[3] Asheim, G., W. Buchholz, J. Hartwick, T. Mitra and C. Withagen (2007), "Constant Savings Rates and Quasi-Arithmetic Population Growth under Exhaustible Resource Constraints," *Journal of Environmental Economics and Management* 53:2, 213-229.

[4] D'Autume, A. and K. Schubert (2008), "Zero Discounting and Optimal Paths of Depletion of an Exhaustible Resource with an Amenity Value", *Revue d'Economie Politique* 119:6, 827-845.

[5] Burmeister, E. and P. Hammond (1977), "Maximin Paths of Heterogeneous Capital Accumulation and the Instability of Paradoxical Steady States," *Econometrica* 45: 853-870.

[6] Cairns, R. and N. V. Long (2006), "Maximin: A Direct Approach to Sustainability", *Environment and Development Economics* 11, 275-300.

[7] Cairns, R. and H. Tian (2010), "Sustained Development of a Society with a Renewable Resource", *Journal of Economic Dynamics and Control* 34:6, 1048-1061

[8] Chichilnisky, G., G. Heal and A. Beltratti (1995), "The Green Golden Rule", *Economics Letters* 49(2): 175-179.

[9] Dasgupta, P. (2009), "The Welfare Economic Theory of Green National Accounts," *Environmental and Resource Economics* 42: 3-48.

[10] Dasgupta, P. and G. Heal (1974), "The Optimal Depletion of Exhaustible Resources," *Review of Economic Studies* 41, Symposium Issue, 3-28.

[11] Dasgupta, P. and G. Heal (1979), *The Economics of Exhaustible Resources*, Nisbet, Cambridge.

[12] Dasgupta, P. and K.-G. Mäler (1990), "The Environment and Emerging Development Issues," *Proceedings of the World Bank Conference on Development Economics*: 101-132, World Bank, Washington DC

[13] Dasgupta, P. and K.-G. Mäler (2000), "Net National Product, Wealth and Social Well Being, *Environment and Development Economics* 5: 69-94.

[14] Doyen, L. and V. Martinet (2010), Maximin, Viability and Sustainability, EconomiX Working Papers No 2010-19, University of Paris West - Nanterre la Dfense, http://econpapers.repec.org/RePEc:drm:wpaper:2010-19.

[15] Frank, R. and R. Hutchens (1993), "Wages, Seniority, and the Demand for Rising Consumption Profiles," *Journal of Economic Behavior & Organization* 21, 3: 251-276

[16] Hartwick, J. (1977), "Intergenerational Equity and the Investing of Rents from Exhaustible Resources", *American Economic Review* 67: 972-974.

[17] Hicks, J. (1946), *Value and Capital*, Oxford, Clarendon Press.

[18] Léonard, D. (1981), "The Signs of the Co-State Variables and Sufficiency Conditions in a Class of Optimal Control Problems", *Economics Letters* 8: 321-325.

[19] Llavador, H., J. Roemer and J. Silvestre (2008), "A Dynamic Analysis of Human Welfare in a Warming Planet," mimeo., Yale University.

[20] Lowenstein, G. and N. Sicherman (1991), "Do Workers Prefer Increasing Wage Profiles," *Journal of Labor Economics* 9, 1: 67-84.

[21] Martinet, V. and L. Doyen (2007), "Sustainability of an Economy with an Exhaustible Resource: A Viable Control Approach", *Resource and Energy Economics* 29: 17-39.

[22] Pezzey, J. (1997), "Sustainability Constraints versus 'Optimality' versus Intertemporal Concern and Axioms vs. Data," *Land Economics* 73, 4: 448-466.

[23] Pezzey, J. (2004), "One-sided sustainability tests with amenities, and changes in technology, trade and population," *Journal of Environmental Economics and Management* 48(1):613-631.

[24] Ramsey, F. (1928), "A Mathematical Theory of Saving," *Economic Journal* 38: 543-559.

[25] Solow, R. (1974), "Intergenerational Equity and Exhaustible Resources," *Review of Economic Studies* 41, Symposium Issue: 29-45.

[26] Solow, R. (1993), "An Almost Practical Step Toward Sustainability," *Resources Policy* 19, 162-172.

[27] World Bank (2006), *Where is the Wealth of Nations? Measuring Capital for the Twenty-First Century*

[28] World Commission on Environment and Development (1987), *Our Common Future* (The Brundtland Report), Oxford University Press.