



<http://economix.u-paris10.fr/>

Document de Travail Working Paper 2010-22

EQUITY RISK PREMIUM AND TIME HORIZON : WHAT DO THE U.S. SECULAR DATA SAY?

Georges PRATS



Université de Paris Ouest Nanterre La Défense
(bâtiments T et G)
200, Avenue de la République
92001 NANTERRE CEDEX

Tél et Fax : 33.(0)1.40.97.59.07
Email : nasam.zaroualete@u-paris10.fr

université
Paris Ouest
Nanterre La Défense

EQUITY RISK PREMIUM AND TIME HORIZON : WHAT DO THE U.S. SECULAR DATA SAY?

Georges Prat *

November 2010

Abstract - An ex-ante equity risk premium is the difference between the expected return of a risky asset at time t for a given future time horizon and an equivalent maturity risk-free interest rate. Using annual US secular data from 1871 to 2008, this study aims to model simultaneously the measures and the explanations of ex-ante equity risk premia for two polar horizons: the one period ahead horizon (i.e. the “short term” premium) and the infinite time horizon (i.e. the “long term” premium). Expectations being represented by traditional adaptive processes, large disparities in the dynamics of the two premia are evidenced. According to the conditional CAPM, each premium is at time t explained by the product of the price of risk by the expected variance of returns, these two magnitudes being horizon dependant. The expected variances depend on the past values of the centered squared returns (we found 5 and 8 years for the one year and the infinite horizon, respectively). For each horizon, the price of risk is determined by a spread of interest rates capturing economic factors of uncertainty and by an unobservable variable determined according to the kalman filter methodology (i.e. a state variable). The state variables are supposed to capture the influence of hidden variables and of non directly measurable psychological effects. The model gives a valuable representation of the “short term” and “long term” premia.

Résumé - Une prime de risque ex-ante est définie par la différence entre la rentabilité anticipée à la date t d'un actif risqué pour un horizon de placement donné, et le taux d'intérêt d'un titre de créance sans risque ayant une maturité égale à cet horizon. En utilisant les données séculaires annuelles américaines sur la période 1871-2008, cet article a pour objectif de modéliser simultanément les mesures et les explications des primes de risque ex-ante des actions pour deux horizons pôliaires : l'horizon d'une période (« prime « de court terme ») et horizon infini (prime « de long terme »). Les anticipations étant représentées par des processus adaptatifs traditionnels, on constate des différences importantes entre ces deux primes au cours de la période. Suivant le MEDAF conditionnel, à la date t , chaque prime est expliquée par le prix du risque multiplié par la variance anticipée des rentabilités, ces grandeurs étant paramétrées par l'horizon. Les variances anticipées sont supposées dépendre des valeurs passées du carré des rentabilités centrées (on trouve une durée de 5 ans et 8 ans pour la prime de court terme et la prime de long terme, respectivement). Pour chaque horizon, le prix du risque est déterminé par un « spread » de taux d'intérêt capturant les facteurs économiques de l'incertitude et par une variable inobservée déterminée suivant la méthode du filtre de Kalman (i.e. une variable d'état). Les variables d'état sont supposées capturer à la fois les influences de variables cachées et d'effets psychologiques non directement mesurables. Le modèle donne une représentation valable des primes de court et de long terme.

JEL classification : D81 ; D84 ; E44 ; G11 ; G12

Key words: **equity risk premium, time horizon**

* EconomX, Centre National de la Recherche Scientifique (CNRS) and University of Paris Ouest Nanterre La Défense, 200 avenue de la République, 92001 Nanterre, France. Email : georges.prat@u-paris10.fr

EQUITY RISK PREMIUM AND TIME HORIZON : WHAT DO THE U.S. SECULAR DATA SAY?

1 – Introduction

A thorough understanding of “the” market equity risk premium is a major stake both for theoretical and empirical reasons.¹ At the theoretical level, a key input in asset allocation models (e.g. the CAPM) is the value of the market equity risk premium; in fact, these models are rather inoperative without a valuable estimate of this premium. At the empirical level, the stock market capitalization is highly sensitive to the risk premium value since one percent shift in this latter could add or subtract one trillion of dollars (i.e. \$ 10^{12} millions) to the US stock market value.²

However, a multitude of premia must in fact be considered. It is indeed a matter of fact that investors in equity markets intervene with different time horizons of decision making. These are intra day or daily traders, individual non-professional portfolio managers, long term institutional investors such as pension funds, ... Since about fifteen years, numerous models of heterogeneity of investors were developed in particular while distinguishing fundamentalists (referring to fundamentals) and chartists (referring to past market price changes).³ Although the fundamentalists are often viewed as reflecting the behaviour of long term investors and the chartists the behaviour of short term investors, the time horizon of the investment was rarely taken into account explicitly for measuring and modeling equity risk premia. The poor attention given to this important source of heterogeneity in the stock market is as much more astonishing than the literature about interest term structure shows that maturity - and then time horizon - matter strongly for decision investment and for the value of required risk (or term) premia.

Concerning the stock market, since there is a single market price for a given equity, this implies that there must exist an equilibrium price revealed by the market to all investors, although multiple horizons of investment prevail. In fact, at any time, the same equity is held and exchanged between agents having different horizons so that the market clearing condition equalizes the observed price to a weighted average of “virtual” prices, each one referring to

¹ The author would like to thank David Le Bris for his useful comments on a first draft of this paper.

² See Graham and Harvey (2003).

³ Among others, see Brock and Hommes (1998), Boswijk et al. (2007).

the price that would prevail on a virtual market characterized by a given single horizon, the weight for each virtual price in the effective market price being represented by the share of equities that investors wish to hold according to the considered horizon.⁴ Anufriev and Bottazzi (2004) discussed the conditions of existence of a market equilibrium price in a multiple horizon framework and show that, under a suitable parameterization, the no-arbitrage market condition leads the fundamental value to be a stable fixed point.⁵ According to any stock valuation model, three factors determine an equity value: the expected return, the riskless rate and the required risk premium measured by the difference between these two last magnitudes. The intuitive assumption underlying our work is that each horizon of investment is characterized by a particular combination of these factors, the stock market price being of course the starting common knowledge information for all agents whatever the horizon of investment.

Using annual US secular data from 1871 to 2008, the present study aims to model ex-ante equity risk premia for two polar horizons: the one period (year) ahead horizon (i.e. the “short term” premium) and the infinite time horizon (i.e. the “long term” premium). Regarding the literature and in addition to the length of the analyzed period, the new contribution of this paper is in several ways. First, the two polar horizons are modeled simultaneously in the CAPM framework. Second, the question of the measurement of ex-ante equity risk premia – implying the knowledge of how expectations are formed – and the question of their explanation are solved in the same model. Third, since the price of risk is not directly observable, it is represented using the Kalman filter methodology, while taking into account information contained in a spread of interest rates. The rest of this paper is organized as it follows. **Part 2** presents a brief survey of the literature on market equity risk premia and shows why it seems relevant to consider ex-ante premia in place of ex-post premia, and why premia can be considered both as time varying and horizon-dependant phenomena. **Part 3** presents the general theoretical framework of our approach based on the conditional CAPM, according to which each market premium is at time t the product of the price of risk by the expected

⁴ Let P_τ^* the virtual price related to investors with an horizon τ and n_τ the number of equities held by this class of investors. With a number h of independent horizons, the market clearing condition is for an equity priced

P may be written as $\sum_{\tau=1}^h n_\tau (P - P_\tau^*) = 0$ with $\sum_{\tau=1}^h n_\tau = N$, where N refers to the total number of equities. This

lead to $P = \sum_{\tau=1}^h a_\tau P_\tau^*$ with $a_\tau = \frac{n_\tau}{N}$ and $\sum_{\tau=1}^h a_\tau = 1$.

⁵ Subbotin (2009) gives a survey of the rare literature about this subject.

variance of returns. **Part 4** presents assumptions concerning the expected variances and the prices of risk determinations, which allow empirical analyses for the two horizons considered. **Part 5** presents the Kalman filter methodology used to estimate jointly the two premia, and gives the empirical results. **Part 6** gives concluding remarks.

2 – The equity risk premia in the literature: surveying concepts, approaches and empirical results

Any equity premium is defined by the difference between a given representation of the expected return of the risky asset at time t for a given time horizon and an equivalent horizon risk-free rate. Two kinds of premia are distinguished: the ex-post premium and the ex-ante one. Unlike the ex-ante premium, the ex-post premium is deduced from the return observed over the future horizon considered, and not from the return expected over this horizon. Since investors cannot consider the ex-post premium to make their financial choices at time t , this magnitude cannot be regarded as a decision-making concept, unless the perfect foresight hypothesis holds, in which case it is clear that there is no risk premium, so that the ex-post excess return could not be viewed as a risk premium. Considering now the rational expectation hypothesis (REH), the so-called ex-post premium equals in fact the rational ex-ante premium plus a white noise representing the forecasting error which is of course unknown at time t . In this instance, the rational expected return and then the ex-ante premium remain unknown variables. Empirical analyses evidence that, because of excessively large error terms, the values of ex-post premia are almost often as negative as positive (among others, see Mpako-Priso (2001)) and this is somewhat disconcerting and likely to generate severe econometric biases, in particular when errors are not white noises. Moreover, many studies in the literature use lagged predictors to forecast the excess equity returns (i.e. the ex-post premium): dividend yield, earnings price ratio, short-term interest rate, payout ratio, term and default spreads of interest rates, inflation rate, book-to-market ratio, consumption, wealth, etc. As a result, no robust predictors are found (Goyal and Welch 2003, 2007), confirming that the ex-post premium is probably more a countable observation than an operational concept. In fact, experts' expected returns derived from surveys convey systematic forecast errors (Abou and Prat (1997)) and are mainly driven by autoregressive processes (Abou and Prat (2000)), which confirm that modeling ex-ante or ex_post premia are two different subjects.

Merton (1969), Samuelson (1969) and Py (1973) early analysed the relation between the time life horizon and the portfolio risk, and showed that, if the financial market is efficient (i.e. returns are not predictable), rational investors can behave myopically if their utility function exhibits constant relative risk aversion.⁶ This implies that the required risk premium is not horizon dependant. However, if returns are predictable, this result does not prevail any more. To illustrate this, Barberis (2000) interestingly builds optimal portfolios made up of stocks and bonds quoted on the US market. The author shows that, taking into account predictable features of stock returns (actual returns depend on past values of dividend yield), the optimum is reached by 40% of stocks for a one-month time horizon and by 100% of stocks for a 10-year time horizon. When return are supposed to be unpredictable, the share of stocks is near 35% whatever the horizon. This result suggests that the horizon of investment intervenes strongly to determine the required equity risk premium. In fact, when returns are partially predictable on the basis of their past values and/or macroeconomic variables, agents do not require a unique risk premium but a set of premia scaled by the time horizon.⁷

So, it is not astonishing to find the existence of a term structure for ex-ante equity premia deduced from survey data revealing experts' stock price expectations. Such studies largely confirm that, despite common trends, substantial discrepancies characterize risk premia according to the time horizon of the investment.⁸ Following papers by Welch (2000, 2001) and Prat (2001), Graham and Harvey (2001-2007) present a set of four studies about the expected equity premia defined as the difference between the experts' mean expected stock returns and an equivalent horizon bonds yields. These studies are based on quarterly surveys conducted since June 2000 by Duke University and CFO Magazine and concern stock market returns expected by about 270 anonymous Chief Financial Officers (CFOs) of U.S. corporations. They found that, in contrast with the 10-year expected risk premium, the one-year risk premium is highly erratic through time, ranging between 1.3 and 6.6% depending on the quarter surveyed. As a result, these studies strongly confirm that ex-ante premia appear to be both time-varying and horizon-dependent.

Concerning the studies relating to the long term view, the usual method to analyse the equity risk premium is to observe historical averages of the difference between stock market returns and a risk-free interest rate. At the theoretical level, this approach refers to the well-

⁶ More recently see Gollier and Zeckhauser (2002) who extended this approach.

⁷ Appendix 1 gives a formal illustration of this point.

⁸ Abou and Prat (2010) give an overview concerning studies about equity premia issued from survey data.

known debate about the so-called “*equity premium puzzle*”: with reasonable preference parameters values (i.e. the risk aversion coefficient and the subjective discount rate), the theoretical risk premium inferred from the consumption asset-based general equilibrium model is far too low (about 1-2% a year) as against the observed market premium, which stand about 6-7% a year on average (Mehra and Prescott (1985)).⁹ Study by Siegel (2005) shows that the premium was substantially lower during the periods 1802-1870 (3.2%) and 1871-1925 (4.00%), while Ibbotson Associates (2006) consider that the relevant historical premium is 7.1% during the period 1926-2005. Dimson, Marsh and Staunton (2003) report that premium was generally higher during the second half of the 20th century. Overall, we can see that these estimations of equity premium are particularly widespread according to the averaged period, which led Shiller (2000) to point out that “*the future will not necessarily be like the past*” and Fernandez (2006, p.12) to conclude that “*... equity premium change over time and it is not clear why capital market data from the 19th century or from the first half of the 20th century may be useful in estimating expected returns in the 21st century ...*”.

Another approach to measure the long term premium is to base the value of the premium on the dividends discount model (DDM). Study by Harris and Marston (2001) is based the Gordon formula according to which the long term premium equals the dividend yields plus the expected long term rate of growth in dividends minus the yield on long-term US government bonds as proxy of the risk-free rate. The five years ahead expected growth in earnings per share issued from surveys is supposed to approximate the expected growth in dividends, allowing to evaluate an ex-ante long term risk premium for US stocks (S&P 500) over the period 1982-98. The authors show strong evidence that this long term risk premium change over time and that a significant part of its dynamics may be explained by readily available forward-looking proxies for risk, as the spread of interest rates, the consumer confidence index reported by the Conference Board, the degree of discrepancy between financial analysts' forecasts, or the implicit volatility issued from options prices. But the average market risk premium is found to be 7.1% which joins the *equity premium puzzle*. However, the period was not large enough to allow reliable conclusions. This is not the case of the paper by Fama and French (2002) who still inferred ex-ante premia on the US stock market (S&P index) from the DDM, using alternatively dividends and earnings per share. The authors infer the expected growth rate of dividends (or earnings) per share and the risk-free

⁹ Papers by Kocherlakota (1996), Cochrane (1997) and Siegel and Thaler (1997) provide comprehensive surveys of the literature related to the equity premium puzzle and tentative to solve this puzzle.

rate from historical mean values of dividends (earnings) and interest rate, respectively. Interestingly, for the period extending from 1951 to 2000, Fama and French found a mean premium around 2.5% a year, which approaches the value predicted by the consumption-based asset-pricing model. The authors suggest an hypothesis to interpret why the DDM ex-ante premium is strongly lower than the historical average of the ex-post premium based on actual returns (see above) : because ex-post returns include “*large unexpected gains*”, the observed equity returns over the past half-century are higher on average than the long term expected returns included in the DDM throughout the expected growth in dividends (earnings). This hypothesis seems to be confirmed by semi-annual S&P industrial expectations 6 and 12-month ahead carried out by J. Livingston’ surveys on a panel of experts. Abou and Prat (2010) calculate indeed risk premia over the period 1952 to 1993 using these data and showed that their mean values are around 2.3% per year. This result joins in another way the Fama and French results, and then confirm the importance in distinguishing the ex-ante premium from ex-post premium.

Concerning the short term view about equity risk premium, as soon as 1987, French et al. showed that monthly risk premia fluctuations on the US stock market are partly driven by conditional variance of returns (ARCH effects), while using daily stock returns for three european countries, paper by Koutmos et al. (2008) gives a recent illustration of this short-term approach. In another way, De Santis and Gerard (1997) analysed the factors of the short-term dynamics of premia by using a conditional multivariate CAPM while study by Kryzanowski et al. (1997) based on the conditional Arbitrage Pricing Theory put into evidence several macroeconomic factors of the time-varying monthly equity premia for a set of 130 mutual funds equities on the Canadian market. According to these studies, there is an implicit assimilation between the frequency of observation in returns and the time horizon of the investment, which is a simplifying assumption since in fact, the frequency of observation can be larger or smaller than the horizon of the investment.¹⁰

3 – Equity risk premia and time horizon: theoretical framework

3.1 - The one-period ahead standard model: recall

¹⁰ For instance, Benartzi and Thaler (1995) suggested that long-term investors can adopt a myopic behaviour since they observe returns over periods shorter than the horizon. Conversely, studies modeling high frequency data using GARCH specifications suggests that the one period ahead expected variance refers to the squared returns generally to a much longer past than one period.

Let us consider a closed economy where no international effects exist on the stock market (i.e. a segmented market), and suppose that this market is efficient in the sense that any portfolio (among them, the market portfolio) is nondiversifiable (i.e. the market allowance of equities is optimal). Expectations can be rational or not, and this means that the stock market price does not necessarily convey all the relevant information (i.e. informational efficiency can prevail or not). In this context, according to the conditional CAPM, the equilibrium required risk premium on any risky asset j is given by

$$E(R_{it+1}^j | \Omega_t) - R_{ot} = \gamma_{it} \text{Cov}(R_{it+1}^j, R_{it+1} | \Omega_t) \quad (1)$$

where R_{it}^j is the return of equity (or portfolio) j in the country i , R_{it} the market return in country i , R_{ot} the one period riskless rate and $\gamma_{it} > 0$ the unit local price of risk at time t in country i . The expected return $E(R_{it+1}^j | \Omega_t)$ of the risky asset j made at time t for $t+1$ is conditional to the set of information available at time t Ω_t . The covariance between the return of asset j and the market return represents the magnitude of the uncertainty taken into account by holders of asset j to determine their required risk premium. When asset j is the market portfolio itself, the relation (1) leads to the following expression of the ex-ante market premium at time t in the country i :

$$\Phi_{it} = E(R_{it+1} | \Omega_t) - R_{ot} = \gamma_{it} \text{Var}(R_{it+1} | \Omega_t) \quad (2)$$

In this case, the market risk premium is measured by the difference between expected one period ahead stock return and the risk-free rate, and is explained at any time by the product of the unit local price of risk¹¹ by the expected variance of the market return. During

¹¹ Consider a representative agent whom wealth is made with a share of riskless asset and a share of a risky asset represented by the non diversifiable market portfolio. The representative agent is supposed to maximise at time t the expected utility of his wealth at time $t+1$. Put in the expectation/variance form, this program is

$$\underset{\Theta}{\text{Max}} \ E_t (R_{it+1}^w | \Omega_t) - \frac{\delta_{it}}{2} \text{Var}(R_{it+1} | \Omega_t)$$

where R_{it}^w is at time t the return of the total wealth which equals a weighted average of R_{it} and R_{ot} (the weight depending on the share $0 \leq \Theta_{it} \leq 1$ of the risky asset in the wealth), and where $\delta_{it} > 0$ is the absolute risk aversion coefficient. Given that $\text{Var}(R_{it+1}^w | \Omega_t) = \Theta_{it}^2 \text{Var}(R_{it+1} | \Omega_t)$, the first order condition of the program gives the expression of the risk premium required by the representative agent, that is $E(R_{it+1} | \Omega_t) - R_{ot} = \Theta_{it} \delta_{it} \text{Var}(R_{it+1} | \Omega_t)$. Comparing this result with (2) leads to the equality $\gamma_{it} = \Theta_{it} \delta_{it}$.

the last century, many studies confirm that the influence of international stock markets on US stock market is negligible. This result suggests that the US market is that approaching more the hypothesis of a segmented market, and this is not astonishing because it represents the more important stock market. Since subscript i will always refers to US, we now remove it from the variables and parameters.

3.2 - Short term versus long term horizons

The time-horizon over which the expected market return $E(R_{t+1} | \Omega_t)$ and the expected variance $Var(R_{t+1} | \Omega_t)$ (equation (2)) may equal a priori one hour ahead, one day ahead, one month ahead, one year ahead or more ... provided that these variables are conditional to information available at time t . We will now consider the one year return $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$ of the US stock market, where P_t and D_t are the S&P composite stock market price index at time t and the corresponding dividends distributed during the last year, respectively. To model the ex-ante risk premium we will consider *at the same* time its *measurement* (left hand side of (2)) and its *explanation* (right hand side of (2)). We will consider two traditional polar time-horizons : the *one period* ahead horizon and the *infinite* time horizon. We will refer to the “short term” ex-ante risk premium for the first one and to the “long term” ex-ante risk premium for the second.

According to (2), the one period ahead ex-ante risk premium is defined by the difference between the expected one period ahead stock return and the risk-free rate R_{ot} one year to maturity:

$$\Phi_{1t} = E(R_{t+1} | \Omega_t) - R_{ot} \quad (3)$$

this premium being explained by the product of the unit US price of risk by the one period ahead expected variance of the market return :

$$\Phi_{1t} = \gamma_{1t} Var(R_{t+1} | \Omega_t) \quad (4)$$

Given that $E(R_{t+1} | \Omega_t) = \frac{P_t - E(P_{t+1} | \Omega_t) + E(D_{t+1} | \Omega_t)}{P_t} = R_{ot} + \Phi_{1t}$, the forward iterative

resolution of this last equation leads to the well known expression of stock price in an infinite time horizon :

$$P_t = E_t \sum_{k=0}^{\infty} D_{t+k} \prod_{j=0}^k \frac{1}{(1+R_{t+j})} \quad \text{with} \quad R_t = R_{ot} + \Phi_{1t} \quad (5)$$

According to (5), the price equals the present value of the expected stream of future dividends.¹² Supposing that the expected rate of growth in dividends and the actualisation rate are uniform between t and all the future periods $t+k$, that are \bar{g}_t and \bar{R}_{ot} respectively, we obtain the “Gordon-Shapiro” stock price valuation formula which allows to define the infinite time horizon ex-ante risk premium Φ_{2t} , this horizon being viewed here as a single long term representative horizon:

$$\Phi_{2t} = \frac{D_t (1 + \bar{g}_t)}{P_t} + \bar{g}_t - \bar{R}_{ot} \quad (6)$$

In accordance with (2), Φ_{2t} is explained by

$$\Phi_{2t} = \gamma_{2t} \operatorname{Var}(R_{t,2} | \Omega_t) \quad (7)$$

where γ_{2t} and $\operatorname{Var}(R_{t,2} | \Omega_t)$ represent the US price of risk and the expected variance of returns (expressed per year) in an infinite time horizon context, respectively.

4 – Expected returns, expected variances and prices of risk : assumptions

4.1 - The expected returns and expected variances

Concerning the measurement of the two ex-ante premia, it is necessary to make hypotheses about the expected stock return for the one period horizon and the expected growth rate of dividends for the infinite horizon. It is worth noting that expected returns revealed by survey data strongly suggest that experts form their expectations mainly according to an autoregressive process¹³, the adaptive model appearing to be a simplified

¹² When expectations are assumed to be rational, (5) gives the “fundamental value” of equities. Under the transversality condition, there is no bubble and the price equals the fundamental value.

¹³ Among others, see Abou and Prat (2000).

form – and the most popular - of such a process. Accordingly, it is supposed that expectations can be represented by simple adaptive processes which are :

$$E(R_{t+1} \mid \Omega_t) = \beta_1 R_t + (1 - \beta_1) E(R_t \mid \Omega_{t-1}), \quad \text{with } R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}, \quad 0 \leq \beta_1 \leq 1 \quad (8)$$

$$\bar{g}_t = \beta_2 g_t + (1 - \beta_2) \bar{g}_{t-1}, \quad \text{with } g_t = \log \frac{D_t}{D_{t-1}}, \quad 0 \leq \beta_2 \leq 1 \quad (9)$$

For given values of coefficient β_1 and β_2 and of initial values of the expected variables, equations (8) and (9) determine the time series of the two expected variables. Provided that indicators of riskless interest rates are given for the two horizons, we get measurements of the one period risk premium Φ_{1t} and of the infinite horizon risk premium Φ_{2t} by reporting (8) and (9) in (3) and (6), respectively.

We now turn to the conditional expected variances intervening in the explanation of the premia. **Appendix 1** illustrates why the expected variance is horizon-dependent when the returns are partially predictable (what is the case in particular for annual returns) and show why, according to the date, the short term premium may be greater or smaller than the long term premium. This leads to consider that the one period ahead expected variance $Var(R_{t+1} \mid \Omega_t)$ and the infinite time horizon expected variance $Var(R_{t,2} \mid \Omega_t)$ do not take the same value. An ARCH-M model would not be appropriate since the conditional volatility would concern the residuals of the risk premium equation and not the total variance of the return as required by our model. An ARCH model where the mean equation specifies the stock return as a constant term plus an error term would give a first step estimation of the conditional expected variance, but this estimation would be disconnected from the estimation of the portfolio model.¹⁴ As a tentative, for the two horizons, the expected variances are assumed to be represented as an m -order (resp. m') weighted averages of the past values of the centered squared returns, this last one representing a proxy of the instantaneous observed variance¹⁵:

¹⁴ Moreover, annual data are generally not appropriate to implement an ARCH process, this latter being generally adapted to at least monthly data.

¹⁵ We also considered the actual value of the instantaneous variance ($i=0$) in (10) and (11), but this assumption lowered the criteria of information of the risk premia model compared to the retained specification ($i=1$) where the expected variance is determined only by the past values of the observed variance. Moreover, we let the possibility to add a constant term in the right hand side of the equations; but, for the two equations, these constants appeared to be insignificant and then were removed.

$$\text{One year horizon : } \quad \text{Var}(R_{t+1} \mid \Omega_t) = \frac{\sum_{i=1}^m \alpha_{1i} \sigma_{t-i}^2}{\sum_{i=1}^m \alpha_{1i}}, \quad \alpha_{11} = 1; \alpha_{1i} > 0 \quad (10)$$

$$\text{Infinite horizon : } \quad \text{Var}(R_{t,2} \mid \Omega_t) = \frac{\sum_{i=1}^{m'} \alpha_{2i} \sigma_{t-i}^2}{\sum_{i=1}^m \alpha_{2i}}, \quad \alpha_{21} = 1; \alpha_{2i} > 0 \quad (11)$$

with $\sigma_t^2 = (R_t - \bar{R})^2$, where \bar{R} is the average annual return.¹⁶ The parameters α_{1i} , α_{2i} , m and m' and are determined in the course of the estimation of the risk premia model. Note that these two equations represent at the same time how agents *observe* the variance and how they *expect* it, and they are in accordance with many other studies.¹⁷ Using CFO's surveys, Graham and Harvey (2001-2007) asked questions designed to determine expert's assessment of market volatility. Interestingly, this latter appeared to be much lower than usual alternative measures, hence suggesting that the expected variance have a lower magnitude than observed variance, which is in accordance with hypotheses (10) and (11).

4.2 – The prices of risk

Concerning the unit US price of risk intervening in the explanation of the premia, γ_{1t} and γ_{2t} are typically unobservable variables. This lead us to estimate a state-space model where for each horizon a signal (or measurement) equation describes the risk premium and a state (or transition) equation contribute to determine the unobservable US local price of risk together with macroeconomic variables. Accordingly, the two state variables are AR(1) processes augmented possibly by a constant term and by the rate of change in CPIs, of the real consumption per capita, of earnings per share and by various spreads of interest rates. In fact, none of them was found to be significant when added in the state equation, excepted the rate of change in corporate earnings per share for the long term price of risk. However, when

¹⁶ Note that the centered squared return σ_t^2 representing the instantaneous variance appeared to be insignificantly autocorrelated over the sample period.

¹⁷ In a forecasting view and following Bollerslev (1987) and Hansen and Lunde (2005) propose to represent the expected variance as an ARMA model for the squared returns plus a constant term. Müller et al. (1997), and for high frequency data, authors refer to an equation analogue with (10) and (11) including a constant term.

considered separately by adding them to the state equation, the relative spread between the 30 years high grade corporate bonds yield and the 10 years government bonds yield appeared significantly in the determination of the price of risk for the two horizons. The prices of risk γ_{1t} and γ_{2t} are then determined according to the following relations:

The price of risk for the one year time horizon

$$\gamma_{1t} = sv_{1t} + \delta_1 S_t + \kappa_1 \quad \delta_1 > 0 \quad (12)$$

$$sv_{1t} = \rho_1 sv_{1,t-1} + b_1 \frac{\Delta B_t}{B} + \eta_{1t} \quad 0 \leq \rho_1 \leq 1, \quad b_1 > 0 \quad (13)$$

The price of risk for the infinite time horizon

$$\gamma_{2t} = sv_{2t} + \delta_2 S_t + \kappa_2 \quad \delta_2 > 0 \quad (14)$$

$$sv_{2t} = \rho_2 sv_{2,t-1} + b_2 \frac{\Delta B_t}{B} + \eta_{2t} \quad 0 \leq \rho_2 \leq 1, \quad b_2 > 0 \quad (15)$$

where $S_t = \frac{\bar{R}_{Ct} - \bar{R}_{ot}}{\bar{R}_{Ct}}$, \bar{R}_{Ct} and \bar{R}_{ot} representing the 30 years corporate bond yield and the 10

years government bond yield respectively, where $\frac{\Delta B_t}{B}$ stands for the rate of change in the earnings per share, where η_{1t} and η_{2t} are *Niid* error terms, and where the values of the constant drifts κ_1 and κ_2 are a priori undetermined. The spread variable S_t captures the risk of default since corporate bonds are more risky assets than government bonds. But S_t also captures risk due to the fact that corporate bonds are 30 years to maturities while government bonds are 10 years to maturity. Our intuition is not that this spread of interest rates influences directly the equity risk premia, but rather that economic factors of uncertainty which are imbedded in S_t intervene in the equity premia determination. Because of these reasons, coefficients δ_1 and δ_2 are expected to be positive.¹⁸ Concerning now the positive influence of change in earnings per share ($b_1, b_2 > 0$), it can be expected that an increase of corporate

¹⁸ Number of contributions have shown that spreads of interest rates are negatively (positively) correlated with stock prices (returns). Concerning the spread between different ratings US bond yields over a long period, see in particular Prat (1982), Chapter IV which is entirely devoted to this relation. Concerning the link between spread between different maturities US bond yields and equity risk premium through the APT, see, among others Chen, Roll and Ross (1986) and Elton, Gruber and Mei (1994) and Kryzanowski et al. (1997).

earnings will incite to rise the share of risky asset in the portfolio, thus implying a rise in the price of risk (see note (11)). Overall, the state variables sv_{1t} and sv_{2t} are supposed to capture the total influence on the price of risk of hidden factors and of non directly measurable psychological effects.

5 - Equity risk premia and time horizon : empirical evidence on US secular data

We used the annual secular data over the period 1871 to 2008 as available on the web site of Robert Shiller. P_t is the Standard and Poor's 500 stock price index, D_t and B_t are the S&P dividends and earnings per share during last year, R_{ot} the one-year interest rate, \bar{R}_{ot} the 10 years government bonds yield and the consumer price index, and the real consumption per capita. These data are in january of each year. We also extended a 30 years high grade corporate bonds yield time series over this period by using Friedman and Schwartz (1982) data ending in 1975.¹⁹ Data about P_t , D_t and B_t were discussed and revised by Wilson and Jones (2002) (WJ) over the period 1870 to 1999. Note that contrary to the Shiller's annual data, the WJ series are for december of each year. Comparing the Shiller's december S&P stock price index, dividends and earnings with the corresponding WJ data, we observed that the series are practically confused from 1957 to 1999. However, during the period 1870 to 1956, the WJ values appeared to be at a level slightly larger compared to the Shiller's data, although the two data sets exhibit very similar movements. To appreciate if there is significant bias to use a data set in place of the other, we compared over the period 1871-1957 the annual stock returns (including dividends) calculated by using the december Shiller' S&P data with the december stock returns deduced from the WJ data. The two measures appeared to be closely correlated. Regressing the Shiller' S&P returns on the WJ returns, the coefficient of regression was found to be insignificantly different from one and the intercept insignificantly different from zero, the residuals being insignificantly autocorrelated. As a result, concerning our study, we can indifferently use the Shiller or the WJ data.

For given values of β_1 and of the initial value of the expected stock return, (8) gives the one period ahead expected return time series and (3) the short term risk premium measurement. Reporting in the structural equation (4) the expected variance given by (10) and

¹⁹ Friedman and Schwartz (1982), Table 6.17, pp. 296-98.

the unit US price of risk given by (12), and adding an error term in (4), we obtain the signal equation of the one year ahead risk premium (16), where the variable sv_{1t} is determined by the state equation (13) :

$$\Phi_{1t} = E(R_{t+1} \mid \Omega_t) - R_{ot} \quad (3)$$

$$E(R_{t+1} \mid \Omega_t) = \beta_1 R_t + (1 - \beta_1) E(R_t \mid \Omega_{t-1}) \quad 0 \leq \beta_1 \leq 1 \quad (8)$$

$$\Phi_{1t} = (sv_{1t} + \delta_1 S_t + \kappa_1) \frac{\sum_{i=1}^m \alpha_{1i} \sigma_{t-i}^2}{\sum_{i=1}^m \alpha_{1i}} + v_{1t} \quad (16)$$

$$sv_{1t} = \rho_1 sv_{1,t-1} + b_1 \frac{\Delta B_t}{B} + \varepsilon_{1t} \quad 0 \leq \rho_1 \leq 1 \quad (13)$$

In a similar manner, for given values of β_2 and of the initial value of the expected growth in dividends, (9) gives the long run expected rate of growth in dividends and then (6) gives the long term risk premium measurement. Reporting in the structural equation (7) the expected variance given by (11) and the unit US price of risk given by (14), and adding an error term in (7), we obtain the signal equation of the infinite time horizon risk premium (17), where the variable sv_{2t} is determined by the state equation (15):

$$\Phi_{2t} = \frac{D_t (1 + \bar{g}_t)}{P_t} + \bar{g}_t - \bar{R}_{ot} \quad (6)$$

$$\bar{g}_t = \beta_2 g_t + (1 - \beta_2) \bar{g}_{t-1} \quad 0 \leq \beta_2 \leq 1 \quad (9)$$

$$\Phi_{2t} = (sv_{2t} + \delta_2 S_t + \kappa_2) \frac{\sum_{i=1}^{m'} \alpha_{2i} \sigma_{t-i}^2}{\sum_{i=1}^m \alpha_{2i}} + v_{2t} \quad (17)$$

$$sv_{2t} = \rho_2 sv_{2,t-1} + b_2 \frac{\Delta B_t}{B} + \varepsilon_{2t} \quad 0 \leq \rho_2 \leq 1, \quad b_2 > 0 \quad (15)$$

For given values of β_1, β_2 and initial values of expected variables, the 4-equations-system (16), (17), (13) and (15), can be estimated jointly as a system using the Kalman filter methodology, where (16) and (17) are the *signal equations* while (13) and (15) are the *state*

equations (see **Appendix 2** for a formal presentation of the state-space model and of the recurrent equations used in the estimation method). The innovations v_{1t} and v_{2t} of the signal equations are supposed *Niid* and independent of the auxiliary residuals ε_{1t} and ε_{2t} of the state equations, respectively. However, a significant correlation may exist between the two signal residuals resulting from the interdependences between the two premia and between their factors, which could cause biases in estimates. This is why a covariance term (*covar*) between v_{1t} and v_{2t} were added to the set of hyperparameters to be estimated. All parameters (including initial values of state variables) are those minimizing the Akaike, Schwarz and Hannan-Quinn criteria of information.

[Insert Table 1]

Table 1 gives descriptive statistics of the two “observed” premia as determined by equation (3) and (8) for the one year premium and by equations (6) and (9) for the infinite horizon premium, with optimal values of the expectation coefficients of $\beta_1 = 7\%$ and $\beta_2 = 3\%$ respectively for the adaptive processes (8) and (9).²⁰ These coefficients show a rather long memory for the two horizons, but with longer for the long run horizon, which is rather intuitive. It is worth noting that, excepted in the beginning of 1980’s, the values of the premia keep positive values. In addition to error measurements, the negative values observed could be connected to the shock due to the Republican Presidential election of Ronald Reagan in 1980 reinforced at the same time by the Republican majority elected at the Senate; if these events had an effect on equity risk premia, it was probably in the direction of a fall. The short term premium appears to be higher in mean since more volatile than the long term premium and these outcomes are rather intuitive. Interestingly, the mean value of the infinite horizon premium (2.3% per year) joins the one of 2.5% obtained by Fama and French (2002). **Figures 1 and 2** exhibit the time pattern of the components contributing to the measure of the premia, for the one year and the infinite time horizon, respectively: interestingly, in both cases, the components appear to be not negligible one compared to others. **Figure 3** compares the values of the measurement of two observed premia : although these two magnitudes are stationary at the 10% level of significance, the horizon is a very discriminant parameter conditioning the dynamics of the premia (the value of $R^2 = 0.25$ is not very high), and this result confirms lesson from survey data reflecting experts’ opinions (see part 2).

²⁰ The optimal initial values of the expected return in (8) and of the expected rate of growth in (9) are found to be 4% and 0% per year.

[*Insert figure 1*]

[*Insert figure 2*]

[*Insert figure 3*]

Table 2 gives the estimates of the structural model of risk premia. Concerning the signal equations, a grid search over the lags m and m' intervening in the expected variances determination led to the optimal values 5 and 8 for the one year and the infinite horizon, respectively. As a result, compared to the short term premium, the long term premium is influenced by the variance over a longer time span, which is rather intuitive since this result contributes to explain why the former is more volatile than the latter, as shown by figure 3. Although the coefficients of the lagged variances decrease from the third lag for both horizons, the lag-coefficients increase before decreasing for the long term premium, and this outcome was already observed in the literature.²¹ However, excepted two weighs for the long term premium, standard errors of estimates suggest that they are not significantly different from 1 at the 5% level, hence suggesting that the expected variances could be approached with a simple arithmetic average of a set of past values of centered squared returns. **Figure 4** compares the two expected variance time patterns and confirms that despite similar trends ($R^2 = 0.67$), the short term expected variance exhibits higher volatility of the long term one. The significant positive value obtained for the covariance between the residuals of the two signal equations (*covar*) results from the correlations between the two premia, between the two expected variances and between the two state variables, as displayed in Figures 3, 4 and 7, respectively. The covariance between the two state residuals is found to be insignificantly different from zero and this is why this parameter has been removed from the estimations.²²

[*Insert Table 2*]

[*Insert figure 4*]

Another point is that, as expected, the coefficients δ_1 and δ_2 of the spread of interest rates S_t are significantly positive what confirms former results of the literature.²³ The larger sensibility to the spread found for the short term premium than for the long term one seems intuitive insofar as S_t represents default risk perceived due to economic factors or default risk

²¹ For instance, see Ederington, and Lee (1993).

²² We also found a zero covariance between the signal residuals and the state residuals for each horizon. This was a condition underlying the updating equations (B5) and (B6) used and presented in Appendix 3.

²³ See note (18).

expected in a rather near future. The constants κ_1 and κ_2 were not found to be significant and therefore have been removed at the final stage of estimation. According to (12) and (14), the state variables sv_{1t} and sv_{2t} given by (13) and (15) contributes with the spread S_t to the representation of the prices of risk for each horizon γ_{1t} and γ_{2t} . Since this paper is concerned by a structural model, the state variables are estimated at each point in time conditional on the whole sample data (*smoothed inference*) rather than using only the past observations (*predicted inference*) or actual and past observations (*filtered inference*). At each point of time, the kalman filter method yields the standard deviations (*std*) of each state variable. For an accepted 5% level of significance, the *std* allows to determine a zone of uncertainty defined at each date by the line ranging between the estimated value plus and minus 1.96 *std*. In the same manner, the *std* of coefficients of S_t , which are constant over time (although horizon dependant), allow to define a zone of uncertainty associated to $\delta_\tau S_t$. These standard deviations allow us to associate upper and lower bounds to the estimated values of the price of risk that are exhibited by **Figures 5 and 6** for the one year and the infinite time horizons, respectively. It is worth noting that the estimated values of γ_{1t} and γ_{2t} are rarely negative, although no constraint was imposed. Moreover, among 128 annual data, in the beginning of 1982's, we find for the short premium only one negative value under the upper value, while we find five values for the long term premium. These results are rather satisfying since the theoretical value of the price of risk is positive. **Figure 7** shows that, although the short term and long term prices of risk are correlated ($R^2 = 0.22$), they exhibit large own fluctuations.

[*Insert figure 5*]

[*Insert figure 6*]

Figures 8 and 9 represent the observed and the fitted values of the premia for the two horizons, respectively: the state-space model fits well the main fluctuations of the long term premium whereas the fit of the 3-month premium is of lower quality because of the higher volatility of the latter. It is noted that the expected variances and the prices of risk can explain sudden changes in the risk premia and periods of high or low premia, what makes it possible to understand why it is not necessary to consider explicitly structural changes over this long period to fit the data. We further checked the goodness of the fits by using the conventional coefficient of determination R^2 and a modified measure, R_D^2 , assessing the relevance of our

model with respect to the simple random walk plus drift process (benchmark).²⁴ The values of R_D^2 reach 0.88 and 0.68 for the long and the short term horizons (table 2) indicate that the residual variance of the measurement equation is 0.12 and 0.32 times the one of the benchmark model for the infinite and 1-year horizons, respectively : clearly, the risk premia model strongly outperforms the benchmark model.

[*Insert figure 8*]

[*Insert figure 9*]

We now examine the statistical properties of the residuals of the signal equations (innovations) v_{1t} and v_{2t} . The diagnostic tests we refer to are presented in **Appendix 3**.²⁵ According to Harvey's (1992) heteroskedasticity test, the null of homoskedasticity of these residuals is rejected for the one year horizon but accepted for the infinite horizon. The appropriate Ljung-Box Q test by Harvey (1992) based on the first 11 autocorrelations applied to the signal standardized smoothed residuals showed a rather weak²⁶ but significant autocorrelation for the two horizons, which suggest that, beyond a possible specification bias, market frictions such as transaction costs²⁷ and risky arbitrage opportunities²⁸ could cause delayed adjustments of premia $\Phi_{\tau t}$ toward their theoretical values. Supposing these adjustments to be represented by an error correction models (ECM), we showed in **Appendix 4** that ECM's residuals are independents and homoscedastic at the 5% level of significance.

6 – Concluding remarks

Any ex-ante equity premium is defined by the difference between a given representation of the expected return of the risky asset at time t for a given future time horizon

²⁴ The two measures of goodness of fit are defined by $R^2 = 1 - \text{SSR} / \sum_{t=1}^T (y_t - \bar{y})^2$ and

$R_D^2 = 1 - \text{SSR} / \sum_{t=2}^T (\Delta y_t - \bar{\Delta y})^2$ where $y_t = \Phi_{\tau t}$ ($\tau = 1, 2$) and SSR is the sum of the squared residuals. A

negative R_D^2 implies that the estimated model is worse than a simple random walk plus drift (Harvey, 1992).

²⁵ The same tests are implemented by Prat and Uctum (2008) for a similar model concerning exchange rates risk premia.

²⁶ In fact, no coefficient of correlation R exceeds 0.20.

²⁷ See Anderson (1997).

²⁸ See Shleifer and Summers (1990).

and an equivalent maturity risk-free interest rate. Using annual US secular date from 1871 to 2008, the challenge of our study with respect to the literature is to model at the same time the measures of ex-ante equity risk premia (which specify how expected returns are formed) and the explanations of these premia in the CAPM framework, by considering simultaneously two polar horizons: the one period (year) ahead horizon (i.e. the “short term” premium) and the infinite time horizon (i.e. the “long term” premium).

Representing expected returns by traditional adaptive processes, large disparities in the dynamics of the two measured premia are evidenced, as expected when the market is not efficient and as shown by ex-ante premia deduced from survey data reflecting experts’ opinion. According to the CAPM, each of the two premia is explained by the product of the unit price of risk by the expected variance of observed returns, each of these time varying magnitudes depending on the horizon. The expected variances are found to depend on the past values of variances during 5 and 8 years for the short term and long term premia, respectively. For each horizon, the price of risk is determined both by a spread of interest rates capturing economic factors of uncertainty and by an unobservable variable determined according to the kalman filter methodology (i.e. a state variable), that is supposed to capture the influence of hidden variables and of non directly measurable psychological effects. The model gives a valuable representation of the “short term” and “long term” premia over the period 1881-2008, and then gives an explanation of the large disparities in their dynamics. Finally, we find a weak but significant autocorrelation in residuals that could be explained by short term adjustments of premia toward their theoretical values, possibly due to transaction costs and risky arbitrage. Overall, these results highlight the existence of a time varying term structure of ex-ante equity risk premia and suggest that it is necessary to solve simultaneously their measurement and their explanation, although, when expectations are not rational, results are conditional to the hypothesis retained for the expected return representation.

Appendix 1 - Why equity risk premia are horizon-dependant when returns are predictable? Illustrations

It is supposed that no dividends are distributed. Let p_t denote the logarithm of the price of an equity or a portfolio at time t and Δ the 1-period change operator. Suppose that the market is efficient in the sense that p_t conveys all available information about the future price (i.e. expectations are rational). The return Δp_t is thus a white noise plus a possible constant drift.²⁹ In this case, we have $E(p_{t+\tau} - p_t) = \tau E(\Delta p_{t+1})$ and $Var(p_{t+\tau} - p_t) = \tau V(\Delta p_{t+1})$, $\tau \geq 1$, that is, the two first moments increase in the same proportion with τ . Since the risk premium depends on the expected variance (equation (2)), the premium averaged per period may be time-varying if the variance is so but does not depend on τ , so that there is a single premium. Conversely, when returns are partially predictable on the basis of their past values and/or macroeconomic variables³⁰, the stock market is not efficient and agents do not require a unique risk premium but a set of premia scaled by the time horizon. For example, suppose that the one period return is related to the variable ΔX_t , according to the simple relation $\Delta p_{t+1} = \Delta X_t + \eta_{t+1}$, where η_{t+1} is a white noise, with $E(\Delta X_t) = E(\eta_t) = 0$, $Var(\Delta X_t) = \theta^2$, $Var(\eta_t) = \omega^2$ and $Cov(\Delta X_{t+1}; \Delta X_t) = Cov \forall t$. Suppose further that $Cov(\Delta X_{t+\tau}; \Delta X_t) = 0 \quad \forall \tau > 1$, the one period variance is $Var(\Delta p_{t+1}) = \theta^2 + \omega^2$ while the τ period horizon variance is $\frac{1}{\tau} Var(p_{t+\tau} - p_t) = \theta^2 + \omega^2 + 2\left(1 - \frac{1}{\tau}\right)Cov$. It can be seen that when $Cov > 0$, the variance and therefore the required premium increase with the horizon, while when $Cov < 0$, the variance and the premium decrease with the horizon.³¹ This implies that a sufficient condition to generate an increasing or decreasing term structure of risk premia is the existence of a

²⁹ Even if we introduce a discount rate with constant variance which is independent of the white noise forecast error, this conclusion is still valid.

³⁰ Lo and MacKinlay (1988) and Cochrane (1999a) give overviews concerning the predictable character of stock returns. Beyond the autocorrelation of returns, economic variables such as spreads of interest rates, change in money supply, production growth, change in corporate earnings, the ratio dividend/earnings and the dividend yield are often shown to be significant predictors.

³¹ Two examples for the sign of Cov are given by Cochrane (1999b) with $\Delta X_t = a \Delta P_t$, $a > 0$: in this case, Cov is positive when the actual return is positively autocorrelated but negative when a mean-reversion describes the dynamics of returns. Here, the condition $a = 0$ corresponds to the efficiency hypothesis according to which returns are a white noise.

serial correlation in returns.³² If the sign or the magnitude of the covariance is time-varying, the slope of the term structure of premia is also time-varying, and this may explain why the short term premium may be greater or smaller than the long term premium. If now we relax the hypothesis $\text{Var}(\Delta X_t) = \theta^2$ and assume that $\text{Var}(\Delta X_t)$ is a conditional AR(n) process, it can be shown that $\text{Var}(\Delta p_{t+1})$ is also an AR(n) process. When $\tau > 1$, an autoregressive structure with order greater than n is preserved for $\text{Var}(\Delta p_{t+\tau})$.³³

Appendix 2 - The equity risk premia model and the Kalman filter equations

The system formed by the four equations (16), (17), (13) and (15) can be written in the following state-space form (see Harvey (1992), Ch. 3; Hamilton (1994), Ch.13):

$$\text{signal equations : } \Phi_t = \sum_{(2,1)} sv_t + \delta_{(2,2)} X_t + v_t, \quad t = 1, \dots, T \quad (\text{B1})$$

$$\text{state equations : } sv_t = \rho_{(2,2)} sv_{t-1} + b_{(2,1)} \Delta B_t / B + c_{(2,1)} + \varepsilon_t, \quad t = 1, \dots, T \quad (\text{B2})$$

$$\text{where } \Phi_t = \begin{bmatrix} \Phi_{1t} \\ \Phi_{2t} \end{bmatrix}, \quad sv_t = \begin{bmatrix} sv_{1t} \\ sv_{2t} \end{bmatrix}, \quad \Sigma_t = \begin{bmatrix} \text{Var}_1 & 0 \\ 0 & \text{Var}_2 \end{bmatrix}, \quad \rho = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix}, \quad c = \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix},$$

$\delta = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}$, $X_t = \begin{bmatrix} S_t \cdot \text{Var}_1 \\ S_t \cdot \text{Var}_2 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $v_t = \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}$ and $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{t2} \end{bmatrix}$. The conditional expected variances $\text{Var}_1 = \text{Var}(R_{t+1} \mid \Omega_t)$ and $\text{Var}_2 = \text{Var}(R_{t,2} \mid \Omega_t)$ in the matrix Σ_t depend on the lag parameters α_{1i} and α_{2i} (see equations (10) and (11) resp.). Σ_t , ρ and c are matrices containing fixed and unknown parameters to be estimated. sv_t is a vector of time-varying unobservable components, with initial value sv_o assumed to have a mean a_o and a covariance matrix M_o . $\Delta B_t / B$ is a variable common to both horizons. The disturbances v_t and ε_t are serially uncorrelated with mean zero and covariance matrices $\text{Var}(v_t) = U$ and

³² Transaction costs do not alter this result: when for example $\text{Cov} > 0$, there always exists a horizon long enough to be profitable.

³³ Complexity increases even more when we consider a vector of predictive variables, each one partially predicting the return. In this case, $\text{Var}(\Delta p_{t+\tau})$ is determined by the actual and past variances and covariances of these variables.

$\text{Var}(\varepsilon_t) = Q$. They are moreover mutually uncorrelated, that is $E(v_t, \varepsilon_{t'}) = 0$ for all t, t' ,³⁴ and also uncorrelated with sv_o . Let $\hat{sv}_{t/t}$ be the optimal estimator (or the update, see below) of sv_t based on all available information up to t , denoted Ω_t . Let $M_{t/t} = E[(sv_t - \hat{sv}_{t/t})(sv_t - \hat{sv}_{t/t})']$ be the covariance matrix of the estimation error. The optimal predictor of sv_t conditional on Ω_{t-1} , is given by :

$$\hat{sv}_{t/t-1} = \rho \hat{sv}_{t-1/t-1} + b \Delta B_t / B + c \quad (\text{B3})$$

and it can be shown that the covariance matrix of the forecast error, $M_{t/t-1} = E[(sv_t - \hat{sv}_{t/t-1})(sv_t - \hat{sv}_{t/t-1})']$, can be written as:

$$M_{t/t-1} = \rho M_{t-1/t-1} \rho' + Q \quad (\text{B4})$$

The equations (B3) and (B4) are the *prediction equations* of the Kalman filter. From (B1) we get the forecast error on Φ_t and its covariance matrix given by $H_t = E[(\Phi_t - \hat{\Phi}_{t/t-1})(\Phi_t - \hat{\Phi}_{t/t-1})'] = \Sigma_t M_{t/t-1} \Sigma_t' + U$. The linear projection of sv_t on Ω_t leads to the following *updating equations*:

$$\hat{sv}_{t,t} = \hat{sv}_{t,t-1} + K_t (\Phi_t - \hat{\Phi}_{t/t-1}) \quad (\text{B5})$$

$$M_{t,t} = M_{t/t-1} - K_t \Sigma_t M_{t/t-1} \quad (\text{B6})$$

where $\hat{\Phi}_{t/t-1} = \Sigma_t \hat{sv}_{t/t-1} + \delta X_t$, and where $K_t = M_{t/t-1} \Sigma_t' H_t^{-1}$ is a correction term, known as the gain matrix of the Kalman filter, applied in (B5) to the forecast error in y_t and in (B6) to the covariance matrix between the forecast errors in Φ_t and sv_t , namely $\Sigma_t M_{t/t-1} = E[(\Phi_t - \hat{\Phi}_{t/t-1})(sv_t - \hat{sv}_{t/t-1})']$. If v_t , ε_t and sv_o are multivariate Gaussian, then Φ_t is $N(\Sigma_t \hat{sv}_{t/t-1}, H_t)$. The parameters in equations (B1) and (B2) can then be estimated by the maximization of the log-likelihood function $L = \sum_{t=1}^T \log f(\Phi_t)$, where

$$f(\Phi_t) = (2\pi)^{-1} |H_t|^{-1/2} \exp\left(-\frac{1}{2} (\Phi_t - \hat{\Phi}_t)' H_t^{-1} (\Phi_t - \hat{\Phi}_t)\right)$$

is the pdf of Φ_t .

³⁴ Note that $E(v_t, \varepsilon_{t'})$ may be equal to some non-zero matrix G if $t = t'$ and 0 otherwise, that is, the residuals may be contemporaneously correlated. In this case the prediction equations (B3) and (B4) are unaltered but the updating equations (B5) and (B6) are modified as described in Harvey (1992, sub-section 3.2.4).

Appendix 3 - Diagnostic residual tests for the Kalman filter inference

We describe Harvey's (1992) autocorrelation and heteroskedasticity tests for the standardized signal residuals \hat{v}_t (that are errors v_{1t} and v_{2t} of (16) and (17)) resulting from the smoothed inference over our sample size $T = 128$. Let γ_θ be the sample autocorrelations in \hat{v}_t at lag $\theta = 0, \dots, p$. We set $p = \sqrt{T} \approx 12$ (see Harvey (1992, p.259)). The null of no serial autocorrelation in the residuals can be tested by using the Ljung-Box Q statistic

$$Q^* = T^*(T^* + 2) \sum_{\theta=1}^p \gamma_\theta^2 / (T^* - \theta), \text{ where } T^* = T - d \quad (d \text{ is the number of non-stationary}$$

elements of the state vector that are associated to a signal equation, equal to 0 in our case). Under the null, Q^* is a $\chi^2(q)$, with $q = p - n$, where n is the number of hyperparameters to be estimated minus one, equal to 7 and 11 for our 1 year and infinite horizon models, respectively. The author suggests to calculate the test for heteroskedasticity as

$$H(h) = \sum_{t=T-h+1}^T \hat{v}_t^2 / \sum_{t=d+1}^{d+1+h} \hat{v}_t^2, \text{ where } h \text{ is the nearest integer to } T^*/3, \text{ equal to 73 with our sample}$$

size. The asymptotic distribution of the statistic $hH(h)$ is then $\chi^2(h)$.

Appendix 4 - Testing the adjustments of premia toward their theoretical values according to an ECM

We consider the two following error correction models:

$$\Delta\Phi_{1t} = \lambda_1 (\bar{\Phi}_{1t-1} - \Phi_{t-1}) + \mu_1 \Delta\bar{\Phi}_{1t} + \omega_1 \Delta\Phi_{1,t-1} + \xi_{1t} \quad (18)$$

$$\Delta\Phi_{2t} = \lambda_2 (\bar{\Phi}_{2t-1} - \Phi_{2t-1}) + \mu_2 \Delta\bar{\Phi}_{2t} + \omega_2 \Delta\Phi_{2,t-2} + \xi_{2t} \quad (19)$$

where $\bar{\Phi}_{1t}$ and $\bar{\Phi}_{2t}$ are the fitted values of the premia given by the signal equations (here, the targets) and where ξ_{1t} and ξ_{2t} are error terms. For both horizons, we found that ω_τ is not significantly different from zero, so that the lagged endogenous variable has been removed. Estimating equations (18) and (19) simultaneously with the SUR method, (18) yields $\lambda_1 = 0.88 (11.3)$, $\mu_1 = 1.01 (19.4)$ with $R^2 = 0.68$, and (19) yields $\lambda_2 = 0.89 (12.8)$,

$\mu_2 = 1.03$ (49.0) with $R^2 = 0.94$. Interestingly, the Ljung-Box Q test and ARCH test showed now that residuals ξ_{1t} and ξ_{2t} are independents and homoscedastic at the 5% level of significance. The fact that coefficients are rather close to unit is due to the slight autocorrelation of innovations in the signal equations.

REFERENCES

- Abou A. and G. Prat (1997), A propos de la rationalité des anticipations boursières : quel niveau d'agrégation des opinions?, *Revue d'Economie Politique*, 5, 647-69.
- Abou A. and G. Prat (2000), *Modelling stock price expectations: lessons from micro data*, in Price Expectations in Goods and Financial Markets, Gardes and Prat Editors, London: Edward Elgar, 313-46.
- Abou A. and G. Prat (2010), "The Dynamics of U.S. equity premia: lessons from professionals' view », *Bankers, Markets and Investors*, n°104, Jan-Feb 2010, 4-20.
- Anderson H.M. (1997), "Transaction Costs and Nonlinear Adjustment Towards Equilibrium in The US Treasury Bill Markets", *Oxford Bulletin of Economics and Statistics*, .59, 465-484.
- Anufriev M. and G. Bottazzi (2004), "Asset pricing model with heterogeneous investment horizons", Laboratory of Economics and Management (LEM) Working Paper Series, 2004/22, Sant'Anna School for Advanced Studies, Pisa, Italy.
- Barberis N. (2000), "Investing for the long run when returns are predictable", *Journal of Finance*, 55 (1), 225-64.
- Benartzi S. and R. Thaler, 1995, Myopic loss aversion and the equity premium puzzle, *Quarterly Journal of Economics*, 110, 73-92.
- Bollerslev T. (1987), "Z conditional heteroskedastic time series model for speculative prices and rates of return", *Review of Economics and Statistics*, 69(3), 542-47.
- Boswijk H.P., C.H. Hommes and S. Manzan (2007), "Behavioral Heterogeneity in Stock Prices", *Journal of Economic Dynamic and Control*, 31(6), 1938-70.
- Brock W.A. and C.H. Hommes (1998), "Heterogeneous beliefs and routes to chaos in a simple asset pricing model", *Journal of Economic Dynamic and Control*, 22, 1235-74.
- Chen N.F., R. Roll and S. Ross S. (1986), "Economic forces and the stock market: testing the APT and alternative asset pricing theories", *The Journal of Business*, 59, 383-403.
- Cochrane J.H. (1999a), "New facts in finance", *Economic Perspectives*, Federal Bank of Chicago Review, Third Quarter, 23 (3), 36-58
- Cochrane J.H. (1999b), "Portfolio advice for a multifactor world", *Economic Perspectives*, Federal Bank of Chicago Review, Third Quarter, 23 (3), 59-78
- De Santis G. and B. Gerard. (1997), "International asset pricing and portfolio diversification with time varying risk", *Journal of Finance*, 52, 1881-1912.

Dimson E., P. Marsh and M. Staunton (2003), "Global Evidence on the risk premium", *Journal of Applied Corporate Finance*, 15(4), pp.27-38.

Ederington, I. and J. Lee (1993), "How markets process information: news releases and volatility", *Journal of Finance*, 48, 1161-1191

Elton E.J, M.J. Gruber and J. Mei (1994), "Cost of capital using Arbitrage Pricing Theory: a case study of nine New York Utilities", *Financial Markets, Institutions and Instruments*, 3, 46-73.

Fama E.F. and K.R. French (2002), "The equity premium", *Journal of Finance*, 57(2), 637-59.

Fernandez P. (2006), "Equity premium: historical, expected, required and implied", IESE Business School, University of Navarra, Working Paper n°661, December.

French K.R., G.W. Schwert and R.F. Stambaugh (1987), "Expected returns and volatility", *Journal of Financial Economics*, 19, 3-29.

Friedman M. and A.J. Schwartz (1982), "Monetary trends in the United States and the United Kingdom, Their relation to income, prices and interest rates, 1867-1975", NBER, University of Chicago Press, Chicago, 192.

Goyal A. and I. Welch (2003), "Predicting the equity premium with dividends ratios", *Management Science*, 49(5), 639-54.

Goyal A. and I. Welch (2007), "A Comprehensive look at the empirical performance of equity premium prediction", *Review of Financial Studies*, 2007.

Gollier C. and R. Zeckhauser (2002), "Time horizon and portfolio risk", *The Journal of Risk and Uncertainty*, 24 (3), 195-212.

Graham J.R. and C.R. Harvey (2001), "Expectations of equity risk premia, volatility and asymmetry from a corporate finance perspective", November 30, available at SSRN <http://ssrn.com/abstract=292623>.

Graham J.R. and C.R. Harvey (2003), "Expectations of equity premia, volatility and asymmetry", Working Paper, Duke University, USA.

Graham J.R. and C.R. Harvey (2005), "The long-run equity premium", September 9, available at SSRN <http://ssrn.com/abstract=795369>.

Graham J.R. and C.R. Harvey (2007), "The equity risk premium in January 2007: evidence from the global CFO outlook survey", January, available at SSRN <http://ssrn.com/abstract=959703>.

Hamilton J.D. (1994), *Time series analysis*, Princeton University Press, Princeton, New Jersey.

Hansen P. and A. Lunde (2005), "A forecast comparison of volatility models: does anything beat a Garch (1,1)?", *Journal of Applied Econometrics*, 20(7), 873-89.

Harris R.S. and F.C. Marston (2001), "The market risk premium: expectational estimates using analysts' forecasts", *Journal of Applied Finance*, 11(1), 6-15.

Harvey A.C. (1992), *Forecasting, structural time series models and the Kalman filter*, Cambridge University Press, Cambridge.

Ibbotson Associates (2006), *Stocks, Bonds, Bills, and Inflation*, Valuation Edition, Yearbook.

Kocherlakota N.R. (1996), "The equity premium: It's still a puzzle?", *Journal of Economic Literature*, 34, 42-71.

Koutmos G., J. Knif and C. Philipatos (2008), "Modeling common volatility characteristics and dynamic risk premia in european equity markets", *Quarterly Review of Economics and Finance*, 48, 567-578

Kryzanowski L., S. Lalancette and M.C. To (1997), "Performance attribution using an APT with pre-specified macro factors and time-varying risk premia and betas", *Journal of Financial and Quantitative Analysis*, 32(2), 205-24.

Lo A.W. and A.C. MacKinlay (1988), "Stock market prices do not follow random walks: evidence from a single specification test", *Review of Financial Studies*, 1, 41-66.

Mehra R. and E. Prescott (1985), "The equity premium: a puzzle", *Journal of Monetary Economics*, 15, 145-61.

Merton R.C. (1969), "Lifetime portfolio selection under uncertainty: the continuous-time case", *Review of Economics and Statistics*, 51(3), 247-57

Mpacko-Priso A (2001), *La prime de risque des actions*, Théories et Applications, Dianoïa, Paris.

Müller U., M. Dacorogna, R. Dave, R. Olsen O. Pictet and J. Von Weizsäcker (1997), "Volatilities of different time resolutions – analysing the dynamics of market components", *Journal of Empirical Finance*, 4, 213-39.

Prat G. (1982), « *La bourse et la conjoncture économique* », Economica, Paris.

Prat G. (2001), « Une analyse de la dynamique des primes de risque des actions suivant l'horizon de placement », *Revue d'Economie Politique*, 2, 291-329.

Prat G. and R.Uctum (2008), "The dynamics of ex-ante risk premia in the foreign exchange market: evidence from the Yen/USD exchange rate using survey data", *Documents de Travail EconomiX*, n°2.

Pye G. (1973), « Lifetime portfolio selection in continuous time for a multiplicative class of utility functions», *American Economic Review*, 63, 1013-16.

Samuelson P.A. (1969), "Lifetime portfolio selection by dynamic stochastic programming", *Review of Economics and Statistics*, 51(3), 239-46.

Shiller R. J. (2000), *Irrational exuberance*, Princeton University Press, Princeton New Jersey, 2000.

Siegel J.J. (2005), "Perspectives on the Equity Risk Premium", *Financial Analysts Journal*, 61(6), 61-71.

Siegel J.J. and R.H. Thaler (1997), "Anomalies: the equity premium puzzle", *Journal of Economic Perspectives*, 11 (winter), 191-200.

Shleifer, A. and L.H. Summers (1990), "The Noise Trader Approach to Finance", *Journal of Economic Perspectives*, 4, 19-33.

Subbotin A. (2009), *Multiple Investment Horizons and Stock Prices Dynamics*, PhD Thesis, university of Pris 1, December 2009. 138 p.

Welch I. (2000), "Views of financial Economists on the equity premium and on professional controversies", *Journal of Business*, 73(4), October, 501-37.

Welch I. (2001), "The equity premium consensus forecast revisited", Cowles Foundation discussion paper n°1325, available at <http://cowles.econ.yale.edu>.

Wilson J.W. and P.J. Jones (2002), "An analysis of the S&P Index and Cowles's Extensions: Price Indexes and stock returns 1870-1999", *Journal of Business*, 75(3), 505-33.

Table 1 – Short term and long term risk premia : descriptive statistics

	Mean Median	Maximum Minimum	Std deviation		Unit root ADF test
			Skewness	Kurtosis	
One year horizon premium	5.54 5.25	12.81 -6.73	4.10 -0.27 2.43	3.24 0.19	t-stat=-2.79
Infinite horizon premium	2.29 2.35	10.03 -3.88	2.16 0.36 4.01	8.17 0.02	t-stat=-2.74

Notes. The risk premia are expressed in percent per month. The sample period is 1881-2008 (128 observations). The asymptotic critical values for the ADF test statistic is -3.48, -2.88 and -2.58 at the 1% , 5% and 10% levels, respectively.

Table 2 : Estimating the one year and infinite time horizon equity risk premia model using the Kalman filter methodology

	one year time horizon ($\tau = 1$)	infinite time horizon ($\tau = 2$)
State equations (13) and (15)		
ρ_τ	0.93*** (21.7)	0.92*** (20.5)
	0.041*** (2.6)	–
b_τ	-9.91*** (-36.3)	-11.65*** (-39.0)
Signal equations (16) and (17)		
δ_τ	0.051** (3.0)	0.030*** (4.9)
$\alpha_{\tau 1}$	1.39*** (4.5)	1.41*** (4.7)
$\alpha_{\tau 2}$	1.40*** (4.4)	2.30*** (4.2)
$\alpha_{\tau 3}$	1.40*** (4.9)	2.80*** (4.8)
$\alpha_{\tau 4}$	0.98*** (3.4)	1.58*** (2.8)
$\alpha_{\tau 5}$	–	1.30** (2.3)
$\alpha_{\tau 6}$	–	0.93*** (2.8)
$\alpha_{\tau 7}$	–	0.58** (2.3)
$c_\tau^{'}$	-1.17*** (-10.4)	-3.86*** (-34.8)
<i>covar</i>	0.31** (1.98)	
R^2	0.925	0.942
R_D^2	0.676	0.881
Q	19.77	19.34
hH	117.01	54.66
AIC	8.56 9.02 8.74	

Notes. Estimations cover the period 1880-2008 (128 years). For $\tau=1, 2$, the two signal equations $\Phi_{\tau t} = sv_{\tau t} Var_\tau + \delta_\tau S_t Var_\tau + v_{t,\tau}$ (where the expected variances Var_τ are given by (10) and (11)) and the two state equations by $sv_{\tau t} = \rho_\tau sv_{\tau t-1} + \varepsilon_{\tau t}$ ($\tau = 1, 2$), have been estimated as a system of equations using the Kalman filter methodology. *covar* stands for the covariance between the two signal residuals. The estimates are obtained by setting to zero the insignificant covariance between the two auxiliary (state) residuals and the insignificant values of intercepts κ_τ . The variances of $\varepsilon_{t,\tau}$ and $v_{t,\tau}$ are estimated as $\exp(c_\tau)$ and $\exp(c_\tau^{'})$, respectively. AIC, SC and HQC stand for Akaike, Schwarz and Hannan and Quinn information criteria for the system estimation. The initial values of $sv_{\tau t}$ have been optimized as 0.01 and 0.05 according to the

minimum information criteria. Numbers in brackets are the t-values. ***, ** and * indicate that estimates are significant at the 1%, 5% or 10% levels, respectively. R^2 and R_D^2 are two goodness of fit measures (see footnote 21). Q and hH represent Ljung-Box serial correlation and heteroskedasticity test statistics (see Appendix 3 for a presentation of these statistics). For the Q -statistic, the asymptotic critical values for a χ^2 with 5 d.f. (one year horizon) and 1 d.f. (infinite horizon) are (9.24, 11.07, 15.09) and (2.71, 3.84, 6.61) for (10%, 5%, 1%) levels of significance, respectively. For the hH statistic, the asymptotic critical values for a χ^2 with 43 d.f. (both horizons) are 57.51, 61.66 and 69.96 for 10%, 5% and 1% levels of significance, respectively.

**Figure 1 - The two components of the short term equity risk premium
1881-2008**

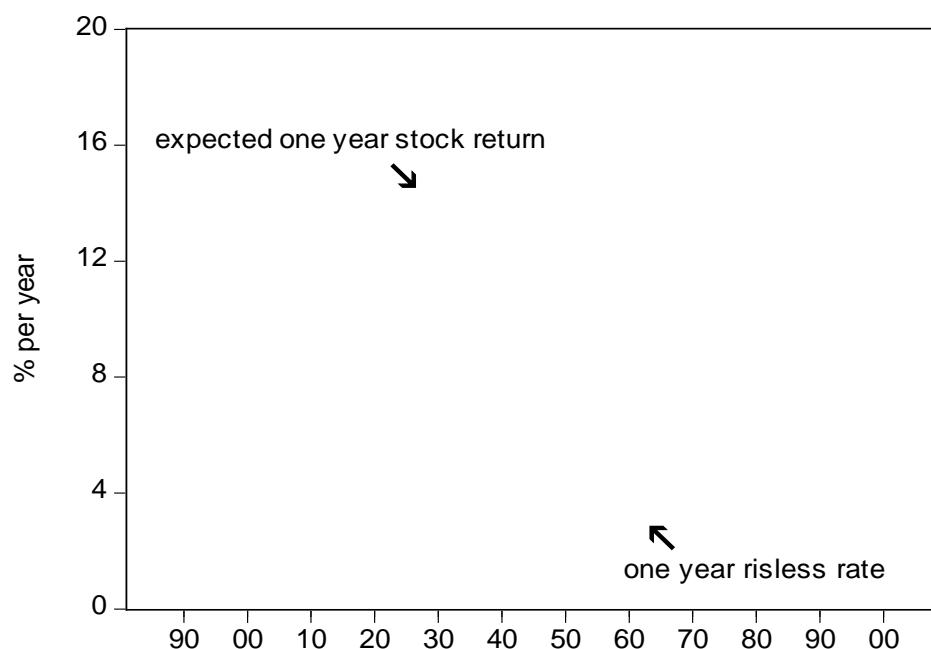
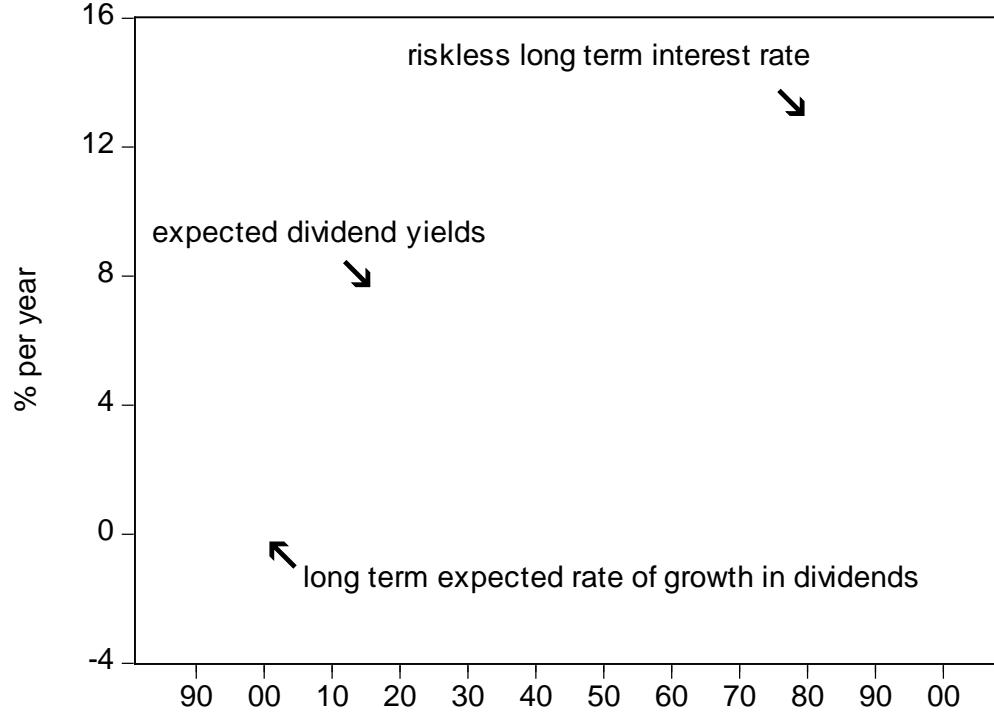
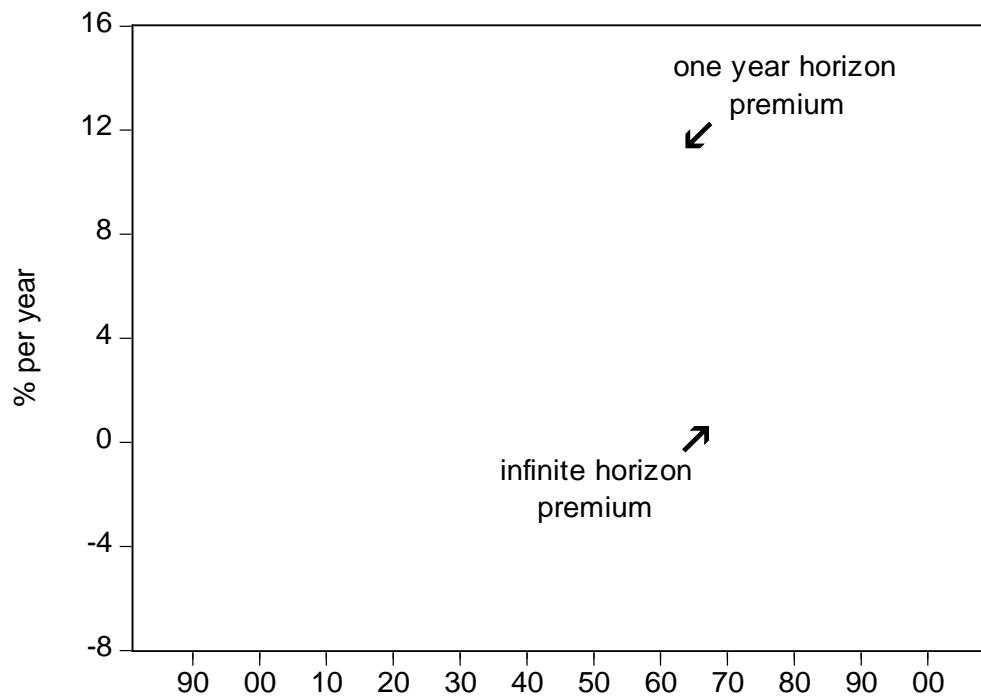


Figure 2 - The three components of the long term equity risk premium

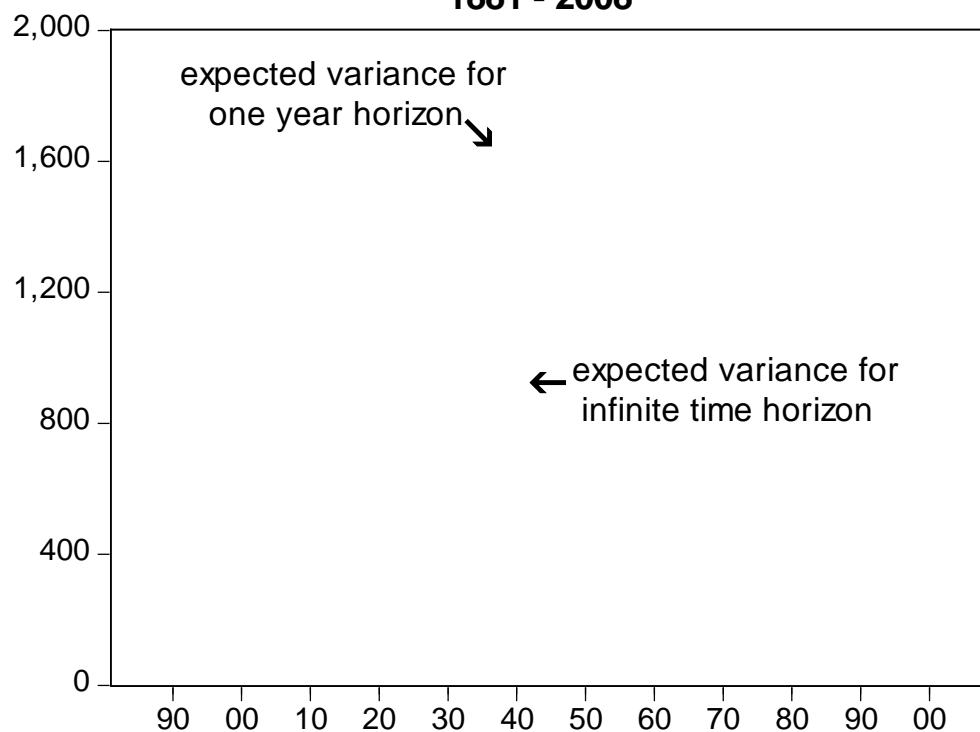
1881-2008



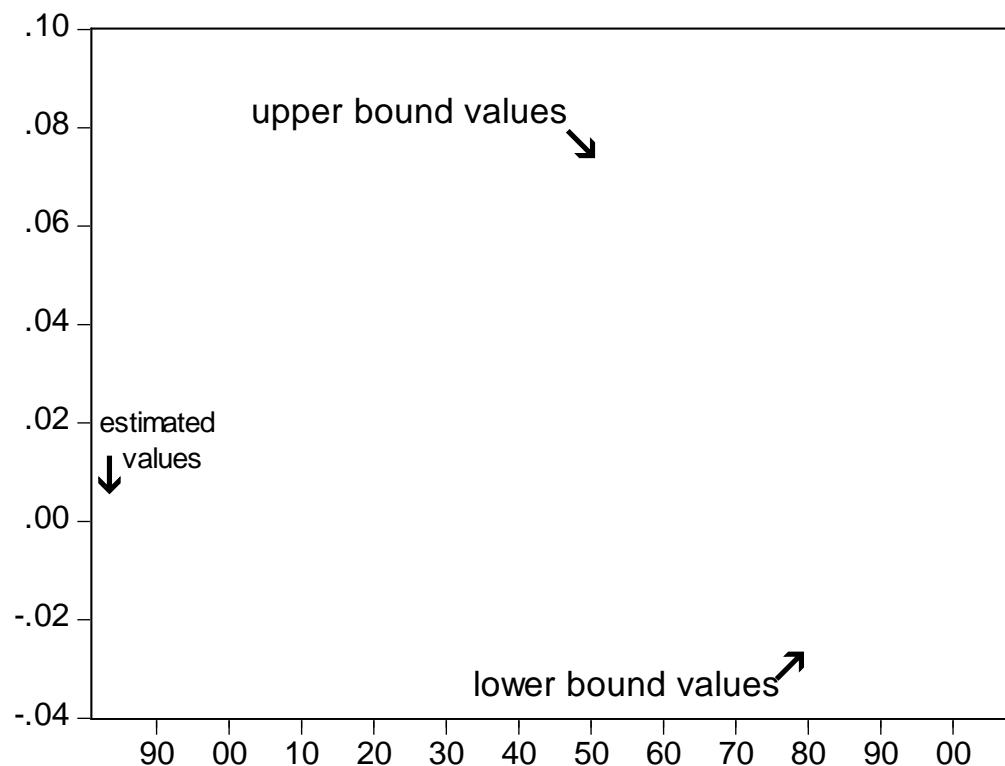
**Figure 3 - The one year horizon and the infinite horizon equity risk premia
1881-2008**



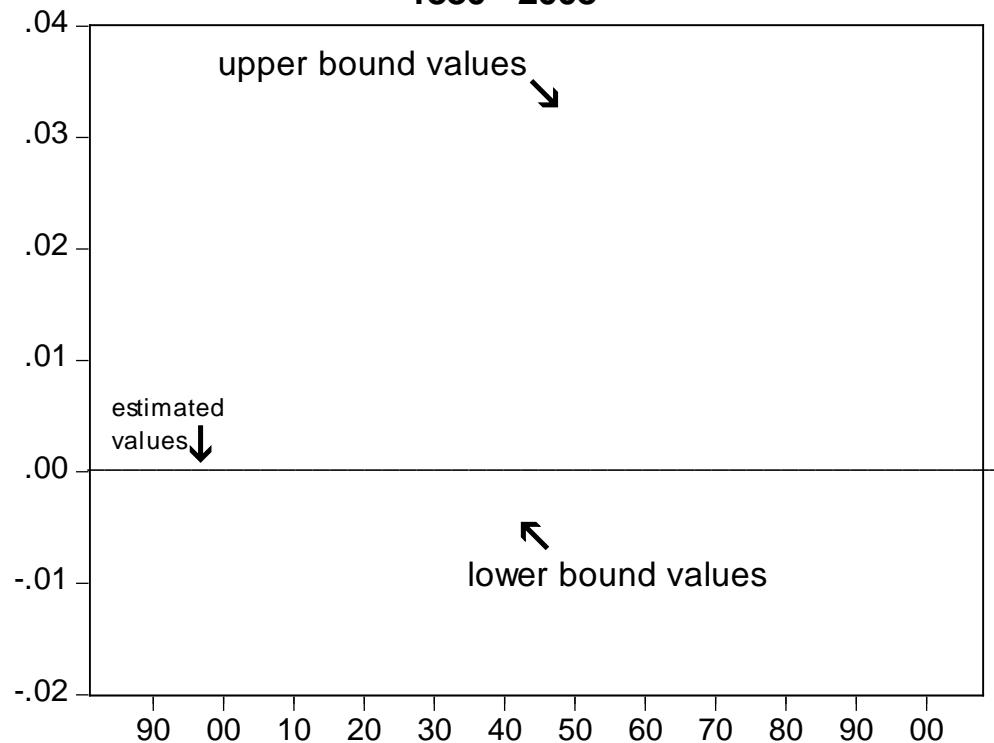
**Figure 4 - Short term and long term expected variances
of stock returns**
1881 - 2008



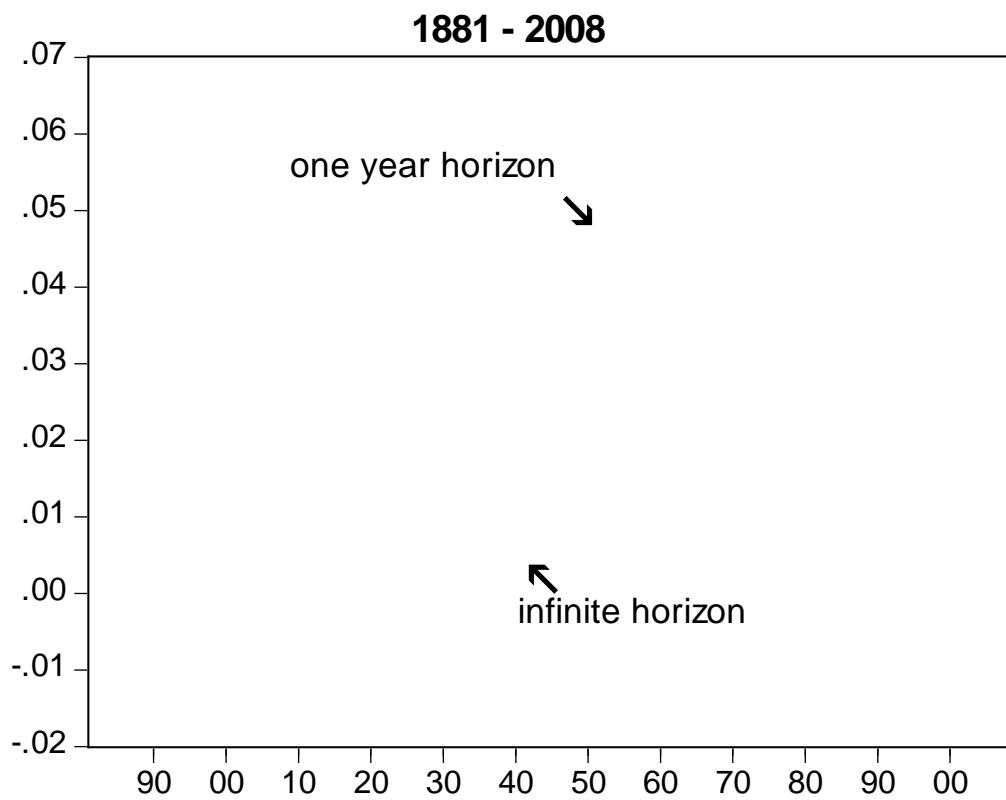
**Figure 5 - Price of risk for the one year horizon
1880 - 2008**



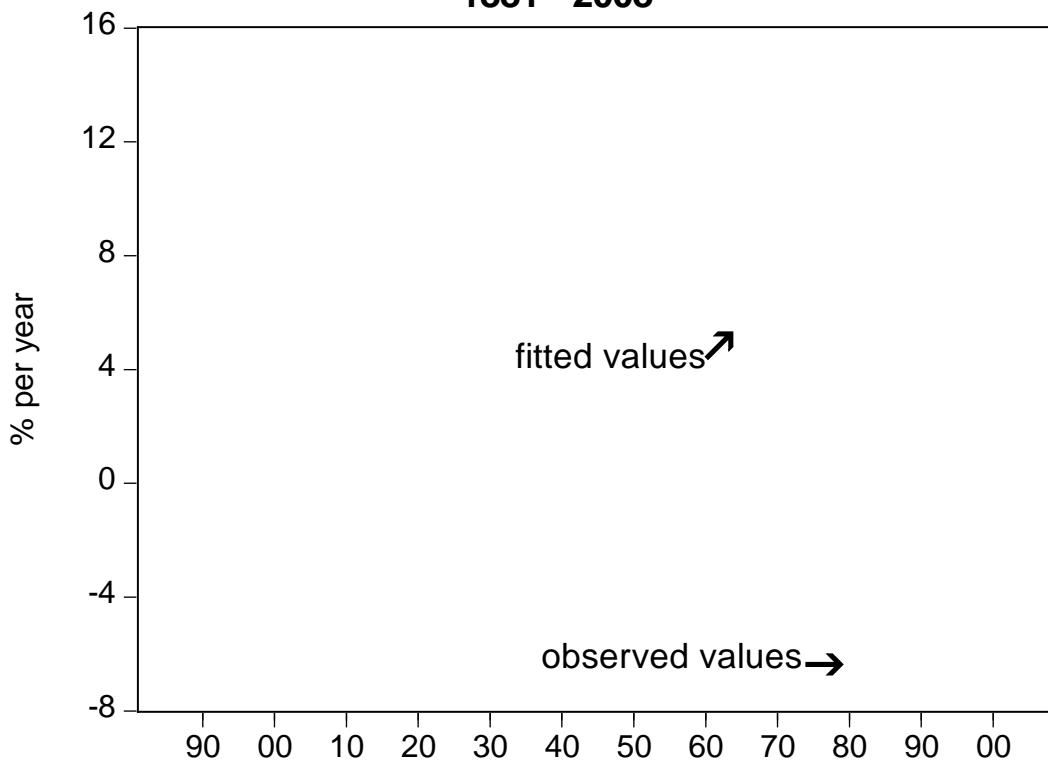
**Figure 6 - Price of risk for the infinite horizon
1880 - 2008**



**Figure 7 - The prices of risk for one year
and infinite time horizons**



**figure 8 - Observed and fitted values of
the one year equity risk premium
1881 - 2008**



**Figure 9 - Observed and fitted values of the infinite horizon equity risk premium
1881 - 2008**

